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XLI. *On the Computation of the Moon's Motion in Right Ascension.* By FRANCIS BAILY, Esq. F.R.S. &c. &c.*

AS the method of determining the longitude of places, by means of moon-culminating stars, is daily coming into more general use, I trust the following table will be acceptable to such of your readers as may have occasion to make calculations connected with inquiries of that kind. I have already shown (in my paper on this subject, inserted in the Memoirs of the Royal Astronomical Society, vol. ii. p. 1), that “in order to deduce, from the observations, the correct difference of meridians between the two places, we require only one element from the lunar tables, viz. the moon’s horary motion in right ascension; or, more properly, the true increase of the moon’s right ascension between the two apparent times of culmination:” and that, for the purpose of determining the correct value of this element, it will be best, when the difference of longitude is very great, to compute the right ascension of the moon for the two given times of observation; using the equation of second, third, and sometimes even fourth differences. But it appears that in many cases, of frequent occurrence, where the difference of meridians does not exceed three hours, we may adopt a much more concise and easy method for the solution of the problem, by computing accurately the semi-diurnal motion of the moon in right ascension, from an ephemeris, by means of differences only.

In No. 33 of Professor Schumacher’s *Astronomische Nachrichten*, M. Bessel has given a series which is applicable to this purpose; together with a short table of the value of that

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series, for every hour of the day. The table, however, which he has computed, being frequently found too limited for general use, I have here enlarged it, by calculating the terms of the series for every ten minutes during the day; whereby the requisite quantities for any intermediate time may be taken out almost on inspection; or at least, in the most essential points, with very little trouble: since it will seldom be found necessary to interpolate, except in the values given in the column B.

M. Bessel's formula is as follows; viz. make a, b, c, d, e respectively equal to the first difference, the mean of the two middle second differences, the third difference, the mean of the two fourth differences, and the fifth difference of the moon's right ascension, as taken from the Nautical Almanac. Then will

$$a + \frac{2n-1}{2} b + \frac{3n^2-3n+\frac{1}{2}}{6} c + \frac{4n^3-6n^2-2n+2}{24} d \\ + \frac{5n^4-10n^3+5n-1}{120} e$$

denote the moon's semi-diurnal motion in right ascension corresponding to that fractional part of the twelve hours, from the *preceding* noon or midnight, indicated by n : and which must always be assumed equal to the middle point of time between the two observations. Or, preserving the same value of c and x as are adopted in my Memoir above mentioned, the value of n must be assumed equal to $\frac{1}{2}(c+x)$. If therefore we express the co-efficients of b, c, d, e by the letters B, C, D, E, respectively, the semi-diurnal motion (M) of the moon in right ascension will be denoted by

$$M = a + B b + C c + D d + E e.$$

The following table contains the logarithms of B, C, D, E, for every ten minutes of the twelve hours from the preceding noon or midnight, as above mentioned; to which must be added respectively the logarithms of b, c, d, e ; the natural numbers thence resulting, being added to the first difference, will give M, or the semi-diurnal motion required. And, retaining the value of the symbols x, s and Δ , as adopted in the Memoir above quoted, the true difference of longitude will be

$$x = \frac{7\frac{1}{2}s}{M} \Delta - \Delta$$

which is (I believe) the most simple form in which the general solution of the problem can be *at present* expressed: but, when the *new* Nautical Almanac appears, $7\frac{1}{2}s$ will be a constant

stant quantity, and equal to 7.5×86636.555 ($\log = 1.4927720$); and other quantities will be given that may still further abridge the arithmetical operations.

Argument $= \frac{1}{2}(c+x)$	Logarithms of				Argument $= \frac{1}{2}(c+x)$
	B	C	D	E	
0 ^h 0 ^m	-9.69897+	+8.92082+	+8.92082-	-7.92082-	12 ^h 0 ^m
10	9.68674	8.88358	8.91452	7.89017	11 50
20	9.67415	8.84404	8.90759	7.85648	40
30	9.66118	8.80626	8.90003	7.81975	30
40	9.64782	8.75663	8.89188	7.78085	20
50	9.63403	8.70776	8.88307	7.73788	10
1 0	9.61979	8.65455	8.87361	7.69085	11 0
1 10	9.60506	8.59603	8.86347	7.64050	10 50
20	9.58983	8.53085	8.85259	7.58207	40
30	9.57403	8.45706	8.84096	7.51555	30
40	9.55764	8.37169	8.82845	7.45244	20
50	9.54061	8.26986	8.81520	7.36098	10
2 0	9.52288	8.14267	8.80121	7.24118	10 0
2 10	9.50440	7.97108	8.78709	7.07557	9 50
20	9.48509	7.70031	8.77180	6.83372	40
30	9.46489	+6.93855+	8.75534	-6.51562-	30
40	9.44370	-7.48946-	8.73650	+5.98986+	20
50	9.42142	7.83556	8.71619	6.73015	10
3 0	9.39794	8.01773	8.69442	6.97498	9 0
3 10	9.37312	8.13964	8.67203	7.10460	8 50
20	9.34679	8.22982	8.64795	7.21709	40
30	9.31875	8.30028	8.62217	7.31246	30
40	9.28880	8.35722	8.59442	7.37997	20
50	9.25661	8.40426	8.56398	7.43406	10
4 0	9.22185	8.44370	8.53085	7.47473	8 0
4 10	9.18406	8.47707	8.49507	7.51249	7 50
20	9.14267	8.50544	8.45486	7.54447	40
30	9.09691	8.52961	8.41021	7.57067	30
40	9.04576	8.55015	8.36245	7.59366	20
50	8.98777	8.56750	8.30477	7.61309	10
5 0	8.92082	8.58200	8.23716	7.62895	7 0
5 10	8.84164	8.59390	8.16105	7.64222	6 50
20	8.74473	8.60340	8.06131	7.65284	40
30	8.61979	8.61064	7.93794	7.66078	30
40	8.44370	8.61575	7.76219	7.66657	20
50	8.14267	8.61878	7.46136	7.66996	10
6 0	- ∞ +	-8.61979-	+ ∞ -	+7.67094+	6 0

I shall now proceed to give an example of the use of this formula and table, a duty which I hold to be incumbent on every person who proposes any new ones for general use. On the 9th of February 1830, the second limb of the moon was

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observed at Greenwich at $13^h 36^m$ apparent time, and at the Cape of Good Hope at $12^h 20^m$ apparent *Greenwich* time; each being estimated to the nearest minute. Consequently the middle point of time, between the two observations, is $\frac{1}{2}(c + \kappa) = 0^h 58^m$ from the preceding midnight: and the successive differences of the moon's right ascension, taken from the Nautical Almanac, will be as follow: viz.

	a	b	c	d	e
Feb. 8. at M = $154^\circ 31' 52''$	$+5^\circ 54' 21''$				
9. at N = $160^\circ 26' 13''$	$+5^\circ 49' 13''$	$-5' 8''$			
at M = $166^\circ 15' 26''$	$+5^\circ 44' 56''$	$-4' 17''$	$+51''$		
10. at N = $172^\circ 0' 22''$	$+5^\circ 41' 37''$	$-3' 19''$	$+58''$	$+7''$	
at M = $177^\circ 41' 59''$	$+5^\circ 39' 19''$	$-2' 18''$	$+61''$	$+3''$	$-4''$
11. at N = $183^\circ 21' 18''$					

Therefore, by entering the table with the argument $\frac{1}{2}(c + \kappa)$, we have the respective logarithms of the several quantities as under: viz.

$$\begin{array}{llll} b = -2.35793 & c = +1.76343 & d = +0.69897 & e = -0.60206 \\ B = -9.62264 & C = +8.66519 & D = +8.87550 & E = -7.70026 \end{array}$$

$$Bb = +1.98057 \quad Cc = +0.42862 \quad Dd = +9.57447 \quad Ee = +8.30232$$

and, taking the natural numbers of these logarithms, we shall find the value of M to be as follows: viz.

$$\begin{array}{rcl} a & = & 5^\circ 44' 56''.000 \\ Bb & = & + 1^\circ 35'.625 \\ Cc & = & + \quad 2'.683 \\ Dd & = & + \quad 0'.375 \\ Ee & = & + \quad 0''.020 \end{array}$$

$$M = 5^\circ 46' 34''.703 \quad (\log = 4.3179526)$$

which is the moon's motion in right ascension for the twelve hours, of which $\frac{1}{2}(c + \kappa)$ is the middle point of time. So that if we had $s = 24^h 3^m 57^s.6$, and $\Delta = + 2^m 26^s.315$, the operation would be as follows: viz.

$$\begin{array}{rcl} 7 \frac{1}{2} s & = & 5.8127677 \\ M & = & 4.3179526 \\ \hline & & 1.4948151 \\ \Delta = 0^h 2^m 26^s.315 & = & 2.1652889 \\ 7 \frac{1}{2} s & & \\ \hline \frac{M}{\Delta} & = & 1.16 \quad 11.976 = 3.6601040 \\ \hline x & = & 1^\circ 13' 45''.661 \end{array}$$

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In this example, it will be seen that I have interpolated for the odd minutes, in order to show the method of obtaining, in all cases, the correct values of the logarithms of the table required: but it is evident that, with the exception of the logarithm of B, this was unnecessary; and that no error would have arisen if we had taken, even in all the quantities, the tabular values opposite to the nearest tenth minute. And I would further remark, that it will seldom be necessary to extend the series to so many terms as are here given: since we may generally stop at the third difference (and sometimes even at the second difference), if we see that the subsequent differences are not sufficiently large to affect the result.

Before I conclude, I would remark that the logarithms of D and E were calculated for every half hour only, and the values interpolated for the intermediate ten minutes, using second differences in the computations.

F. B.

XLII. *Errata in Weisse's Planetary Tables.* By A CORRESPONDENT.

THE number of individuals who in this country are engaged in science, except in what may not inaptly be designated as fancy-work [?], is so very limited, that considering the heavy duty on the importation of books*, it is a most hazardous speculation for a bookseller to introduce foreign scientific works into England. The consequence is, that many of the highest value in point of utility, but of which the sale would necessarily be limited, are either altogether unknown in this country, or have fallen into the hands of individuals without the means, or possibly the inclination, to bring their merits before the public. The industrious zeal of Mr. Baily and the hitherto discreditable state of our Nautical Almanac, led to the speedy adoption of Encke's Ephemeris; but the inestimable *Tabulæ Regiomontanæ Reductionum* of Bessel† have scarcely been heard of. The *Formeln der Geometrie und Trigonometrie*, published at Berlin in 1827, and Hesterman's most useful *Leges Trigonometriæ*, have met with no better fate. The

* While in Russia, a nation which we are accustomed to condemn as something more than semibarbarous, no duty, and in France a duty equivalent to only six shillings per hundred weight, is levied upon the importation of books;—in England, they are subjected to five pounds per hundred weight.

† This work, indispensable to practical astronomers, consists of one large octavo volume, printed with a degree of clearness which should serve as a model for all books of tables, and is published by Bachelier of Paris, and Treuttel, London.

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