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# XLII. Notes on quaternions

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the magnet, the plates of soft iron become the seat of sufficiently intense sounds for a great number of persons to be able to hear them simultaneously.

176. Finally, I tried to induce at the same time electricity of tension and magnetism in the steel and iron discs. For this purpose, the bar which supports them was placed in the centre of a glass tube $0^{m}\cdot02$ in diameter, to which was fixed a horizontal circle of wood, $0^{m}\cdot18$ in diameter by $0^{m}\cdot018$ in thickness. This circle, entirely covered with tinfoil, communicates with a good electric machine. Parallel to the surface which is made to vibrate, it is thus brought as near as possible to it without the spark being emitted to it. The acoustic properties of the three discs remained indifferent to this new action. The plate of tempered steel had acquired a permanent magnetism which did not at all interfere with its musical properties.

177. It results from these experiments, that electric or magnetic induction has no *appreciable* action on the elasticity of different sonorous bodies, such as glass, copper, brass, soft iron, and steel tempered or untempered. The number of vibrations executed by them in the unity of time remains the same. But this conclusion must probably not be accepted in too absolute a manner. It might be that extremely energetic and very durable causes of induction determine an action which, in my experiments, has been too weak to be observed*.

Geneva, April 15, 1848.

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## XLII. *Notes on Quaternions.* *By* George Boole, *Esq.*†

### *Interpretation of Quaternions.*

MR. CAYLEY'S ingenious researches, published in the last Number of the Philosophical Magazine, have recalled to my mind some speculations of my own upon the same subject. To the purely mathematical treatment of it I have indeed little to add. What I shall say will rather have reference to its philosophy.

It were much to be desired that the general principles which govern the use of signs, as instruments of reasoning, were re-

* M. G. Wertheim has found that no modification of elasticity is perceptible in an iron or steel wire occupying the centre of an electro-magnetic bobbin, when the current has only traversed it for a short time. According to that ingenious experimentalist, the magnetization does not act directly upon the elasticity, but produces a new molecular arrangement.—*Annales de Chimie et de Physique*, December 1844, vol. xii. p. 623.

† Communicated by the Author.

duced to a consistent theory; for there undoubtedly exists a theory of signs applicable as well to the signs of common discourse as to the signs of mathematics. Without attempting to exhibit any complete doctrine on the subject, I will venture to state one or two views which I have been led to form, and apply them to the subject of quaternions.

Signs employed as instruments of reasoning may, in one point of view, be considered as the representatives of operations. This is not indeed the interpretation that we *necessarily* attach to them, but it is one which it is probable that in all cases we *may* attach, and from which their laws may in all cases be deduced. If we employ a particular symbol or combination of symbols A to represent a given operation, and another symbol or combination of symbols B to represent another given operation of the same kind, then, according to the Arabic order of writing, AB will represent the successive performance of the operations denoted by B and A. In order that such signs may be really available as instruments of deduction, it is necessary that the laws of the symbols entering into B and A should be such, that the expression AB developed according to those laws may, *in agreement with the conventions* established for the interpretation of A and B, represent an operation, the effect of which is equivalent to the combined effects of the operation A and B performed in the order (proceeding from right to left) AB.

In strict accordance with this principle, we may assign an interpretation to a quaternion $w+ix+jy+kz$, *subject to the condition*

$$w^2+x^2+y^2+z^2=1 \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

For if we write

$$w=\cos\frac{\theta}{2} \quad x=\sin\frac{\theta}{2}\cos\phi \quad y=\sin\frac{\theta}{2}\cos\psi \quad z=\sin\frac{\theta}{2}\cos\chi,$$

and assume the quaternion $w+ix+jy+kz$ to represent a rotation through an angle $\theta$ round an axis whose direction cosines are $\phi$, $\psi$, $\chi$, then representing this quaternion by $\Lambda$ and any other quaternion subject to a similar condition by $\Lambda'$, we shall have

$$\Lambda\Lambda'=\Lambda'',$$

$\Lambda''$ being a quaternion which, according to the same conventions, will represent a single rotation equivalent in effect to the two rotations represented by $\Lambda'$ and $\Lambda$ performed in the given succession.

A quaternion which does not satisfy the condition (1.) cannot be directly interpreted in geometry. Such expressions may nevertheless be employed, with the understanding that certain factors shall be rejected or other reductions performed, and thus they may lead to correct results. But the employment of such a form of the process seems to involve a departure from the principle, that the laws of the sign shall constitute in every respect an exact counterpart to the laws of the thing signified.

Sir William Hamilton's theory of the application of quaternions appears to be based upon the relations which their elements bear to the angles and angular points of spherical polygons; and similar to this is the basis which Prof. Graves has adopted for the not less interesting theory of triplets and multiplets. In all these systems we meet with such theorems as the following, viz. *that the product of two given points is a certain third point*, &c., by which it is meant that a certain expression, having a determinate reference to the latter point, is the product of two expressions having a similar determinate reference to the two original points. I believe that upon examination it will be found that these systems of interpretation are founded upon a principle of Naming, as the one which I have proposed is founded upon a principle of Operation. And I think it not foreign to the subject to remark, that the symbolical forms of common language as exhibited in the calculus of logic may indifferently be referred to the one or the other of these modes of conception.

*Laws of Quaternions.*

The laws of quaternion multiplication are founded, as is well known, upon the following relation:

$$i^2=-1 \quad j^2=-1 \quad k^2=-1 \quad ij=k \quad jk=i \quad ki=j$$
$$ji=-k \quad kj=-i \quad ik=-j;$$

but it may be shown that the three last of these laws are consequences of the former ones considered as of universal application.

For if $ij=k$ universally, let the subject be $jy$, then

$$ij\,jy=kjy,$$

or

$$ij^2y=kjy;$$

but $j^2=-1$, therefore

$$-iy=kjy,$$

and similarly for the others.

Lincoln, September 3, 1848.