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“The Relation of the Constants of the Elongation Equation to Contraction of Area.”

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THE record of a series of ordinary tension-tests of metals usually contains, in addition to the items referring to maximum tensile stress, the ascertained magnitudes of the ultimate elongation on a certain constant length between gauge-points, and of the contraction of the cross-sectional area. Frequently, however, circumstances arise in which it is desirable to estimate the elongation on some different length between gauge-points, associated perhaps with changed dimensions of cross-section of the specimens. In a Paper¹ recently read before the Institution, Professor Unwin has clearly shown that a knowledge of the elongation equation enables such an estimate to be made with considerable accuracy, but the direct method employed in that Paper for determining the constants of the equation in question will not suffice in cases where all the known elongations are measured on one and the same length between gauge-points.

The problem presented, therefore, is to determine the elongation equation, having been given the elongation on a specified constant length, together with the contraction of area. About 2 years ago the Author endeavoured to solve this problem and applied the results obtained, with a measure of success, to the particular circumstances which had called for its investigation.

Of the total contraction of area, part is the result of general plastic extension, and a final part is connected with more or less local extension. Dividing the total plastic extension into that part which is approximately uniformly distributed and that part which is not—briefly termed local extension—it would appear to be

¹ “Tensile Tests of Mild Steel; and the Relation of Elongation to the Size of the Test-bar.” Minutes of Proceedings Inst. C.E., vol. clv. p. 170.

possible to find some relation between the contraction of area and the quantities termed general and local extensions respectively.

Fig. 1.

Referring to Fig. 1, let s_1 denote the average local strain and s_2 the general strain; and let A_1 , A_2 and A_0 represent respectively the contracted area, the area in the region of general extension, and the original area. In Fig. 2, A is the area of a normal section lying in the region of local extension, distant x from A_2 , and it may be assumed that

$$A = A_2 - ax^n \quad \dots (1)$$

where a and n are quantities to be determined. If $2j$ is the length throughout which local extension obtains, the volume of this part may be expressed by

$$V = 2 \int_0^j A \delta x = 2 \left\{ A_2 j - \frac{a}{n+1} j^{n+1} \right\}.$$

But

$$A_1 = A_2 - aj^n;$$

therefore

$$V = 2j \left\{ A_2 - \frac{A_2 - A_1}{n+1} \right\} \quad \dots (2)$$

Hence the average area in the contraction is

$$A_{av} = \frac{V}{2j} = A_2 - \frac{A_2 - A_1}{n+1} \quad \dots (3)$$

If h be the original length of bar which becomes the region of local extension, the original volume is $h A_0$ and the volume after strain is $h (1 + s_1) A_{av}$. Equating these volumes,

$$\begin{aligned} h A_0 &= h (1 + s_1) \left\{ A_2 - \frac{A_2 - A_1}{n+1} \right\} \\ &= h (1 + s_1) \left\{ \frac{n}{n+1} A_2 + \frac{1}{n+1} A_1 \right\} \quad \dots (4) \end{aligned}$$

If c is the fractional contraction of area,

$$c = \frac{A_0 - A_1}{A_0}; \quad \dots \quad (5)$$

and also, from the principle of constant volume,

$$A_0 \times l = A_2 (1 + s_2). \quad \dots \quad (6)$$

Hence

$$A_1 = A_0 (1 - c)$$

$$A_2 = A_0 \frac{1}{1 + s_2}.$$

Substituting in equation (4) —

$$1 + s_1 = \frac{1}{\frac{n}{n+1} \cdot \frac{1}{1+s_2} + \frac{1-c}{n+1}} \quad \dots \quad (7)$$

The Author has tried various values of n as applied to mild steel, but up to the present he has not found any marked advantage to follow on deviation from the simplest possible value, namely $n = 1$, which corresponds with the assumption of a parabolic outline. Adopting $n = 1$, equation (7) becomes

$$1 + s_1 = \frac{2(1 + s_2)}{1 + (1 - c)(1 + s_2)}$$

or

$$s_1 = \frac{s_2 + c(1 + s_2)}{2 + s_2 - c(1 + s_2)} \quad \dots \quad (8)$$

Taking l as the length between gauge-points and A as the sectional area of the specimen, the original length of the contracted region may be denoted by $m\sqrt{A}$, where m is some number or function to be determined; and hence if e is the fractional total elongation or strain the elongation equation may be expressed thus:—

$$le = (l - m\sqrt{A})s_2 + m\sqrt{A}s_1$$

or

$$e = s_2 + m(s_1 - s_2) \frac{\sqrt{A}}{l} \quad \dots \quad (9)$$

It may be remarked that expressed as fractions e , s_2 and $m(s_1 - s_2)$ are respectively identical with Professor Unwin's e , b and c .

Substituting in equation (9) the value of s_1 given by equation (8), and for brevity denoting $m \frac{\sqrt{A}}{l}$ by q ,

$$(1 - c)(1 - q)s_2^2 + \{2 - c + q(2c - 1) - e(1 - c)\}s_2 = e(2 - c) - qc \quad \dots \quad (10)$$

Assuming m to be determinate and knowing e and c in any given instance, the value of s_2 may be calculated, which, in turn, fixes the value of s_1 by equation (8). Thus the constants of the elongation equation become known.

Restricting the purview to mild steel and the harder steels used for axles and tires, and regarding the quantity m as a mere number, the lowest and highest values which the Author has obtained are about 0.9 and 2.0 respectively. Certain functions of e and c have been substituted for m , but, although the results have led in some cases to a marked improvement in the constancy of value of m on a limited range of observations, the Author has not by that means hitherto secured any great advantage over a numerical average value for m when the expressions were applied to all the experiments which he has made and collated.

In Professor Unwin's Paper an exceedingly interesting series of tests on axle- and tire-steel, performed by Mr. F. C. Fairholme, Assoc. M. Inst. C.E., for Messrs. C. Cammell and Company of Sheffield, is set forth and analyzed. The figures for the contraction of area are not given in the Paper, but by the courtesy of Mr. Fairholme, Professor Unwin, and the Secretary of the Standards Committee (Mr. Leslie Robertson, M. Inst. C.E.), the Author has had access to them. The tests were made in duplicate on successive pairs of sets of specimens, each pair of sets being formed from and representing one and the same kind of material; and there were as many pairs of sets as there were different kinds of material. Thus the first set of a pair comprised

Two specimens of 2 inches gauge-length

"	"	"	4	"	"
"	"	"	6	"	"
"	"	"	8	"	"

all 0.5 square inch in sectional area; and the second set comprised

Two specimens of 2 inches gauge-length

"	"	"	4	"	"
"	"	"	6	"	"

all 0.25 square inch in sectional area.

For the purposes of an example it will be assumed that the elongations and contractions of the specimens of 2 inches gauge-length are given, and that it is required to estimate the constants of the elongation equation. Then, having determined the elongation equation merely from the particulars referring to the specimens of 2 inches gauge-length, the elongations on the 4 inches,

6 inches, and 8 inches gauge-lengths will be calculated and compared with the experimental results. The value of m adopted is 1.37, which does not materially differ from the average for all the experiments which the Author has dealt with.

The results of this investigation are given in the Appendix. Tables I. and II. refer to the first part of the process above-mentioned, and to facilitate comparison the values of the constants of the elongation equation deduced by Professor Unwin from the whole range of each set of experiments are interpolated in the Tables. The second part of the investigation, namely, the comparison of the calculated with the observed elongations, is shown in Tables III. and IV.

The Paper is accompanied by diagrams, from which the Figures in the text have been prepared, and by the following Appendix.

[APPENDIX.

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TABLE I.—AXLES.

Mark.	Average e 2 Inches Gauge- Length observed.	Average c 2 Inches Gauge- Length observed.	s_2 Calculated from Eq. (10).	Professor Unwin's Value of $b = s_2$	s_1 Calculated from Eq. (8).	Coefficient of $\frac{\sqrt{A}}{l}$ $= m(s_1 - s_2)$	Coefficient of $\frac{\sqrt{A}}{l}$ Professor Unwin.
A ₁	0·365	0·518	0·211	0·215	0·530	0·437	0·420
A ₂	0·335	0·524	0·224	0·211	0·546	0·441	0·500
B ₁	0·320	0·526	0·157	0·181	0·495	0·463	0·390
B ₂	0·300	0·536	0·182	0·193	0·526	0·471	0·420
C ₁	0·330	0·508	0·178	0·220	0·492	0·430	0·310
C ₂	0·292	0·536	0·174	0·215	0·520	0·474	0·310

TABLE II.—TIRES.

Mark.	Average e 2 Inches Gauge- Length observed.	Average c 2 Inches Gauge- Length observed.	s_2 Calculated from Eq. (10).	Professor Unwin's Value of $b = s_2$	s_1 Calculated from Eq. (8).	Coefficient of $\frac{\sqrt{A}}{l}$ $= m(s_1 - s_2)$	Coefficient of $\frac{\sqrt{A}}{l}$ Professor Unwin.
A ₂	0·235	0·392	0·168	0·150	0·366	0·271	0·340
B ₁	0·195	0·270	0·145	0·118	0·247	0·140	0·210
B ₂	0·190	0·280	0·154	0·126	0·260	0·145	0·250
C ₁	0·225	0·378	0·129	0·140	0·327	0·271	0·240
C ₂	0·200	0·392	0·129	0·120	0·338	0·286	0·320

TABLE III.—AXLES.

Mark.	—	Elongation on			
		2 Inches.	4 Inches.	6 Inches.	8 Inches.
A ₁	{ Observed . . .	0·365	0·305	0·264	0·235
	{ Calculated . . .	0·365	0·288	0·262	0·251
A ₂	{ Observed . . .	0·335	0·269	0·257	..
	{ Calculated . . .	0·335	0·279	0·261	..
B ₁	{ Observed . . .	0·320	0·250	0·228	0·215
	{ Calculated . . .	0·320	0·238	0·212	0·198
B ₂	{ Observed . . .	0·300	0·260	0·212	..
	{ Calculated . . .	0·300	0·241	0·221	..
C ₁	{ Observed . . .	0·330	0·277	0·267	0·234
	{ Calculated . . .	0·330	0·254	0·229	0·216
C ₂	{ Observed . . .	0·292	0·258	0·236	..
	{ Calculated . . .	0·292	0·234	0·214	..

TABLE IV.—TIRES.

Mark.	—	Elongation on			
		2 Inches.	4 Inches.	6 Inches.	8 Inches.
A ₂	{ Observed . . .	0·235	0·195	0·175	..
	{ Calculated . . .	0·235	0·202	0·190	..
B ₁	{ Observed . . .	0·195	0·165	0·137	0·135
	{ Calculated . . .	0·195	0·170	0·161	0·157
B ₂	{ Observed . . .	0·190	0·155	0·145	..
	{ Calculated . . .	0·190	0·172	0·166	..
C ₁	{ Observed . . .	0·225	0·180	0·177	0·155
	{ Calculated . . .	0·225	0·177	0·161	0·153
C ₂	{ Observed . . .	0·200	0·160	0·147	..
	{ Calculated . . .	0·200	0·165	0·153	..