ART. III.—*On the Intensity of Sound.*—II. *The Energy used by Organ Pipes*; by CHARLES K. WEAD.

[Read in abstract at the Philadelphia meeting of the American Association. 1884.]

IN a former paper* the case of a vibrating tuning fork has been considered as an important example of sounding bodies that gradually expend the store of energy originally imparted to them. We have now to consider one of the class that can store up little or no energy, viz: an organ pipe; and have therefore to determine, not the rate of loss as with the fork and piano string, but the rate at which energy is supplied to the system from without. The experimental problem is very simple, and it seems strange that it has not been completely worked out.

The literature of the subject is very slight. Lord Rayleigh,† in an oft-quoted experiment, measured the pressure and volume of air supplied to a whistle of 2740 d. v., and so found the rate of consumption of energy. Several years earlier Mr. Bosanquet in a very interesting and valuable paper‡ discussed the relative amount of energy supplied to the several pipes of an

* This Journal, xxvi, 177, Sept., 1883.

‡ Phil. Mag., xlv, 1872.

† Proc. Roy. Soc., xxvi, 248.

Open Diapason stop in an organ, but gave no absolute amount. He assumes as a matter of general knowledge that an organ builder furnishes a series of pipes of sensibly equal loudness (and quality) throughout the scale; he quotes what he calls Töpfer's law, that the consumption of wind by pipes belonging to the same stop varies directly as the length of the pipe, and confirms it approximately by experiments; and so he concludes that the amount of energy per second necessary to produce sounds of equal loudness under similar conditions varies inversely as the vibration-frequency. On the other hand M. Allard* makes the assumption that the energy per second needed to maintain a sound just audible at a given distance varies directly as the vibration-frequency, and finds a satisfactory confirmation of his views in the experiments on the range of fog-horns made by various lighthouse boards. But the condition of the observer will be very different in the two cases; so they are scarcely comparable.

The experiments now to be detailed and discussed are sufficiently numerous and exact to disprove this alleged law of Töpfer's, so far at least as one organ is a fair sample of all. They were performed on a Hook and Hastings No. 11 Organ in the Congregational Church of Ann Arbor, Mich.; this instrument has two manuals of 58 keys each from C to a''', the great organ having 9 stops—the seven to be named in table I, a 2 $\frac{2}{3}$ ' twelfth and a 3 rank mixture. The pressure of wind was very exactly 3 inches of water, and the total capacity of the bellows about 35 cubic feet; this quantity of wind would leak out in about 3 minutes.

The only method of experiment available, unless one has a very large gas-meter at his disposal, is to fill the bellows and determine the time needed for the whole or any definite part of its contents to leak out; then determine similarly the time when one or more pipes are sounding. For example, 12 cu. ft. (=A) of air are used; if this leaks out in 60 seconds the leakage is $A \div 60 = 20$ cu. ft. per sec.; if when a pipe is sounding the time is 24 sec., the flow is then $A \div 24 = 50$ cu. ft. per sec., and the pipe consumes the difference, that is 0.30 cu. ft. per second; if this is supplied under a pressure of 3 inches of water = 15.6 lbs. per sq. ft., the energy used by the pipe = $30 \times 15.6 = 47$ ft. lbs. per sec. In this way the computations have been made for the tables.

Mr. Bosanquet limited his work to observing the times, and finding the difference of their reciprocals, thus getting the desired relative values. He used a string pendulum, finding the time needed for the bellows to empty itself, the "feel" of the blowing lever indicating when the bellows is full or empty.

* *Comptes Rendus*, xcv, 1662.

But the numbers he gives, especially for leakage, show such wide variations as to throw great doubt on the accuracy of the method. Therefore two modifications were made: first, a stop-watch indicating eighths of a second was used; and second, the movement of the wind-indicator above the keyboard was observed through a space of 50^{mm}; to be sure only about one-third of the wind was used, but it is absolutely necessary to allow 10 to 20 seconds to elapse for the subsidence of the strong vibrations set up in the top of the bellows by the act of pumping. It was sometimes found that though no key was pressed the leakage was different according as the stop was drawn or closed, especially with one of the pedal stops.

One further modification of method was made: since the leakage is more than the amount of wind consumed by any single pipe, except a few of the largest, the influence of errors of observation was diminished by combining several pipes so that they might all sound at once; two ways of doing this were tried:

1. A single stop was drawn, and several consecutive white keys, usually eight, were held down by a loaded block; thus we find the relative consumption of wind by different stops, or by pipes of the same stop in different parts of the scale. See table I.

2. Several stops were drawn as in ordinary playing, and a single key held down by a wedge. In this case each pipe receives less wind than when no unison pipe is near, a fact long known and further established by these experiments; but we may still find the relative wind-supply in different parts of the scales. See table II and part of III.

Most of the results of the work can be given best in tabular form. In the tables the names of the stops need no explanation; where 9 stops were drawn they comprise all the stops drawn by the *forte* composition pedal, including the 7 named separately, a twelfth and a 3-rank mixture. The notation of the keys is used consistently always referring to the key, not the pitch, *c'* corresponding to middle *c* (=268 d. v.) when an 8' stop is drawn; it will therefore be seen that the absolute pitch of all the notes in the lower part of table I is the same; while in the next table pipes of five different lengths, besides the mixtures will respond to a single key. L of course means leakage. The time given is the mean of from 3 to 8 observations: these agreed so well that the probable error of the mean is very rarely 1 per cent: take two examples at random;

Table III C $22\frac{1}{4}$, $22\frac{6}{8}$, $22\frac{5}{8}$, $22\frac{5}{8}$; mean 22.66 sec.

C-c $6\frac{3}{8}$, $6\frac{3}{8}$, $6\frac{3}{8}$, $6\frac{3}{8}$, $6\frac{3}{8}$; mean 6.375 sec.

The following columns contain respectively $1 \div t$, and this quantity diminished by the leakage: this remainder represents

TABLE I.—ONE STOP DRAWN; SEVERAL KEYS DEPRESSED.

1. Stop Drawn.	2. No. Keys.	3. Highest. Lowest.	4. t.	5. l ÷ t.	6. V. C C. $3.4 \times 10^5 \times$	7. Ratio.	8. V'. $3.4 \times 10^5 \times$	9. (6-8) ÷ 8. per cent.	10. V''. $3.4 \times 10^5 \times$	11. (6-10) ÷ 10. per cent.	12. Energy. ergs. sec. $10^6 \times$
Leakage	0	---	sec. 56.2	.0178	---	---	---	---	---	---	---
8' Open Diapason	8	C-c	5.44	.1838	.1660	.569	.1635	+ 1.5	.1741	- 4.7	415
"	8	c-c'	8.73	.1145	.0967	.569	.1014	- 4.6	.1035	- 6.5	242
"	8	c'-c''	11.94	.0838	.0660	.683	.0628	+ 5.1	.0615	+ 7.3	165
"	8	c''-c'''	17.83	.0561	.0383	.580	.0389	- 1.6	.0366	+ 4.6	96
"	8	a''-a'''	21.04	.0475	.0297	---	---	---	---	---	74
8' Open Diapason	0	---	65.0	.0154	---	---	---	---	---	---	---
"	8	C-c	5.10	.1961	.1807	---	---	---	---	---	452
"	4	C-F	8.60	.1163	.1009	---	---	---	---	---	252
"	4	G-c	11.0	.0909	.0755	---	---	---	---	---	189
8' Trumpet	8	C-c	10.25	.0976	.0822	---	.0766	+ 7.	---	---	206
"	8	c-c'	16.00	.0625	.0471	.573	.0549	- 14.	---	---	118
"	8	c'-c''	18.21	.0549	.0395	.839	.0377	+ 5.	---	---	99
"	8	c''-c'''	23.71	.0421	.0267	.660	.0264	- 1.	---	---	67
2' Fifteenth	8	C-c	9.75	.1026	.0872	---	.0815	+ 7.	.0896	- 2.7	218
"	8	c-c'	16.16	.0619	.0465	.533	.0533	- 13.	.0533	- 13.	116
"	8	c'-c''	18.96	.0527	.0373	.802	.0365	+ 2.	.0317	+ 18.	93
16' Bourdon	8	c'-c''	12.54	.0797	.0643	---	---	---	---	---	161
"	8	c''-c'''	16.29	.0614	.0460	.71	---	---	---	---	115
16' Bourdon	8	c'-c''	16.29	.0614	.0460	---	---	---	---	---	115
8' Open Diapason	8	c'-c''	11.06	.0904	.0750	---	---	---	---	---	188
8' Melodia	8	"	14.50	.0690	.0536	---	---	---	---	---	134
8' Dulciana	8	"	17.83	.0561	.0407	---	---	---	---	---	102
8' Trumpet	8	"	18.06	.0554	.0400	---	---	---	---	---	100
4' Octave	8	c-c'	14.28	.0700	.0546	---	---	---	---	---	137
2' Fifteenth	8	C-c	9.75	.1026	.0872	---	---	---	---	---	218

that fraction of the total volume of wind which goes to the pipe: the total volume was found by measuring the bellows and the distance it fell while the indicator moved 50 mm.; this was 11.9 cu. ft. = 337000 cc. (say 3.4×10^5 cc.) with an uncertainty of 1 or 2 per cent on account of the folds: this uncertainty, however, does not affect the relative values given in the tables, for it is an uncertainty in our knowledge, not in the action of the bellows.

TABLE II.—NINE STOPS DRAWN; ONE KEY DEPRESSED.

July 21.

1. Key.	2. <i>t</i> .	3. $1 \div t$.	4. V.	5. Comp. V.	6. $4 \div 5$.	7. $6 \div 5$.	8. Energy.
	sec.		c. c. $3.4 \times 10^5 \times$	$3.4 \times 10^5 \times$	$3.4 \times 10^5 \times$	per cent.	ergs. sec. $10^6 \times$
L	67.37	.0148	-----	-----	-----	-----	-----
C	12.35	.0810	.0662	.0652	+ .0010.	+ 1.5	165
C #	12.94	.0773	.0625	.0625	0	0	156
D	13.32	.0751	.0603	.0598	+ 5	+ .8	151
D #	13.03	.0767	.0619	.0573	+ 46	+ 8.0	155
E	15.25	.0656	.0508	.0549	— 41	— 7.6	127
F	14.56	.0687	.0539	.0526	+ 13	+ 2.5	135
F #	15.69	.0637	.0489	.0504	— 15	— 3.0	122
G	16.00	.0625	.0477	.0483	— 6	— 1.3	119
G	16.79	.0596	.0448	.0463	— 15	— 3.2	112
A	17.08	.0586	.0438	.0443	— 5	— 1.1	110
A #	17.21	.0581	.0433	.0424	+ 9	+ 2.1	108
B	18.65	.0536	.0388	.0407	— 19	— 4.7	97
c	17.58	.0569	.0421	.0390	+ 31	+ 8.0	105
L	67.43		Mean			± 3.4	----

July 28.

1. Key.	2. <i>t</i> .	3. $1 \div t$.	4. V.	8. Energy.	1. Key.	2. <i>t</i> .	3. $1 \div t$.	4. V.	8. Energy.
			c. c. $3.4 \times 10^5 \times$	ergs. sec. $10^6 \times$				c. c. $3.4 \times 10^5 \times$	ergs. sec. $10^6 \times$
L	77.37	.0129	-----	-----	<i>g</i>	24.29	.0412	.0286	71
c	17.87	.0560	.0431	108	<i>g</i> #	24.37	.0410	.0285	71
c #	18.69	.0535	.0407	102	<i>a</i>	26.04	.0384	.0259	65
<i>d</i>	20.54	.0487	.0359	90	<i>a</i> #	22.83	.0438	.0313	78
<i>d</i> #	21.21	.0471	.0344	86	<i>b</i>	27.33	.0366	.0242	60
<i>e</i>	23.25	.0430	.0303	76	<i>c'</i>	26.79	.0373	.0249	62
<i>f</i>	24.21	.0413	.0286	71	L	80.81	.0124	----	----
<i>f</i> #	24.10	.0415	.0289	72					

The energy is found by multiplying the volume by the pressure, which was 15.6 lbs. per sq. ft. or 7.6 gms. per sq. cm.

∴ Total energy = $11.9 \times 15.6 = 186$ ft. lbs. = 2.5×10^9 ergs.

By this multiply the fractions in columns headed V, as col. 6 table I.

The numbers in the column ratio (except in table IV) are found by dividing the number against which they are placed by the preceding one; if Töpfer's law were true these ratios would be for the octave everywhere $\cdot 500$.

TABLE III.—MISCELLANEOUS.

1. Stops drawn.	2. Key.	3. <i>t</i> .	4. $1 \div t$.	5. V.	6. Ratio.	7. Energy.
		sec.		c. c. $3 \cdot 4 \times 10^5 \times$		ergs. sec. $10^6 \times$
16' Bourdon -----	L	69·44	·0144		----	
	C	22·66	·0441	·0297	----	74
	D	28·59	·0350	·0206	----	52
	E	33·22	·0301	·0157	----	39
	F	33·50	·0299	·0155	----	39
	C	35·59	·0281	·0137	----	34
	A	38·04	·0263	·0119	----	30
	B	39·58	·0253	·0109	----	27
	C	32·37	·0309	·0165	----	41
	C-c	6·375	·1569	·1425	----	356
8' Open Diapason, Melodia, Dulciana }	L	69·62	·0144		----	
	C	18·21	·0549	·0405	----	101
	<i>c</i>	26·62	·0376	·0232	·572	58
	<i>c'</i>	35·25	·0284	·0140	·603	35
	<i>c''</i>	43·69	·0229	·0085	·608	21
9 stops -----	L	70·6	·0142		----	
	C	12·37	·0808	·0667	----	167
	<i>c</i>	17·96	·0557	·0415	·632	104
	<i>c'</i>	26·00	·0385	·0243	·586	61
	<i>c''</i>	34·25	·0292	·0150	·619	38
9 stops -----	<i>c'''</i>	44·62	·0224	·0083	·550	21
	L	80·62	·0124		For 17th.	----
	C	12·42	·0805	·0681	----	170
	<i>e</i>	23·25	·0430	·0306	·449	77
	<i>g' #</i>	37·46	·0267	·0143	·467	36
	<i>c'''</i>	47·92	·0209	·0085	·592	21

In several cases the results have been discussed mathematically and thus a computed value of V' is found (T. I, col. 8; T. II, col. 5): in these cases an exponential formula was assumed similar to that for the vibration-frequency of the tones in a tempered scale, and the logarithmic formulæ derived from it were combined by the method of least squares; thus

$$y r^n = V$$

$$\log y + n \log r - \log V = 0$$

Whence $(n+1) \log y + \Sigma(n) \log r - \Sigma(\log V) = 0$

$$\Sigma(n) \log y + \Sigma(n^2) \log r - \Sigma(n \log V) = 0.$$

The values of r thus found are collected in table IV. The difference between the computed and observed values of V is divided by the former and the quotient, as a per cent, is placed

in a following column. To obtain V'' in cols. 10 and 11, table I, the ratio was assumed as $\sqrt[4]{\frac{1}{8}}$.

Observations of the same quantity on different days agree to within a few per cent (e. g. key C with 9 stops, .0662, .0667, .0681) but since they differ more than the probable error of a single day's observations the results in the different tables should not be combined if accurate relative values are desired, nor should results in the same table be combined unless they are based on the same value of L. The data for table I were obtained in April and May, 1883, and are not quite as accurate as the data obtained in July, 1884, for the later tables.

Conclusions.—The results of experiments with *different stops* are shown in table I. It is very clear from them that no exact or important conclusions can be drawn from the loudness of the sound as to the relative quantity of wind required to blow pipes of different construction: thus, the soft Dulciana takes more than half as much wind as the comparatively loud Open Diapason ($102 \div 188$). Again, the Trumpet stop in this organ is voiced very loud, yet its pipes require absolutely less energy than any others that sound the same note: this is a conclusive proof that a reed-pipe has a much higher efficiency as a wave-producing mechanism than a flue pipe.

The results on *different pipes* of the same stop or of the same combination of stops are shown in all the tables; in table I for the eight notes of an octave taken together in various parts of the scale, a single stop being drawn; in table II for each of the twenty-five notes in a range of two octaves, nine stops being drawn; in table III for various combinations of stops. Some of the conclusions from these are very clear, and some curious. We must assume with Mr. Bosanquet that a set of pipes gives us a series of sounds of the same quality and of nearly the same loudness as judged by the ear of an expert, and also assume that all pipes of the same stop are equally efficient sound-producers. Now if we recall Töpfer's law, that the consumption of wind varies inversely as the length of the pipe, we should expect to find for the octave approximately the ratio .500, or a little less, since the higher pipes are relatively larger than the lower ones and so must be relatively shorter. But not a single ratio can be found in the tables to confirm this view; everywhere the ratio is considerably greater than .5. The tables give the values in a dozen cases (not including the Trumpet stop) and from table II a dozen more values can readily be found.

To some of the observations it seemed worth while to apply the method of least squares as already said; the several ratios found for the octave are given in table IV.

If a smooth exponential curve be drawn with these ratios for the experiments of table I, where eight keys were depressed at once, it will be found to fall below the experimental curve in the first and third octaves, and above it in the second and fourth octaves in every case examined; the magnitude of the difference is shown in column 9; but this alternating deviation is not great, and is probably not of importance; it does not appear in table III, where several stops are combined.

TABLE IV.

Table.	Stop drawn.	No. keys.	Range, Octaves.	Number equations.	Ratio.	Av. Difference per cent.
I	Trumpet.	8	4	4	·701	± 7
I	Open Diapason.	8	4	4	·620	± 3·2
I	Fifteenth.	8	3	2	·654	± 10
I	Bourdon.	8	2	--	·71	--
III	Three stops.	1	3	4	·595	± 1·4
III	Nine stops.	1	4	5	·595	± 2·4
III	"	1	4	4	·591	± 7
II	"	1	1	13	·5975	± 3·4

The latter half of table IV shows that when the stops were combined as in ordinary playing, but a single key being pressed, there is a remarkable constancy in the value of the ratio for the octave however it is determined, and its value for the Open Diapason differs little from these latter values. This constancy demands an explanation. According to Töpfer's law we should have $\cdot50 = \sqrt[3]{\frac{1}{8}}$; we do have very nearly $\sqrt[4]{\frac{1}{8}} = \cdot5946 = 1 \div 1\cdot682$. This I believe to be an excellent illustration of the unconscious recognition by the artist of the physical or mathematical laws underlying his art. At present we cannot explain the law, any more than the laws of the scale could be explained before the subject of harmonic overtones was understood; we can only correlate this with the following fact relating to organ-pipes—to their diameter, or "scale" as organ-builders call it. It is a matter of experience that to produce the proper loudness of sound it is necessary to increase the ratio of the diameter to the length as the pipe becomes shorter, so when the pitch rises an octave and the theoretical length becomes one-half that of the fundamental the diameter is greater than half that of the fundamental; usually we must go to the seventeenth pipe, as from C to c, to find the one of half the diameter. This is equivalent to saying that in rising 4 octaves the theoretical length becomes $(\frac{1}{2})^4$, but the diameter $(\frac{1}{2})^3$; if we assume an exponential series all the way up the ratio of diameters of pipes an octave apart is therefore $(\frac{1}{2})^{\frac{3}{2}}$ or $\sqrt[4]{\frac{1}{8}}$,—the ratio already found; the corresponding ratio for the semitone

is of course $(\frac{1}{2})^{\frac{1}{16}} = \cdot 9576$. This ratio for the diameters is only a mathematical expression of a mechanical *fact*, there is no theory about it. Such a "scale" gives convenient rules in practice for laying out the pipes, and satisfies the ear, or it would not have found such general adoption. In this organ Open Diapason c' has an internal diameter of 57 min., the e'' of $29\frac{1}{2}$; the Dulciana c' of 31, the e'' of $15\frac{1}{2}$ *. It is not for a moment to be assumed that the amount of wind required is directly determined by the diameter of the pipe; for the organ builder would point out that the shape of the mouth is an important factor, and that the voicer or finisher varies the amount of wind by plugging the holes through the feet of wood pipes, cutting out or closing the feet of metal pipes, varying the width of the slit for the wind, etc., till his ear is satisfied with the loudness and quality of the sound. But in the light of these experiments we must conclude that for similar pipes the volume of air used per second, and therefore the energy expended per second, varies as the $\frac{2}{3}$ -power of the wave-length of the note, or inversely as the $\frac{3}{2}$ -power of the vibration-ratio; and further conclude that the voicer unconsciously strives to secure this ratio just as the tuner unconsciously strives to get the familiar vibration-ratios in the tuning of any instrument. It is to be remembered that we cannot recognize small differences of intensity with much accuracy. Volkman could always detect a difference of 25 per cent; Renz & Wolf† one of 28 per cent; the latter experimenters

*In Clarke's little book on "The Pipe Organ" a simple construction is given for finding the diameters of intermediate pipes when the diameters are given for two pipes 16, 8, 4, &c. semitones apart. At the ends of any convenient base line AB erect perpendiculars AC, BD proportional to the given diameters and join the ends C, D: Draw the two diagonals of the trapezoid thus formed and erect through their point of intersection a perpendicular to the base line. The part of this perpendicular between AB and CD is proportional to the diameter of the pipe midway between the given extremes. By continuing the construction the diameters of the other pipes will be obtained.

A little calculation shows that this gives a harmonic series, and if the first diameter be 2, and the seventeenth 1, the series is 32 ÷ 16, 17, 18 . . . 31, 32. All of the intermediate quotients are slightly less than the numbers derived from the exponential series whose ratio is the 16th root of $\frac{1}{2}$, the value for the 8ve being $\frac{1}{2}^{\frac{1}{16}} = \cdot 571$ instead of $\cdot 5946$. The maximum difference is about 5 per cent—a quantity entirely negligible to ordinary ears.

If a series of pipes were made on this harmonic scale and the quantities of wind could be accurately adjusted in the ratio of the diameters, an exponential curve deduced from experiments on them would show an "alternating deviation" similar to that referred to above. The sign of the deviation in a given 8ve would depend on where the starting point of the harmonic scale was taken.

The sum of 8 terms of the harmonic series corresponding to the key of C, the lowest term being 1, is 5.95: of the same terms of the exponential series 6.19; of 13 terms in the exponential series 9.4. Therefore to find the amount of wind (or of energy) used by the lowest pipe of any group of eight in the tables divide by 6 the amount given for the group.

† Pogg. Ann., xcvi, 595, 1856.

were correct in their judgments about the loudness of sound of a watch when held at different distances in only 55 per cent of their trials if the ratio of loudness in the two cases (computed from the law of inverse squares) was 100:92. In the light of such experiments the numbers headed per cent of difference in our tables are strikingly small in nearly every case—partly of course because of the method of averages we have followed, a number of pipes sounding at once in most cases.

It is interesting to compare the energy used here with that of a tuning fork. From table IV of my former paper (p. 186) it appears that the maximum energy I could give by bowing to König's forks of the middle octave mounted on their cases was considerably less than 0.5×10^6 ergs.; and the maximum rate at which energy was lost was about 0.1×10^6 ergs. per sec. But the Open Diapason pipes of this range ($c' - c''$) used each from 18 to 30×10^6 ergs. per sec., some 250 times as much as the fork giving its maximum sound, or from 1,000 to 6,000 times as much as the fork when giving an ordinary sound. About one-millionth of one horse-power would maintain in ordinary vibration one of these forks; and a tenth of this amount gave a sound loud enough to be heard 200 feet in the open air.

There remains one question of some interest: Do all parts of the scale *seem to the ear* to be of equal loudness, especially the scale of an organ for which we have found the relative intensity of vibration. I find few musicians who have any definite impression on the subject; the question is certainly difficult; and is perhaps indefinite. If, for example, we call that sound the louder which can be heard at the greater distance and then compute the energy passing through the unit of surface at the limit of hearing, we make the violent assumption that the efficiency of the two sound-producers is the same. If we place the two bodies at the same distance Mayer* has shown that the sensation of one sound may be obliterated by a lower one that could not be heard as far off as the first. And there are other physiological difficulties. In fact the problem before us is analogous to the long-standing one of the comparison of two lights of different colors. The problem must, therefore, be left as insoluble with our present knowledge; but two statements of musicians are of interest in this connection. One organist points out that if a piece of music is played on a two or three manual organ, the left hand on the swell keyboard, while the melody is played by the right hand on the great, and the swell-boards are opened to give a proper balance

* Phil. Mag., ii, 500.

of tone, the boards will be found to be too widely opened when the left hand plays the melody on the great key-board and the right on the swell, the stops remaining unchanged. Another points out that if, with a single stop or combination of stops, one runs over the key-board ascending the effect is of a *crescendo*. This would indicate that the organ-builder intentionally makes the higher pipes louder instead of keeping them of equal loudness as assumed previously, and also shows that the ear is more sensitive to high notes than to low ones under the conditions in which music is heard, whatever the case may be with foghorns heard at sea and reported by Allard.

Some time after the preceding experiments had been discussed and reported on Mr. F. H. Hastings kindly furnished the writer with a copy of Töpfer's great work,* and a summary of his views may fitly be connected with this paper. Through many hundred pages the author discusses the theory of organ pipes and gives formulæ for their dimensions, and for the quantity of wind they require. He determined this last experimentally by the method already described, using a bellows of 63 cubic feet capacity; 9 min. 57 sec. were required for this volume of air to leak out under a pressure of 3.2 Weimar inches of water ($= 76^{\text{mm}}$), (II, 95). The experiments must have been very tedious with so large a bellows; they are open to the criticism that the leakage is greater than the wind-consumption of any pipe, except a few of the largest ones; so errors of observation make large errors in the final result, as previously pointed out.

The author's theory on the subject is curious. He says (II, 65) pipes of equal length consume volumes of wind proportioned to the squares of their diameter, and those of the same diameter quantities inversely proportional to the square root of their length, [or directly proportional to the square root of the vibration frequency].

$$\text{Therefore} \quad Q = K \frac{D}{\sqrt{L}}$$

$Q\sqrt{L} \div D^2 = K' =$ coefficient which measures "intensity of vibration."

$Q\sqrt{L} \div \text{area of mouth} = K'' =$ coefficient which measures "sharpness of tone."

Q is expressed in Weimar cu. in. per sec.

K' is found to range from 68 to 110, average 85 :

K'' from 394 to 536, average 450.

* J. G. Töpfer: *Lehrbuch der Orgelbaukunst*. Weimar, 1855. About 1,800 pages and 130 plates folio. Mr. Hastings calls it "by far the most complete book on organ building."

For all the pipes of any given stop K' and K'' should remain constant (pp. 99, 112). This assumption underlies his elaborate table of "normal scales." But his experiments do not seem to establish this constancy. Thus, for the 16 ft. Principal:

	Length.	Q.	K' .	K'' .
e'	20"	61.5	91.7	496
e°	40"3'''	99	72.5	408
E°	82.6	177	58	352
E'	100.4	162	25.6	165 tone dull.

In other cases the values of K' and K'' vary considerably without showing any regular increase.

The second constant, K'' , appears to be in some sense a measure of the quality of the note, the note being duller as K'' is smaller. For pipes of the same length obviously K' is proportioned to the mass of air used per unit section of the pipe, and so to the energy of vibration at any point *within* the pipe, if we make the violent assumption of equal efficiency for pipes of all diameters. In the same way K'' is proportioned to the energy of vibration at the mouth. But we are not concerned with the intensity of vibration *in* the pipe; we want the external effect due to the total cross-section.

The introduction of the square root of the length has no physical meaning or justification that I can discover; but it is needed to make all parts of Töpfer's theory hang together. This may be shown as follows: Assuming equal temperament and that diameters double at the 17th pipe, and putting a for the diameter of any pipe, the diameter of the n th pipe above becomes:

$$D = a \left(\frac{1}{2}\right)^{\frac{n}{16}}$$

Similarly for length

$$L = b \left(\frac{1}{2}\right)^{\frac{n}{12}}$$

And for quantities of wind on Töpfer's assumption

$$Q = c \left(\frac{1}{2}\right)^{\frac{n}{12}}$$

$$\therefore Q \div D^2 = \left(\frac{1}{2}\right)^{-\frac{n}{24}} c/a^2. \quad Q L^{\frac{1}{2}} \div D^2 = c b^{\frac{1}{2}}/a^2 = K'.$$

Evidently it is necessary to introduce $L^{\frac{1}{2}}$ to obtain a constant factor.

Töpfer then goes on to establish a "scale" or series of diameters for a set of pipes. He has found in tables published by Dom Bedos in 1766, on whose work his own treatise is largely based, that the ratio of sections of pipes differing an 8ve in pitch ranges between 1:4 and 1:2; experience shows that these are extreme; so it is safe to take their mean 1:√8.

Again, it is found he tells us (p. 153-4) that in practice the quantities of wind used are nearly as 1:2 for the 8ve, sometimes less, and fortifies himself by a quotation from Chladni (*Akustik*, p. 233): that if two tones of different pitch are to have equal effect the forces which each vibration exerts must be inversely as its vibration-frequency; but this force is proportional to the mass of air used; therefore Q varies inversely as n .

By the proceeding formula

$$Q = K'D^2L^{-\frac{1}{2}}; \quad q = K'd^2l^{-\frac{1}{2}}$$

If the pipes are an 8ve apart $L = 2l$ and $Q = 2q$:

$$\text{Then} \quad \frac{Q}{q} = \frac{D^2}{d^2} \sqrt{\frac{l}{L}} = 2 = \frac{D^2}{d^2} \sqrt{\frac{1}{2}}$$

$$\therefore D^2 = d^2 \sqrt{8} = 2.83d^2; \quad D = d \sqrt[4]{8} = d \times 2^{\frac{3}{4}}$$

This proof is clearly very unsatisfactory; but the "scale" thus determined, and published by Töpfer in 1832, has been largely used by organ builders. By it pipes 4 8ves, 48 semitones apart, have diameters in the ratio of 1:8, or pipes 16 semitones apart, a major 10th, are in the ratio of 1:2.

Another scale may be had by letting the 16th pipe (15 semitones) have the double diameter; the ratio for the 8ve is then 1:3, or more accurately $1:\sqrt[5]{16} = 1:3.032$. But the bass pipes have too little wind.

If, on the other hand, the 18th pipe (17 semitones) have the double (or half) diameter, the ratio is $1:4^{\frac{1}{17}}$ or 1:2.661; the higher pipes are relatively "sharper." This defect may be corrected by cutting their mouths lower, and conversely for the low pipes, remembering that for "gleiche Klangstärke" the quantity of wind and therefore the area of mouth must be in the ratio of $1:\sqrt{8}$ for the 8ve (p. 244). If, in the last case, the ratio of height of mouth to breadth be for c' 0.25, it will be for c 0.23, for C 0.41.

Another scale might be formed doubling the diameter at the 19th pipe; the same correction is to be made but its execution is doubtful. A uniform quality is the first condition in a stop (p. 295).

The author then goes on to apply his theories to the laying down of several "normal scales;" these all have 121 pipes, 60 each way from No. 61 assumed $27'''$ (53^{mm}) diameter. In these tables we find, for example, with the ratio of sections:

$1:\sqrt{8} = 1:2.83$	diam. No. 1, $363.2'''$	No. 61, $27'''$	No. 121, $2'''$
$1:8/3 = 1:2.67$	311.8	27	2.3
$1:5/2 = 1:2.5$	272.1	27	2.7

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From these 121 theoretical diameters for each scale a consecutive series to be chosen for each stop. He finds in practice that for a large number of stops the first ratio is suitable; for an ordinary Principal c' should have the diameter of No. 62; for a Wide Principal, No. 58 is suitable. But for many stops this ratio is not satisfactory; the tone must be fuller in the upper parts; for a given list of stops the ratio $1:2.67$ is better; the diameter of c' ranges between No. 55 for the Wide Principal Bass, and No. 77 for the Viola d'Amour. For the Pedal stops and some others he advises the ratio $1:2.5$.

This is a very brief summary of the portions of Töpfer's voluminous work that relate to the physical side of organ building. It does not appear that the questions involved have been thought out from the standpoint of the physicist, or that the author's views are entirely consistent. Mr. Bosanquet credits to Töpfer the law that the quantities of wind belonging to the same stop vary as the lengths of the pipes. I have not been able to find any statement to this effect more definite than the one already quoted from p. 153-4, that the quantities of wind used are nearly as $1:2$ for the 8ve, sometimes more, sometimes less; but on p. 200 he gives the theoretical quantities for the successive C's through 9 octaves, and the ratio for the 8ve I find to be $1:1.99$. But on the other hand he states positively, as already quoted, that the quantity of wind must be as $1:\sqrt{8}$ for the 8ve; that is, as the sections of the pipes in his first "normal scale." In other words, his most definite statement makes the quantities for the 8ve as $1:\sqrt{8}$; his alleged law and Mr. Bosanquet's experiments give the ratio $1:\sqrt[3]{8}$, and my experiments give the ratio $1:\sqrt[4]{8}$; these ratios are as $1:\sqrt[4]{64}$, $\sqrt[4]{16}$, $\sqrt[4]{8}$. While it is unsafe to dogmatize on a matter that must vary according to the circumstances of the case, the character of the stop, the location of the pipes, the size of the hall where they are to sound, the judgment of the finisher, etc., etc., I have no hesitation in expressing my belief that the last ratio, or one still nearer unity, will usually be found nearer the truth than either of the others.

Feb., 1891.