284. Note on the Problem: "Given a Parallelogram, Construct a Parallel to a Given Line through a Given Point"

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Proof. If n is a positive integer,

$$\frac{x^n - y^n}{x - y} = x^{n-1} + x^{n-2}y + \dots + y^{n-1}.$$

Hence, if x, y have the same sign,

$$\frac{x^n - y^n}{x - y} \text{ lies between } nx^{n-1} \text{ and } ny^{n-1}.$$

It readily follows that (*Gazette*, p. 321),

$$\frac{x^n - y^n}{x^{n-1} - y^{n-1}} \text{ lies between } \frac{n}{n-1}y \text{ and } \frac{n}{n-1}x.$$

Hence by multiplication it follows that (1) is true when p, q are any positive integers.

It then readily follows from (1) that

$$\frac{x^p - y^p}{x^{p-q} - y^{p-q}} \text{ lies between } \frac{p}{p-q}y^q \text{ and } \frac{p}{p-q}x^q. \dots\dots\dots(2)$$

From this it follows that (1) is true when one of the integers p, q is negative.

Using (2) again it then follows that (1) is true when the integers p, q are both negative.

The extension of (1) to fractional values of p, q follows at once by writing $x = X^m, y = Y^m$, where m is the L.C.M. of the denominators of p, q .

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284. [K. 21. a. a.] *Note on the problem: "Given a parallelogram, construct a parallel to a given line through a given point."*

This problem is a well-known one, and solutions of it based on principles of homology are given in Cremona's *Projective Geometry* and Mr. J. W. Russell's *Treatise on Elementary Geometry*. The construction can however be effected as an immediate deduction from the ruler construction for the fourth harmonic to three given lines. Thus let l be the given line, P the point, and $ABCD$ the parallelogram.

Let AC, BD , the diagonals of the given parallelogram meet in O .

Draw PQ the fourth harmonic to PA, PO, PC , meeting l in Q .

Then since $AO = OC$, PQ is parallel to AC .

Draw PS the fourth harmonic to PD, PO, PB , meeting l in S .

Then since $DO = OB$, PS is parallel to DB .

Similarly, draw QR the fourth harmonic to QD, QO, QB , meeting SR the fourth harmonic to SC, SO, SA , in R .

Then $PQRS$ is a parallelogram.

The required line is PL the fourth harmonic to PQ, PR, PS , for since PR bisects QS , PL is parallel to QS , i.e. to l .

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