On the Latitude Problem with three relative Altitudes. By *H. C. Plummer*.

This problem, which has been discussed by Dr. *Wilkins* (A. N. 4580) and by Professor *Harzer* (A. N. 4591), admits of another simple form of solution which may deserve passing note.

For the altitudes we take $h+d_1$, $h+d_2$, $h+d_3$. The differences are known from observation and if we make $d_1+d_2+d_3=0$, h is the mean of the three altitudes. We also write

$$\begin{aligned} c_1 &= \cos S_2 S_3 = \sin \delta_2 \sin \delta_3 \\ &\quad + \cos \delta_2 \cos \delta_3 \cos(\alpha_3 - \alpha_2 - t_3 + t_2) \\ c_2 &= \cos S_3 S_1 = \sin \delta_3 \sin \delta_1 \\ &\quad + \cos \delta_3 \cos \delta_1 \cos(\alpha_1 - \alpha_3 - t_1 + t_3) \\ c_3 &= \cos S_1 S_2 = \sin \delta_1 \sin \delta_2 \\ &\quad + \cos \delta_1 \cos \delta_2 \cos(\alpha_2 - \alpha_1 - t_2 + t_1). \end{aligned}$$

$\alpha_1, \alpha_2, \alpha_3$ being the three right ascensions, $\delta_1, \delta_2, \delta_3$ the three declinations, and t_1, t_2, t_3 the sidereal times of observation (with a common correction). Then from the six arcs connecting the four points S_1, S_2, S_3 and Z , the Zenith, the equation which gives h can be written down immediately in the form

$$A = \begin{vmatrix} 1 & c_3 & c_2 & \sin(h+d_1) \\ c_3 & 1 & c_1 & \sin(h+d_2) \\ c_2 & c_1 & 1 & \sin(h+d_3) \\ \sin(h+d_1) & \sin(h+d_2) & \sin(h+d_3) & 1 \end{vmatrix} = 0.$$

It is convenient to express the determinant

$$\begin{vmatrix} 1 & c_3 & c_2 & \cos d_1 \\ c_3 & 1 & c_1 & \cos d_2 \\ c_2 & c_1 & 1 & \cos d_3 \\ \cos d_1 & \cos d_2 & \cos d_3 & 1 \end{vmatrix}$$

by the symbol $[D, c, c, 1]$, where D stands for the matrix common to A and the latter determinant, c for the cosines in either border and 1 for the corner constituent. Similar determinants in which sines replace the cosines in either border are indicated by writing s for c . With this notation the result of expanding the border terms in A and then resolving A into determinants with simple constituents may be written

$$[D, 0, 0, 1] + [D, c, c, 0] \sin^2 h + [D, s, s, 0] \cos^2 h + 2[D, s, c, 0] \sin h \cos h = 0$$

$$\text{or } [D, c, c, 1] \sin^2 h + [D, s, s, 1] \cos^2 h + 2[D, s, c, 0] \sin h \cos h = 0$$

where the meaning of the coefficients will be readily understood. This is equivalent to *Harzer's* equation

$$A - 2B \operatorname{tg} z + C \operatorname{tg}^2 z = 0.$$

We may, however, write the equation in the form

$$m \cos(2h + M) = n$$

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where

$$\begin{aligned} m \cos M &= [D, c, c, 1] - [D, s, s, 1] \\ &= -\Sigma(1-c_1^2) \cos 2d_1 + 2\Sigma(c_1-c_2c_3) \cos d_1 \\ m \sin M &= 2[D, s, c, 0] \\ &= -\Sigma(1-c_1^2) \sin 2d_1 - 2\Sigma(c_1-c_2c_3) \sin d_1 \\ n &= [D, c, c, 1] + [D, s, s, 1] \\ &= 4c_1c_2c_3 - c_1^2 - c_2^2 - c_3^2 - 1 \\ &\quad + 2\Sigma(c_1-c_2c_3) \cos(d_2-d_3). \end{aligned}$$

In the original problem of *Gauß*, $d_1 = d_2 = d_3 = 0$. It is needless to point out the simplification of the above expressions in this case.

When h has been found the absolute altitudes are known and the problem has been reduced from that of three relative altitudes to that of two absolute altitudes. The solution may be completed accordingly. Or we may follow *Harzer's* method and solve the three linear equations

$$\sin(h+d_i) = a \sin \delta_i + b \cos \delta_i \cos(t_i - \alpha_i) + c \cos \delta_i (\sin t_i - \alpha_i) \quad (i = 1, 2, 3),$$

where $a = \sin \varphi$, $b = \cos \varphi \cos \Delta t$, $c = -\cos \varphi \sin \Delta t$ with the control equation $a^2 + b^2 + c^2 = 1$.

The corresponding method of finding the latitude when two absolute altitudes h_1, h_2 are known, and the interval of time between them, is to use the equation connecting the six arcs joining the four points P, S_1, S_2, Z , namely

$$\begin{vmatrix} 1 & \sin \delta_1 & \sin \delta_2 & \sin \varphi \\ \sin \delta_1 & 1 & \cos S_1 S_2 & \sin h_1 \\ \sin \delta_2 & \cos S_1 S_2 & 1 & \sin h_2 \\ \sin \varphi & \sin h_1 & \sin h_2 & 1 \end{vmatrix} = 0$$

The development of the determinant gives

$$\begin{aligned} 1 - \sin^2 \delta_1 - \sin^2 \delta_2 - \cos^2 S_1 S_2 + 2 \sin \delta_1 \sin \delta_2 \cos S_1 S_2 \\ - \sin^2 h_1 \cos^2 \delta_2 - \sin^2 h_2 \cos^2 \delta_1 \\ + 2 \sin h_1 \sin h_2 (\cos S_1 S_2 - \sin \delta_1 \sin \delta_2) \\ + 2 \sin \varphi (\sin h_1 (\sin \delta_1 - \sin \delta_2 \cos S_1 S_2) \\ + \sin h_2 (\sin \delta_2 - \sin \delta_1 \cos S_1 S_2)) \\ - \sin^2 \varphi \sin^2 S_1 S_2 = 0 \end{aligned}$$

or

$$\begin{aligned} \sin^2 \varphi \sin^2 S_1 S_2 - 2 \sin \varphi \sin S_1 S_2 (\sin h_1 \cos \delta_2 \cos P S_2 S_1 \\ + \sin h_2 \cos \delta_1 \cos P S_1 S_2) \\ - \cos^2 \delta_1 \cos^2 \delta_2 \sin^2 S_1 S_2 + \sin^2 h_1 \cos^2 \delta_2 + \sin^2 h_2 \cos^2 \delta_1 \\ - 2 \sin h_1 \sin h_2 \cos \delta_1 \cos \delta_2 \cos S_1 P S_2 = 0 \end{aligned}$$

or again, with a little reduction,

$$(\sin \varphi \sin P / \sin S_1 \sin S_2 - (\sin h_1 \operatorname{ctg} S_2 + \sin h_2 \operatorname{ctg} S_1))^2 = 4 \cos \sigma \cos(\sigma - S_1 S_2) \sin(\sigma - h_1) \sin(\sigma - h_2)$$

where P, S_1, S_2 represent the angles of the triangle $PS_1 S_2$ and $2\sigma = h_1 + h_2 + S_1 S_2$. This furnishes a direct, symmetrical solution, unlike those given by *Chauvenet*.

H. C. Plummer.

Über das Zweihöhenproblem. Von *Paul Harzer*.

Der vorstehende Aufsatz des Herrn *H. C. Plummer* veranlaßt mich, seine Bemerkung über das Zweihöhenproblem zu ergänzen, indem ich eine anscheinend in Vergessenheit

geratene Lösung dieses Problems in Erinnerung bringe. Es kommt darauf an, bei gegebenen Werten der zwei Größentripel $\delta_\alpha, \tau_\alpha, z_\alpha$ die Unbekannten $\varphi, \Delta u$ aus den Gleichungen