The Homographic Treatment of the Symmetrical Optical Instrument

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III. *The Homographic Treatment of the Symmetrical Optical Instrument.* By
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(COMMUNICATED BY DR. D. OWEN.)

ABSTRACT.

This Paper gives a homographic treatment of the perfect, symmetrical optical instrument; discusses its focal planes, principal planes and nodal points; and shows that these latter are sufficient to define uniquely the optical properties of the instrument.

THE object of this Paper is to replace the rather cumbrous algebra of Abbe's* theory of the symmetrical optical instrument by the more elegant methods of modern geometry.

DEFINITION OF THE INSTRUMENT.

The perfect, symmetrical, optical instrument is defined by two conditions—the condition for stigmatism, and the condition for axial symmetry.

1. *Stigmatism.*—The effect of the instrument must be to form a point image of a point object. Now, to any ray through the point object, there corresponds a ray through the point image. Hence, rays coplanar in the object space correspond to rays coplanar in the image space. Hence the instrument sets up a homographic, or (1, 1) correspondence between the points, lines and planes of the object and image spaces.†

2. *Axial Symmetry.*—Associated with the instrument there must be a certain line—the optic axis—such that to rays in a plane π containing the axis, there correspond rays in the same plane; and to lines perpendicular to the axis, there correspond lines also perpendicular to the axis. Hence to points on the axis there correspond points on the axis, so that the axis is a self-corresponding or "double" line. Also since all lines perpendicular to the axis in the plane π meet in the same point W at infinity, W is a double point. Now the instrument sets up a (1, 1) correspondence between the points or the lines of the plane π . Therefore the plane π contains three double points, the vertices of a triangle whose sides are three double lines. Hence the other two double points lie on the axis and are either both real or both imaginary.

FOCAL PLANES.

The line at infinity, w , of any plane perpendicular to the axis contains all the double points, W , and is therefore itself a double line. Now the focal planes ϕ , ϕ' , of the image and object space are defined as the planes corresponding to the plane at infinity w regarded as belonging to the object or image space respectively. But the double line w lies on ω . Therefore it also lies on ϕ , ϕ' . Hence the focal planes ϕ , ϕ' are perpendicular to the axis.

* Drude: "The Theory of Optics," pp. 14-28 (1902). v. Rohr: "The Formation of Images in Optical Instruments," pp. 83-124 (1920). Gapski: "Theorie des Optischen Instrumente nach Abbe" (1894).

† Jessopp: "The Line Complex."

PRINCIPAL PLANES AND NODAL POINTS.

These are introduced so as to exhibit their duality.

Principal Planes.

Consider a pair of corresponding planes η , η' whose intersection is the line w .

There is a (1, 1) correspondence between the lines on η , η' and the common line w is self-corresponding.

Hence corresponding lines on η , η' are in perspective.

When the centre of perspective is at infinity η , η' are principal planes.

Then the join of corresponding points of η , η' meets the axis at infinity.

If μ , μ' be another pair of principal planes, any line l meeting the axis at infinity, is met by η , η' , μ , μ' in two pairs of corresponding points. Hence l is a double line.

But this is only true for telescopic systems of unit magnifying power. Hence in general there is only one pair of principal planes or nodal points.

Nodal Points.

Consider a pair of corresponding points N , N' whose join is the axis.

There is a (1, 1) correspondence between the lines through N , N' , and the common line the axis is self-corresponding.

Hence corresponding lines through N , N' are in perspective.

When the axis of perspective is at infinity N , N' are nodal points.

Then the intersection of corresponding lines through N , N' is at infinity.

If P , P' is another pair of nodal points any point L at infinity is joined to N , N' , P , P' by two pairs of corresponding lines. Hence L is a double point.

CONSTRUCTION OF IMAGE.

Let A , A' be corresponding points ; and let AN , $A'N'$ meet φ , φ' in F , F' , and η , η' in H , H' .

If Ω' is the point at infinity on $A'N'$, since A , A' ; N , N' ; F , Ω' ; H , H' are pairs of corresponding points, $A'N'$ is parallel to AN and $[ANFH] = [A'N'\Omega'H]$. (using the notation of Mobius for cross-ratios). Hence, φ , φ' ; η , η' , N , N' suffice to determine A' corresponding to A .

MIRROR SYSTEMS.

The distinction between mirror systems and lens systems is that whereas in a lens system, a collineation is effected between the points, lines and planes of two different spaces ; in a mirror system, the collineation is effected between the points, lines and planes of the same space.

Hence in a mirror system, the corresponding pairs of points on the axis form an involution, of which the pole of the mirror P is evidently one double point. Hence the other double point Q is also real, and the centre of the involution (*i.e.*, the point corresponding to the point at infinity, or the focus) is midway between P and Q ; and any two corresponding points are harmonically separated by P and Q . The principal points obviously coincide at P , and the nodal points at Q .

DISCUSSION.

Mr. T. SMITH congratulated the author on the brevity with which his method covered ground that is usually treated at great length, especially in German textbooks. The importance of the perfect instrument lies not in its realisation in practice, which is unattainable except for the plane mirror, but in that it is the ideal to which actual instruments must approximate. In practice perfection is attainable with respect to a single plane object, but perfection with regard to more than one plane implies complete perfection. Clerk Maxwell employed a method independent of constructional details and applied it to the practical case where the principal and nodal points coincide; this case might also be of interest to the author. Clerk Maxwell further introduced one physical property, *viz.*, the ratio of the velocities of light in the object and image spaces, and deduced hence the properties of the focal lengths. In practical cases there is a (1, 1) correspondence between rays, but not between points, and this aspect of the matter has been elegantly treated by the method of characteristic functions, notably by Hamilton.

Mr. F. J. W. WHIPPLE said that the Paper was extremely concise, and might be amplified with advantage. In presenting it the author had stated that correspondence between points implies correspondence between planes. Would he give the proof of this theorem?

The AUTHOR, in reply to Mr. Smith, said it had occurred to him that the consequences of a (1, 1) correspondence between rays might be deduced by means of the geometry of the line complex. The proof asked for by Mr. Whipple was as follows. Let A, B, C, D be four coplanar points in the object space and A', B', C', D' four corresponding points in the image space. Then the latter four points are also coplanar. For let AD meet BC in X . Since X lies on both AD and BC , the corresponding point X' must lie on both $A'D'$ and $B'C'$. Hence these must be coplanar.

ADDENDUM BY THE AUTHOR.

The following example bears upon Mr. T. Smith's remark as to focal lengths:—

Let the axis of the instrument meet the focal planes ϕ, ϕ' in F, F' ; and the principal planes η, η' in H, H' .

Then the geometrical expression of the fact that the refractive indices of the material of those parts of the object and image spaces outside the instrument, are the same, is that H, H' respectively coincide with the nodal points N, N' .

Let K be any point on ϕ . Draw KLM parallel to the axis to meet η, η' in L, M .

Now F' corresponds to the point at infinity on the axis, which is the same as the point at infinity on KLM .

Hence the ray KL passes through F' , and since L, M are corresponding points, it also passes through M .

Hence the ray KL corresponds to the ray MF' .

Hence the point K corresponds to the point at infinity on MF' .

Join KH , *i.e.*, KN . Then K also lies at infinity on a line through $H' (N')$ parallel to $KH (N)$. Therefore KH, MF' are parallel.

But $KF = MH'$.

Therefore $FH = H'F'$, *i.e.*, the focal lengths are numerically equal.