268. Historical Note (220, 221, 222)

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Source: *The Mathematical Gazette*, Vol. 4, No. 73 (Jul., 1908), p. 316

Published by: [Mathematical Association](#)

Stable URL: <http://www.jstor.org/stable/3604863>

Accessed: 28-02-2016 14:58 UTC

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$$\begin{aligned} \therefore f(1) &= z, \\ \text{but } f(1) &= a^m; \\ \therefore z &= a^m; \\ \therefore a^{mx} &= (a^m)^x. \end{aligned}$$

(No assumption is made as to the values of the quantities a, m, n .)

Cor. Demoivre's Theorem.

$$\begin{aligned} \text{Let } f(\theta) &= \cos \theta + i \sin \theta. \\ \text{Then } f(\theta) \cdot f(\phi) &= \cos(\theta + \phi) + i \sin(\theta + \phi) \\ &= f(\theta + \phi); \\ \therefore f(\theta) &= z^\theta; \\ \therefore \{f(\theta)\}^n &= \{z^\theta\}^n \\ &= z^{n\theta} \\ &= f(n\theta); \end{aligned}$$

$$\therefore \{\cos \theta + i \sin \theta\}^n = \cos n\theta + i \sin n\theta \text{ (with the usual restrictions).}$$

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268. [v. 2. 10.] *Historical Note* (220, 221, 222).

To draw from any two given points A, B two straight lines to a point R on a given circle, having centre at C , so that they may be equally inclined to CR .

This problem, obviously intimately connected with Optics, is found in the treatise on Optics by Alhazen, circ. 1100 A.D. It was solved geometrically by Alhazen, Vitellio and Barovius, and algebraically by Huygens, Slusius and Catalan. The solution given in Milne and Davis' *Conics*, p. 155, is due to R. Simson, who discovered it prior to 1729. The similar one published by Robins in 1739 Simson seemed to think was inspired by his own, which depends on the construction of a rectangular hyperbola given two conjugate diameters.

In Prof. Hudson's second solution, line 4, we are told to fit, mechanically, a line of given length between two given lines so that its direction shall pass through a given point. This, which belongs to the directive (*vevovis*) class of problems and is mentioned by Pappus, is dealt with by Newton in his *Universal Arithmetic* in the chapter on the linear construction of equations, where he shews that the line can be constructed mechanically either by the use of the conchoid or by means of a hyperbola and a circle, and he employs it in the graphic solution of cubic equations, and also in the trisection of an angle and the finding two mean proportionals between two given quantities. The whole chapter is very interesting, as Newton there discusses the question as to what lines besides the straight line and circle should be admitted in the solution of geometrical problems. If a mechanical solution is allowed, I would suggest the following for Alhazen's Problem:

Describe in the ordinary way, by means of two pins, string and pencil, a conic having A and B as foci, and adjust the length of the string so as to make the conic touch the circle in R (suppose). Then AR and BR are equally inclined to the common tangent at R , and $\therefore$ also to the radius CR .

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269. [v. 4. b. β.] *On Note* 235.

Line 6 from bottom; after 'most,' insert "at which the values of r and θ are the same for both curves."

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270. [v. 4. b. β.] *On the intersections of the Limaçon and Quatrefoil used in the graphical solution of a biquadratic.* (*Math. Gazette*, Jan. 1907, p. 22.)

Although, using polar coordinates, there are only 4 pairs of values of r, θ that satisfy both equations, it is easy to draw a figure shewing 9 points of