On an Instrument for Measuring the Lateral Contraction of Tie-bars, and on the Determination of Poisson's Ratio

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1903 Proc. Phys. Soc. London 18 582

(<http://iopscience.iop.org/1478-7814/18/1/358>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 130.179.16.201

This content was downloaded on 28/08/2015 at 06:57

Please note that [terms and conditions apply](#).

period might be made very great or excessively small, but there would probably be difficulties in either case.

Prof. EVERETT said it was obvious that the records could not easily be interpreted unless there was a great difference between the periods. He suggested the use of two booms having different periods.

Dr. WATSON pointed out that the boom would respond equally well to any period if it was heavily damped. Fluid damping was unsatisfactory because of convection currents, and he suggested that the plate at the end of the boom should be made of copper and allowed to move between the poles of a strong horse-shoe magnet.

Dr. CHREE said that if the period of the boom was increased beyond 18 seconds, the apparatus became too sensitive and was unstable.

LVIII. *On an Instrument for Measuring the Lateral Contraction of Tie-bars, and on the Determination of Poisson's Ratio.* By J. MORROW, M.Sc. (Vict.), Lecturer in Engineering, University College, Bristol*.

THE ratio of the lateral to the longitudinal strain in a bar under the action of a simple pull or push in the direction of its axis has been the subject of much controversy and a considerable amount of research. Some French mathematicians, including Poisson and Navier, held that this ratio was $\frac{1}{4}$ for all isotropic substances, their arguments were based on a theory of the constancy of the ratio of the elastic coefficients for all materials.

Wertheim endeavoured to show that this theory was false. He filled hollow prisms with liquid and subjected them to a tensile force. The interior of the prism communicated with a capillary tube, and the change of volume of the interior was measured by the distance the fluid advanced or receded in the tube. If Poisson's ratio be denoted by σ , change of unit volume = tensile strain $(1 - 2\sigma)$ very approximately. He maintained that the ratio σ should really be

* Read May 22, 1903.

$\frac{1}{3}$ for all materials; but it appears doubtful whether the specimens dealt with were sufficiently isotropic to justify any definite conclusion either for or against the uniconstant form.

Practical methods for the determination of Poisson's ratio may be divided into three classes. First, those by which two coefficients of elasticity are obtained, and the required ratio inferred by calculation. Second, those which depend on the deformation of the section of a beam when bent. And, lastly, methods by which the tensile and lateral strains are actually measured in specimens of the material under direct tensile or compressive stress.

Kirchhoff experimented in 1859 upon cylinders, which by means of a weight attached to a lever were put simultaneously under the action of bending and twisting forces and the strains produced were measured accurately. The values of Poisson's ratio were calculated from the ratio of the observed displacements, and were

Mean for three steel rods	·294.
„ hard drawn brass rod . . .	·387.

Many others have worked on similar lines.

Cornu (and later Straubel) explored the anticlastic surface of a rectangular beam by means of the interference-fringes produced between it and a plate of glass laid on it, thus obtaining data for the calculation of σ .

Mallock (1879) also examined the anticlastic curvature. By means of a microscope he measured the movement of four fine steel wires fastened to opposite diameters of a circle on the surface, obtaining data for the radii.

Another method of investigation would be to find the displacements in the sides of a beam, either by measuring the lateral strains at any section, or by attaching a mirror to the side and finding the angle through which it turns when the beam is loaded.

If f = the maximum tensile stress, the lateral contraction is $\sigma \frac{f}{E} b$ (b being the breadth of the beam and E Young's modulus) and the angle turned through by the side of the beam is $\tan^{-1} \sigma \frac{f}{E} \frac{b}{d}$.

In all the above methods the practical accuracy of the theory of elastic materials is assumed.

Coming now to actual measurements of the lateral contraction of a tie-bar, it appears that Bauschinger (*Der Civilingenieur*, vol. xxv. 1879, pp. 81-124) was the first to construct an instrument which would measure the alteration of cross dimensions of a metal test-piece. His apparatus is complicated. It is essentially a pair of levers, the small arms of which are the diameter of small caoutchouc cylinders, and the greater the double distance of the scale from a mirror. To read the lateral contraction requires two separate observations by means of two telescopes and mirrors on opposite sides of the specimen.

Stromeyer (Proc. Roy. Soc. vol. lv. no. 334, p. 373) measured directly the lateral contraction of a bar under tension by interference methods; a dark glass attached to the specimen advanced or receded from the surface of a prism as the diameter of the specimen altered, thus producing change in the positions of the interference-bands.

Elastic Constants.

If α is the longitudinal strain produced by unit stress, and β the linear lateral strain in the material, then

$$\sigma = \frac{\beta}{\alpha}, \quad E = \frac{1}{\alpha}, \quad n = \frac{1}{2(\alpha + \beta)}, \quad \text{and} \quad k = \frac{1}{3(\alpha - 2\beta)};$$

where E , n , and k are Young's modulus and the moduli of rigidity and of bulk respectively.

If any two of the elastic coefficients be known Poisson's ratio can be calculated.

Also we have that:—

if $\sigma = \frac{1}{2}$ the material is incompressible, and the extension and lateral contraction are such that the volume remains constant.

if $\sigma > \frac{1}{2}$ k would be opposite in sign to E and n , the material not being homogeneous and isotropic.

if $\sigma = 0$ to $\frac{1}{2}$ we have an ordinary elastic body.

if $\sigma = 0$ either there is no lateral strain or the material is perfectly plastic.

$\sigma = \frac{1}{4}$ is the value of the uniconstant theory which requires $n = \frac{3}{5}$ $k = \frac{2}{5}$ E .

The New Form of Apparatus.

With the object of measuring, in the most direct way possible, the lateral contraction of a tensile specimen or the dilatation of a compressive piece when loaded in an ordinary testing-machine, the author devised the instrument shown in the figure.

Two hardened set-screws A and B, on opposite sides of the specimen S, are pressed inwards by stiff springs C, C. The relative motion of the points of these set-screws is transmitted by the bars E, F, and the pivot P, to the extremities H, G, of the pieces K, L.

SCALE HALF SIZE

It will be noticed that the instrument touches the specimen only at the points between which the alteration of dimension is to be measured. When the specimen extends A and B approach one another, and consequently G falls with reference to H. The relative displacement of G and H is measured optically. A mirror M rests by three steel points (two of which are on G and the other on H, as shown in the plan of the instrument) on glass plates fixed to G and H. A second mirror N is attached to G in a vertical position, it

is capable of a small amount of rotation about a vertical axis by the fine adjustment screw Q. The two mirrors are close together but not touching. A scale is placed some distance away, and by means of a telescope two images of the scale are seen, one in each mirror. These images are brought together in the telescope and the one is read against the other. Thus a certain convenient reading on the N image is taken as the index, and as the load is applied to the specimen the observer reads the values on the M image coinciding with this index.

Every precaution has been taken to ensure the greatest possible accuracy. The screws A and B have spherical points in order that they may not cut into the specimen. The instrument is balanced by the weight W. The mirrors are of specially worked optically plane glass. The joint at P is made by centring the screws in the inner piece, and is constructed with the greatest care.

In order to test the instrument a specially selected bar of mild Bessemer steel was used. It was turned to one inch diameter as given by a Whitworth gauge. After placing in the testing-machine it was loaded several times up to 5 tons. The instrument was attached and allowed to stand for a short time before readings were taken.

The load was then applied by successive tons between the limits 1 and 5 tons and the scale-readings observed. The results for three series of increasing and decreasing loads are given in Table I.

It was thus seen that the readings could be repeated with considerable accuracy. The above are some of the first readings taken, further experience in the use of the instrument led to still more uniform results.

Experimental Determination of Poisson's Ratio.

In these experiments the load on the specimen was in some cases varied from 1 to 5 tons, and in others from 1 to 3 tons. In the former the readings were taken at intervals of 1 ton, and in the latter at intervals of one-half ton. In every case the specimens were loaded several times previous to the readings being taken. In order to insure complete immunity from local strains (due to the method of gripping the specimen)

TABLE I.

Load.	Scale-Readings.			Differences.		
	1st Loading.	2nd Loading.	3rd Loading.	1st Loading.	2nd Loading.	3rd Loading.
1	195.0	201.2	201.8	2.9	2.8	2.9
2	197.9	204.0	204.7	2.8	2.9	2.9
3	200.7	206.9	207.6	2.9	2.9	2.8
4	203.6	209.8	210.4	2.5	2.8	2.8
5	206.1	212.6	213.2	2.7	2.6	2.5
4	203.4	210.0	210.7	2.9	2.6	2.8
3	200.5	207.4	207.9	2.8	2.8	2.8
2	197.7	204.6	205.1	2.9	2.8	2.9
1	194.8	201.8	202.2			
Mean diff. per ton ...				2.80	2.78	2.80

an unusually large distance was allowed between the ends of the specimen and the portion of it under observation.

The instrument used was carefully measured under a reading microscope and found to give a magnification of 4.106. The distance between the needle-points of the tilting mirror, measured in the same way, was .2960 in. The normal distance from the mirrors to the line through the scale parallel to the tilting mirror was measured several times in different ways and the mean value taken as correct.

Under these circumstances the total magnification obtained was about 2800, but the exact value of course varied with the position of the telescope and scale.

Table II. contains the results of experiments on specimens of various materials.

The extensions and lateral contractions were measured simultaneously, an extensometer having been specially arranged so that both instruments could be attached to the specimen at the same time and used throughout the experiment.

The calculated means of the observed tensile and lateral strains for half-ton of load are given in the table, and the values of the modulus of elasticity and Poisson's ratio are obtained from these. The modulus is given in lbs. weight

TABLE II.

Material.	Strains per $\frac{1}{2}$ ton of Load.		Young's Modulus.	Poisson's Ratio.	
	Tensile.	Lateral.			
Mild Steel.	·000,0464	·000,0128	$30\cdot75 \times 10^8$	·276	
	·000,0469	·000,0130	30·46	·277	
	·000,0465	·000,0127	30·70	·273	
Sheffield Spindle Steel.	·000,0488	·000,0135	29·60	·277	
	·000,0505	·000,0142	28·33	·281	
	·000,0488	·000,0132	29·23	·271	
Wrought Iron.	·000,0531	·000,0144	29·08	·271	Dia. = ·761 „ = ·707
	·000,0826	·000,0239	29·80	·289	
	·000,0928	·000,0251	30·73	·270	
Drawn Brass (Muntz Metal).	·000,1047	·000,0335	13·26	·320	
	·000,0978	·000,0345	14·23	·353	
	·000,0960	·000,0337	13·61	·351	
Drawn Copper.	·000,0770	·000,0262	17·61	·340	
	·000,0726	·000,0241	18·70	·332	
	·000,0822	·000,0255	16·88	·310	

per sq. inch. It can be expressed in grammes weight per sq. cm. by multiplying the numbers in the table by 70·31.

The specimens were not annealed, and all the above were approximately one inch in diameter except in the cases noted in the table.

Cast Iron.—Two sets of cast iron specimens were carefully cast in a vertical position. The first was from a mixture of about equal parts of No. 1 Govan, No. 2 Govan, and Heavy Scrap, and the second from 3 parts Redcar Middlesborough, and one part Scrap.

The material was therefore in each case such as might be used for engine and machine castings.

The specimens were brought to a state of ease by continued application and removal of load until permanent set was completely eliminated.

The values given in Table III. are those obtained from the elastic state of the bar, that is, the strains do not include permanent set.

TABLE III.—Cast Iron.

Specimen.	Diameter.	Strains per $\frac{1}{2}$ ton of Load.		Young's Modulus.	Ratio, $\frac{\beta}{\alpha}$.
		Tensile.	Lateral.		
1st Series.					
No. 1	·752	·000,1552	·000,0374	$16\cdot25 \times 10^6$	·241
No. 2	1·025	·000,0904	·000,0206	15·01	·228
No. 3	·752	·000,1556	·000,0420	16·21	·270
2nd Series.					
No. 1	·997	·000,0877	·000,0223	16·36	·254
No. 2	·990	·000,0947	·000,0245	15·37	·259
No. 3	·993	·000,0900	·000,0220	16·07	·244

A long series of observations was made for the purpose of determining whether there was any permanent set in the lateral strain. Readings were taken for loads varying by 0·1 of a ton up to 0·5 ton on a C.I. bar approximately 1 inch in diameter. The readings were in $\frac{1}{40}$ of an inch, and an estimation of $\frac{1}{10}$ of these was made. The total magnification was 2700. The bar had not been previously loaded, and it appeared to be perfectly elastic as regards lateral strain. On increasing the load up to 1·0 ton, however, there appeared to be a small amount of permanent set, and

TABLE IV.

Material.	Value of Poisson's Ratio.		
	Bauschinger.	Stromeyer.	From Table II.
Mild Steel.....	·29	·273 to ·300	·271 to ·281
Wrought Iron ...	·26 to ·31	·279 to ·301	·270 to ·289
Brass Rod.....		·283 to ·357	·320 to ·351
Copper Rod		·325	·310 to ·340
Cast Iron	·16 to ·19 (Tension) ·32 to ·38 (Compression)	·148 to ·269	·228 to ·270

between 1·0 and 1·5 tons the set was very marked. It increased in amount up to 3 tons, which was the limit of load applied.

Table IV. is a comparison of the results of other experimenters with those given in this paper.

In conclusion I must acknowledge my indebtedness to my colleague, Mr. E. L. Watkin, M.A., for his valuable assistance in carrying out these experiments, which were made in the Engineering Laboratory at University College, Bristol.

DISCUSSION.

Prof. EVERETT said he had made determinations of Poisson's ratio by comparing the twist produced in a cylindrical rod by a torque, with the bend produced by an equal torque. The twist was always greater than the bend and the ratio diminished by unity was Poisson's ratio, the material being supposed isotropic. This method was inapplicable to fibrous material: for instance, with wooden rods he had found the twist to be about five times the bend. Mr. Morrow's method had the advantage of being direct, and could be applied to fibrous materials as well as to those which were isotropic. The experiments described in the paper were also on a far larger scale than had heretofore been employed. The mode of attachment of the mirror, which was a point of great importance in the experimental arrangements, seemed to be quite satisfactory.

Dr. CHREE indicated several differences between the author's terminology and that usual in mathematical textbooks. He also pointed out the expediency of explaining the units in which the values obtained for Young's modulus were expressed, and of adding the equivalent values in grammes weight per square centimetre. He referred to the fact that in the discussion of Wertheim's experiments on the value of Poisson's ratio given in Todhunter & Pearson's *History of Elasticity*, it was pointed out that the materials employed seemed hardly to be isotropic, and that if this were the case the experiments could not be regarded as decisive. He also pointed out that more than one experimentalist had

obtained results for cast iron which indicated that whilst it might be elastic, in the sense that strain disappeared on the removal of stress, it did not obey Hooke's law; and that if this were the case the application of the term Poisson's ratio to the material was not in accordance with mathematical usage. He dwelt on the importance of corroborative evidence in all cases that the materials dealt with were really elastic and isotropic.

Dr. R. T. GLAZEBROOK said it was difficult to assure oneself that the materials used were isotropic. The author's figures showed different values for Young's modulus for different specimens of the same material. He referred to the importance of stating the units employed, and to the fact that experiments had been carried out on an engineering scale.

Mr. MORROW said the units used in the paper were lbs. and inches. His instrument was not dependent on the ordinary theory of elasticity. He pointed out that the values obtained for the Young's modulus of cast iron depended upon the method employed, elongation and bending giving different results for the same specimen.

LIX. *Single-Piece Lenses.* By THOMAS H. BLAKESLEY*.

[Plate XII.]

IF the definition of a lens is taken to be a mass of refracting material contained between two spherical surfaces, then any lens is completely determined in shape and size when the radii of curvature of its two end surfaces and the distance between them are given. The dimensions which are measured at right angles to the axis do not affect the focussing properties of the lens except in the second order of small quantities, and such dimensions are not brought into consideration in what follows.

If the kind of material in reference to any kind of light, as defined by the index of refraction of the material for that

* Read May 22, 1903.