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LIV. *On a Simple Apparatus for determining the Thermal Conductivities of Cements and other Substances used in the Arts.* By CHARLES H. LEES, D.Sc., and J. D. CHORLTON, B.Sc., *Joule Scholar of the Royal Society**.

THE following method of determining the thermal conductivities of bad conductors has been designed with the object of simplifying as much as possible the apparatus and observations required in making determinations which are not required to be of the highest order of accuracy, but in which errors of more than 2 per cent. are to be avoided. When the constants of the apparatus have once been determined, the only observations necessary to determine a conductivity are the thickness of a sheet of the substance used and the temperatures of three thermometers. It is hoped that this simplification will lead those who require bad conductors of heat for structural purposes to carry out their own tests of the materials they have available.

Method.—The apparatus consists of a flat cylindrical metal box, of 11·4 cms. diameter and 3 cms. depth, through which steam can be passed. The bottom of the box consists of a circular brass plate 1·3 cm. thick in which a radial hole reaching to the centre is bored. In this hole a thermometer is placed with its bulb at the centre of the plate. The top and sides of the box are covered with green baize to prevent loss of heat as far as possible. This vessel is supported on a circular plate of the material to be tested, which in its turn is supported on a brass disk similar to the one forming the base of the heating vessel, and like it provided with a radial hole and thermometer.

The lower disk is suspended horizontally from a support by three strings attached to three short pegs projecting from the edge of the disk, and hangs about 30 centims. above the table. About 10 centims. above the table a thermometer is placed horizontally, with its bulb under the centre of the lower plate to give the temperature of the air ascending to the disks. The bulb of the thermometer is protected against radiation from the lower disk by a bright metal screen.

The two surfaces of the disks which come into contact with the material to be experimented on are amalgamated, so that if the material is a solid, contact over its entire surface may be obtained by using a thin mercury film between the solid and disks. In order to determine the thickness of the

* Communicated by the Authors.

material used, two short pegs of brass, one vertically above the other, project from the disks, and their distance apart is measured by means of a wire gauge or calipers, when the disks are in contact and when the plate to be tested is between, the difference is the thickness of the plate of the material tested. The under surface of the lower disk may be kept polished, or it may be painted, in order that the heat radiated from it may remain constant during a series of experiments.

When steam is passed through the upper cylinder the temperature of the upper disk is raised to nearly 100° C. ; heat flows through the plate of material experimented on to the lower disk, the temperature of which is therefore raised above that of the surrounding air. It begins in consequence to lose heat by radiation and conduction to the air, and eventually a stage is reached when this loss of heat is equal to the heat received from the material experimented on.

Hence if the amount of this loss is found by a separate experiment, a determination of the temperature gradient in the material experimented on at its surface of contact with the lower plate, will enable the thermal conductivity of the material to be found.

Theory.

If θ is the temperature at a point x above the under surface of the lower plate, θ_0 the temperature of the air, k_1 the internal conductivity, h_1 the external conductivity or "emissivity," Newton's law being supposed to hold for the limits of temperature used, p the perimeter, q the area, of cross section of the plate, the differential equation for the motion of heat in the plate, the isothermals being assumed plane, is satisfied if

$$\theta - \theta_0 = A_1 \cosh \sqrt{\frac{ph_1}{qk_1}} \cdot x + B_1 \sinh \sqrt{\frac{ph_1}{qk_1}} \cdot x,$$

where A_1 and B_1 are constants.

The condition for continuity of flow at the under surface is

$$k_1 \frac{d\theta}{dx} = h_1 \theta \text{ for } x=0,$$

i. e.

$$k_1 \sqrt{\frac{ph_1}{qk_1}} \cdot B_1 = h_1 A_1;$$

$$\therefore \theta - \theta_0 = A_1 \left\{ \cosh \sqrt{\frac{ph_1}{qk_1}} \cdot x + \frac{h_1}{k_1 \sqrt{\frac{ph_1}{qk_1}}} \sinh \sqrt{\frac{ph_1}{qk_1}} \cdot x \right\}$$

Now for the lower brass disk

$$p/q = 2\pi r/\pi r^2 = 2/5 \cdot 7 = \cdot 351,$$

while k_1 for brass = $\cdot 25$, and it will be shown presently that $h_1 = \cdot 0003$ about. Hence $\sqrt{\frac{ph_1}{qk_1}} = \cdot 02$, and since the thickness of the plate is $1 \cdot 3$ cm. the maximum value of the expression in brackets is

$$\cosh \cdot 026 + \frac{\cdot 0003}{\cdot 25 \times \cdot 02} \sinh \cdot 026 = 1 \cdot 0018;$$

or the temperature of the lower plate varies less than $\frac{1}{5}$ per cent., and for the purpose of the present experiment may be taken to be uniform throughout and equal to the indication θ_1 of the thermometer in the centre of the disk. The temperature of the lower disk is therefore given completely by the equation

$$\theta - \theta_0 = \{\theta_1 - \theta_0\} \left\{ \cosh \sqrt{\frac{ph_1}{qk_1}} \cdot x + \frac{h_1}{k_1 \sqrt{\frac{ph_1}{qk_1}}} \cdot \sinh \sqrt{\frac{ph_1}{qk_1}} \cdot x \right\}.$$

The rate of flow of heat into the lower disk, the thickness of which is $1 \cdot 3$ centim., is given by $k_1 \frac{d\theta}{dx}$ for $x = 1 \cdot 3$. It is therefore

$$\begin{aligned} &= k_1 \sqrt{\frac{ph_1}{qk_1}} \{\theta_1 - \theta_0\} \left\{ \sinh \cdot 026 + \frac{h_1}{k_1 \sqrt{\frac{ph_1}{qk_1}}} \cdot \cosh \cdot 026 \right\} \\ &= (\theta_1 - \theta_0) \left(\cdot 00026 h_1 + \cdot 026 k_1 \sqrt{\frac{ph_1}{qk_1}} \right) \\ &= (\theta_1 - \theta_0) (h_1 + \cdot 00013) \\ &= (\theta_1 - \theta_0) (h_1 + h_1'), \text{ say, where } h_1' = \cdot 00013. \end{aligned}$$

If k is the internal, h the external conductivity of the medium under test, the temperature θ at a point x above its under surface is given by:—

$$\theta - \theta_0 = (\theta_1 - \theta_0) \left\{ \cosh \sqrt{\frac{ph}{qk}} \cdot x + B \sinh \sqrt{\frac{ph}{qk}} \cdot x \right\},$$

where B is a constant, the value of which is fixed by the *Phil. Mag.* S. 5. Vol. 41. No. 253. June 1896. 2 M

condition for continuity of flow at the surface of contact of the material and the lower plate, *i. e.*

$$k\sqrt{\frac{ph}{qk}}.B=(\theta_1-\theta_0)(h_1+h_1').$$

Hence in the material under test

$$\theta-\theta_0=\{\theta_1-\theta_0\}\left\{\cosh\sqrt{\frac{ph}{qk}}.x+\frac{h_1+h_1'}{k\sqrt{\frac{ph}{qk}}}\sinh\sqrt{\frac{ph}{qk}}.x\right\}.$$

If b is the thickness of the material, at $x=b$ we must have $\theta=\theta_2$, the temperature indicated by the thermometer in the upper brass plate, *i. e.*

$$\theta_2-\theta_0=(\theta_1-\theta_0)\left(\cosh\sqrt{\frac{ph}{qk}}.b+\frac{h_1+h_1'}{k\sqrt{\frac{ph}{qk}}}\sinh\sqrt{\frac{ph}{qk}}.b\right).$$

The maximum value of $\sqrt{\frac{ph}{qk}}.b$ for the materials experimented on is $\cdot 2$; hence the hyperbolic functions can be expanded in ascending powers of $\sqrt{\frac{ph}{qk}}.b$, and terms involving cubes can be neglected. Thus:—

$$\theta_2-\theta_0=(\theta_1-\theta_0)\left(1+\frac{ph}{2qk}b^2+\frac{h_1+h_1'}{k\sqrt{\frac{ph}{qk}}}\sqrt{\frac{ph}{qk}}.b\right),$$

$$\text{or } \frac{\theta_2-\theta_1}{\theta_1-\theta_0}=\frac{b}{k}\left(h_1+h_1'+\frac{ph}{2q}.b\right).$$

The last term in the bracket never exceeds 4 per cent. of the first two terms, and the final result will be affected only to a very small extent if for h , h_1 is substituted.

Hence we have

$$k=\frac{\theta_1-\theta_0}{\theta_2-\theta_1}b\left(h_1+h_1'+\frac{ph_1}{2q}.b\right),$$

which enables k to be calculated from the result of the experiment if h_1 has been previously determined.

Determination of h_1 .—To determine the value of this quantity the two plates were separated by a layer of air 3 mm. thick, enclosed by a ring of badly conducting material upon

which the upper plate rested; steam was passed through the box at the top, and the temperature of the upper plate raised to nearly 100° C. The lower plate was next heated a few degrees above 100° C. by means of a gas-flame and then allowed to cool, and as soon as its temperature fell to 100° C. the supply of steam to the upper can was cut off and the hot water in the can allowed to run out.

Both plates were then found to cool at almost the same rate; the difference in the temperatures of the two plates never exceeded 1° C., the upper plate being always the warmer. Since the conductivity of air is only about ·00005 C.G.S. units, the amount of heat conducted from the upper plate to the lower is negligible, so that each plate cools independently of the other.

The thermometer in the lower plate was watched as the temperature fell, and the times at which the mercury passed every alternate degree were observed; the results are given in the table below.

The temperature of the air was steady at 18° C.

Temperature of lower plate.		Time.	Fall of temp. per second.	Loss of heat, gram-degrees per second.	Loss of heat per sq. cm., gram-degrees per second.	k_1 .
θ_1 .	$\theta_1 - \theta_0$.					
° C.	° C.	h m s				
96	78	3.17.42				
94	76	18.37	·0370	3.92	·0255	·000336
92	74	19.32	354	75	43	28
90	72	20.30	340	60	34	25
88	70	21.30	325	44	23	19
86	68	22.33	308	26	12	12
84	66	23.40	301	14	04	09
82	64	24.46	290	07	·0199	11
80	62	25.58	274	2.90	86	03
78	60	27.12	263	79	81	02
76	58	28.30	256	71	76	03
74	56	29.48	250	65	72	07
72	54	31.10	244	56	68	11
70	52	32.32	235	49	62	12
68	50	34.0	220	33	51	02
66	48	35.34	208	20	43	·000298
64	46	27.12	197	08	35	93
62	44	38.57	187	1.98	29	93
60	42	40.46	180	91	24	95
58	40	42.40	169	79	16	90
56	38	44.42	156	65	07	82
54	36	46.57	146	55	·0101	81
52	34	49.15	140	48	·0096	82
50	32	51.43				

The numbers in the 4th column are found by subtracting alternate times in the 3rd column from each other and dividing 4°C. by the differences.

The numbers in the 5th column are obtained from those in the 4th by multiplying by 1140 grams, the mass of the lower disk, and by $\cdot 093$, the specific heat of brass, *i. e.* by 106.

The numbers in the 6th column are obtained by dividing by the total radiating surface of the lower disk, plus half the radiating surface of the nonconducting ring $= 148\cdot 5 + 5\cdot 5 = 154$ sq. centims.

The values of h are obtained from these numbers by dividing by the excess of the temperature of the lower disk over that of the air.

Experiments.

The following account of an experiment on a plate of glass will show the method of treatment in each case:—

A disk of plate-glass of diameter equal to that of the brass plates— $11\cdot 4$ centims.—was placed between them, and contact made by means of thin mercury films. Good contact was easily secured between the glass and the lower plate; but with much more difficulty in the case of the upper plate. Contact was eventually obtained by covering the surface of the upper plate with mercury, so that it adhered in pendent drops, at the same time placing a few drops of mercury on the surface of the glass, and then carefully lowering the upper plate on to the glass and allowing the excess of mercury to run out at the edges.

Steam was then passed through the box attached to the upper brass plate, and thermometers inserted in the upper and lower plates, the temperature of the air being indicated by a third thermometer placed as described above.

After a time, the duration of which depended on the conductivity and thickness of the substance experimented upon, the thermometers became steady and the temperatures were observed.

θ_2 = temperature of upper plate = $96\cdot 1^{\circ}\text{C.}$

θ_1 = „ „ lower „ = $92\cdot 4^{\circ}\text{C.}$

θ_0 = „ „ air = $20\cdot 0^{\circ}\text{C.}$

Thickness = $\cdot 292$ centim.

The thickness was obtained by measuring with a micro-meter-gauge the distance between the two pegs, first, when the plates were in contact with each other, and, secondly, when the glass was between them.

The following table gives the observations taken during each experiment, and shows the method of determining the conductivities from the observations :—

Remarks on Specimens Experimented on.

- (a) The plates of Portland cement and plaster of Paris, &c., were made by pouring the liquid cement on to a glass plate, on which three equal beads strung together were laid, a second glass plate was pressed down upon these beads, after a few hours the glass plates could be removed and a plate of cement of uniform thickness was obtained.

The plate of plaster of Paris and sand consisted of two parts by weight of plaster of Paris to one part by weight of sand.

- (b) When the substance experimented upon was a powder, the brass plates were kept apart by three stops of wood, and the powder was prevented from falling out at the edges by a very narrow circular ring of fibre.
- (c) The three values given for the conductivity of garden-soil refer to three specimens of soil of the same kind but containing different amounts of moisture; the first value refers to dry soil, the second to slightly moist, and the third to damp soil. The results show how greatly a small amount of moisture affects the conductivity.
- (d) The calico in the first experiment was dry, and weighed 8.91 grams. In the second experiment it was exposed for some time to a damp atmosphere, and its weight increased to 9.03 grams, so that the conductivity of calico increases 20 per cent. for 1.6 per cent. increase in weight due to moisture absorbed.
- (e) In the three experiments on old flannel, the flannel contained different amounts of moisture. In the first experiment the flannel was quite dry and weighed 8.21 grams, in the second it was exposed to a damp atmosphere, and its weight was 8.29 grams, and in the third experiment was damped with water and its weight increased to 9.05 grams. Thus the conductivity of flannel increased about 11 per cent. for 1 per cent. increase in weight due to absorbed moisture, and afterwards increased 25 per cent. more for a further increase of 10 per cent. in weight.

The flannel in the third experiment is almost as good a conductor as dry calico.

The new flannel, flanelette, silk, and linen were dry.

Substance.	Thick- ness in cm.	Temperatures.			$\frac{ph_1b}{2q}$ = '00005 b .	$k_1 + k_2 + \frac{p^2b}{2q}$ = '00044 $\left\{ \begin{smallmatrix} 45 \\ 46 \end{smallmatrix} \right\} +$	$\theta_2 - \theta_1$, $\theta_1 - \theta_0$.	$\frac{\theta_1 - \theta_0}{\theta_2 - \theta_1}$.	$\frac{\theta_1 - \theta_0 b}{\theta_2 - \theta_1}$.	k .
		Upper plate θ_2 .	Lower plate θ_1 .	Air θ_0 .						
Slate	·388	96·1	92·4	20·0	·00002	·00047	° C. 87 72·4	19·6	7·60	·00357
Glass	·292	94·6	89·8	18·5	2	47	4·8 71·3	14·8	4·32	·00203
Poreclain	·418	98·0	92·2	19·0	2	47	5·8 73·2	12·6	5·27	·00248
Fibre	·157	96·8	83·8	19·0	1	45	13·0 64·8	4·98	·6782	·00035
Leather, Cowhide	·403	97·0	72·3	19·6	2	46	24·7 52·7	2·13	·916	·00042
Leather, Chamois	·183	97·2	69·3	19·0	1	45	27·9 50·3	2·81	·514	·00015
Indiarubber	·183	97·8	88·0	17·5	1	46	9·8 70·5	7·19	1·316	·00061
Blotting-paper	·104	97·1	78·4	20·3	1	45	18·7 58·1	3·11	·323	·00015
Asbestos-paper	·285	96·3	78·2	17·8	1	45	18·1 60·4	3·34	·962	·00043
(a) Portland cement	·316	96·3	83·2	19·2	2	46	13·1 64·0	4·89	1·54	·00071
Plaster of Paris	·336	97·2	83·0	18·8	2	46	14·2 64·2	4·52	1·152	·00070
Plaster of Paris with sand	·321	96·6	86·7	19·3	2	46	9·9 67·4	6·81	2·19	·00101
(b) Plaster of Paris powder ...	·600	98·0	57·0	17·3	3	45	41·0 39·7	9·68	5·81	·00261
Sand	·540	96·9	71·8	17·7	3	47	25·1 54·1	2·16	1·17	·00055

Brickdust, yellow	·540	97·7	67·0	19·8	·00003	·00046	30·7	47·2	1·54	·832	·00038
Brickdust, red	·550	97·1	69·7	20·0	3	46	27·4	49·7	1·83	1·01	·00046
Sawdust.....	·540	98·0	52·5	20·8	3	44	45·5	31·7	·697	·376	·000165
Salt	·540	98·4	75·0	19·2	3	46	23·4	55·8	2·39	1·29	·00059
(c) Garden soil 1	·550	97·5	63·7	19·0	3	46	33·8	44·7	1·32	·716	·00033
" 2	·550	96·7	81·9	18·5	3	47	14·8	63·4	4·28	2·35	·00110
" 3	·630	95·6	83·7	19·3	3	47	11·9	64·4	5·41	3·41	·00160
(d) Calico 1.....	·221	97·5	69·4	17·2	1	44	28·1	52·2	1·86	·411	·00018
" 2.....	·221	96·8	71·8	16·2	1	44	25·0	55·8	2·23	·493	·00022
(e) Old flannel 1	·316	97·0	56·4	18·0	2	44	40·6	38·4	·947	·299	·00013
" 2	·316	97·8	59·9	19·6	2	44	37·4	40·3	1·07	·338	·00015
" 3	·316	97·2	62·9	18·0	2	44	34·3	44·9	1·31	·414	·00018
New flannel	·397	97·8	50·1	16·4	2	43	47·7	33·7	·706	·280	·00012
Flanellette	·298	98·4	61·6	18·8	1	43	36·8	42·8	1·16	·346	·00015
Baize	·356	97·0	53·8	18·1	2	43	43·2	35·7	·827	·278	·00012
Linon	·196	96·7	73·3	16·5	1	44	23·4	56·8	2·43	·476	·00021
Silk.....	·080	96·2	74·9	17·2	0	44	21·3	57·7	2·71	·217	·00095