Mortality Experience of Assured Lives and Annuitants in France.
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THE publication of the tables of mortality based upon the experience, collected by the Committee representing the four principal insurance companies of France, puts us in possession of an extensive and valuable body of statistics bearing upon the mortality of assured lives and annuitants in that country, and at the same time places the Actuarial profession under a considerable obligation to the Committee of French Actuaries to whom we are indebted for the collection and publication of the data and the resulting monetary tables. As in this country, the life assurance companies on the Continent had for a long time, necessarily, to depend upon statistics of mortality based upon observations of the population generally, which, whatever the skill with which they were prepared, were not only affected by the uncertainty attaching to all such statistics, but were not in any case specially suited to represent the probable mortality, either of assured lives or annuitants. The earliest of these tables, to which any value could be attached, are those of Deparcieux and Duvillard, corresponding in point of date, approximately, to the Northampton and Carlisle Tables in England, and it was not until 1860 that the three companies, the Compagnie d'Assurances Generales, Union, and Nationale, published their annuity experience, nor until 1874 that the first of these companies published a table representing the mortality experience

of their assurants. These latter tables, however, were based upon a relatively small number of facts, and in November 1876, a committee of six (subsequently reduced to four) French Companies determined to construct a mortality table representing their joint annuity experience, and that of a seventh company, "le Monde", this work being entrusted to the Actuaries of the six companies under the presidency of M. de Kertanguy, Actuary to the "Generale." Eleven years later, the same committee decided upon the construction of a second table, based upon the experience of assured lives. The data in the case of the annuitants extended in its final form, from 1819 down to the close of the year 1889, and in the case of the assurants, from 1819 to 1887. The general results of the experience for both annuitants and assured lives, were published at the time of the Great Exhibition of 1889, together with an adjustment of the mortality tables by Woolhouse's formula. This adjustment was subsequently replaced by an elaborate graduation by Mr. Makeham's formula, upon which all the subsequent monetary tables have been based.

EXTENT OF THE DATA.

The extent of the data, both as to annuitants and assured lives, is very considerable, as will be seen from the following figures:

Table	Lives	Years of Life	Deaths
Annuitants, Male ...	} 67,247 {	244,292	16,410
" Female ...		391,617	20,506
Assurants, Male ...	} 229,143 {	1,408,398	18,926
" Female ...		382,258	3,691

In respect to the assured lives, therefore, the experience is rather more extensive than that of the Institute, while the annuity experience is somewhat larger than that of the Government Annuity Experience in this country, published in 1884.

MODE OF COLLECTION.

Without discussing all the details of the methods employed in the collection and tabulation of the experience, it will be useful to indicate generally the lines followed, which were practically identical in respect of both tables. A separate tabulation was made in both cases for male and female lives, and in the case of

assured lives for the various classes of assurances most usually granted by the French Companies, although the figures for these special groups have not yet been separately published. In both cases the experience has been based upon lives, and not policies.

In computing the ages of the exposed to risk, it was assumed that all persons born between 1 July in any year and the following 30 June, were born on the intervening 1 January, and that, similarly, all lives entering under observation between 1 July and the following 30 June entered on the intervening 1 January. This latter assumption, however, appears to have been modified in the case of entrants, between the 1 July and 31 December, who were assumed to be at risk for a quarter of a year in the calendar year of entry. This course would seem to be inconsistent with the original assumption, and to have the effect of over-estimating the exposed to risk throughout the table, by approximately one-eighth of the entrants at each age, thus somewhat under-estimating the average rate of mortality at the insuring ages.

In each case the data was arranged in a form permitting of the calculation of "select" tables for each age at entry. The tables at present published, however, consist simply of the aggregate mortality tables for all ages at entry combined, being similar in this respect to the H^M and H^F Tables.

Although the exposed to risk and deaths are shown separately for males and females, the graduated tables upon which the monetary results are based refer only to both sexes in combination.

With respect to the assured lives, in which females represent about one-fifth of the whole experience, this step is perhaps less open to objection than in the case of the annuitants. Assuming that it is usual to assure female lives on precisely the same terms as male lives, and that the proportion of the former to the latter among new entrants is, on the whole, fairly constant, it may be argued with much plausibility that the proper table to employ for the calculation of premiums or reserves is one based upon the experience of both sexes. The principal objection, I apprehend, that can be urged against this proposition is that, owing to the differing incidence of the mortality at the various periods of life among female as compared with male lives, the effect of such a step would probably be to lower somewhat both the net annual premiums and the reserves, and that therefore it is an error on the safe side to exclude the female lives from the

experience and to base all calculations upon the male mortality table only.

Whatever may be said, however, as to the utility of an H^{MF} table in relation to assured lives, among whom it is common, although not universal, to treat the sexes upon the same terms, there can be no doubt, I think, as to the desirability of distinguishing between the sexes in the case of annuitants, among whom the factor of sex appears so powerfully to influence the mortality.

With the view of testing how far the final tables, which represent fairly the aggregate mortality of the two sexes, depart from the original facts when the sexes are treated separately, I have computed the expected deaths of both males and females at various ages amongst the annuitants and assured lives, respectively, the figures being given in Tables 5 and 6. From these it will be seen that while the deaths among the male annuitants exceeded the expected number by about 15 per-cent, those among the female annuitants fell short by nearly 10 per-cent, and that, therefore, annuities granted upon the lives of females, upon the basis of the mixed table, must result, in the long run, in considerable loss to the companies; leaving out of account, of course, any counter-vailing profit from interest.

METHOD OF GRADUATION.

As already stated, both tables were ultimately graduated upon the basis of Mr. Makeham's formula. The constants were determined in each case by a very elaborate process. In the first instance, approximate values of the constants were obtained by the so-called "aggregate" method described in the paper by Mr. King and myself in vol. xxii, *J.I.A.* Corrections to these constants were then obtained by the method of "least squares", an equation of condition being formed at each age between the observed and adjusted value of $\text{Log } p_x$ and the corrections to the three constants in the formula. Thus, starting from the initial equation $\log p_x = a + \beta c^x$, we have $\log p_x = a_0 + \beta_0 c_0^x + \delta a_0 + c_0^x \delta \beta_0 + \beta_0 x c_0^{x-1} \delta c_0$, where p_x is the observed value of the probability of surviving a year at age x , a_0 , β_0 , &c., are the first approximations to the value of the three Makeham constants and δa_0 , &c., the corrections to such first approximations, to be obtained by the method of least squares.

These equations were weighted in proportion to the square

root of the "number of observations" at the given age. It is not clear whether by the "number of observations" is meant the number of lives exposed to risk, or the number of observed deaths. The latter would, of course, be the more correct proceeding, as it has been variously shown that the mean error in the value of q_x and hence, generally, in the value of $\log p$ in any series of observations is nearly proportionate to $\frac{\sqrt{nq}}{n}$ where n is the number of exposures, and nq the adjusted or expected deaths, which, for this purpose, may be taken as equal to the observed deaths.

Improved values of the constants having been thus obtained, the process was again repeated, and the finally adopted values were brought out as under, using the customary symbols for the constants.

Constant	Annuity	Assured Lives	Constant	Annuity	Assured Lives
$c \dots$	1.1001136	1.0916817	$\log c \dots$	.0414376	.0380960
$g \dots$	0.9993868	0.9984400	$\log g \dots$	-.0002664	-.0006780
$s \dots$	0.9944272	0.9949930	$\log s \dots$	-.0024270	-.0021800

It will be seen that the value of $\log c$ is somewhat below the average value found for this quantity, in the case of the Assured Lives table, and somewhat above that value in the case of the Annuity table.

It is to be remarked that Makeham's formula was not applied below age 25 in either table, and that, up to that age, the two tables were made identical, on the ground that on the one hand the observations of annuity at these ages were very few in number, and on the other hand that the lives assured at these ages relating principally to contingent and deferred assurances, might be considered to be of the same class as annuity.

Below age 25 the adjusted rates of mortality were obtained on the assumption that q_x could be represented for this portion of the table as an algebraical function of the sixth degree,

$$q_x = m_0 + m_1x + m_2x^2 + m_3x^3 + m_4x^4 + m_5x^5 + m_6x^6.$$

Seven equations were then obtained to determine the seven constants by equating the above expression to the values q_{26} , q_{25} , $(q_0 + q_1)$, $(q_2 + \dots + q_7)$, $(q_8 + \dots + q_{12})$, $(q_{13} + \dots + q_{18})$, and $(q_{19} + \dots + q_{24})$.

In determining the constants, the original facts for ages 23 to 70 only in the case of assurers, and from 40 to 89 only in the case of annuitants, were used. It is not quite clear why the data beyond age 70 should not have been utilized in determining the constants for the "Assured Lives" table; as a matter of fact, however, the graduated table represents the observed rates of mortality above age 70 fairly well, the adjusted number of deaths being 1,215, as against 1,200 observed. At ages over 82, it is true these numbers become 102 and 80 respectively, but such a deviation at these ages in a table relating to Assured Lives, cannot be considered as of moment. Of greater consequence, perhaps, is the fact that, at ages over 89, the data for which were neglected in determining the constants for the annuity table, the adjusted deaths were 1,101 to 899 observed, a feature, no doubt, due to the unusually large value of the constant c used in the formula.

It may be as well here to refer to the use of the method of "least squares" in this connection. The method is, of course, not so strictly applicable to the construction of a mortality table as in the determination of physical constants, as the positive and negative deviations from the mean results are not equal in amount, nor equally probable. The assumption, however, that they are so, is a very convenient one, and sufficiently near the truth to justify its employment in practice.

In the use of the method, it is clear that two alternative assumptions may be made which lead to different modes of procedure.

In the first place, we may assume that the mortality during adult life, among a group of persons similarly constituted to those under observation, but so numerous as to practically eliminate accidental irregularities, would strictly follow Makeham's law. Upon this assumption, the differences between the observed rates of mortality and those given by the formula, are merely accidental differences, due to the limited number of facts observed at individual ages, and the constants should be so determined as to make the sum of the squares of these deviations a minimum, the equations of condition at each age being multiplied by the square root of the number of deaths, so as to make the probability of a deviation of given amount in such a function as $\log p_x$ equally likely at all ages.

Secondly, it may, in the alternative, be assumed that the true law of mortality only approximately follows that given by

Mr. Makeham's formula. The deviations are then of two kinds; on the one hand the usual accidental irregularities, and on the other real divergencies between the law of the series when divested of these irregularities and the law represented by the formula. From this point of view, the true law of the series should be approximately ascertained by removing the accidental irregularities by some suitable method of graduation, and such constants should be adopted for the formula as will make the monetary values (for example, the annuities) based thereon correspond as nearly as possible to those deduced from the preliminary graduation, special weight being given to those ages at which such correspondence is of greatest importance in practice.

With respect to the use of Mr. Makeham's formula at all for purposes of graduation, it has to be borne in mind that the object of an adjusted mortality table cannot, strictly speaking, be to exhibit the results of any past experience. These are already given in the unadjusted table, and, of course, are not capable of alteration. The object in general of graduation is to produce a table which it is believed will be applicable to a similar body of lives in the future. As it is quite impossible that any two bodies of assured lives or annuitants can precisely resemble one another, when separated by at least a generation in point of time, and subject to all the changes of circumstance and condition involved, it is clearly out of the question to expect that the rates of mortality observed in the past will be reproduced with precision in the future, and it is, therefore, sufficient, for all practical purposes, if the adjusted table, upon which future results are to be estimated, accords, within a reasonable degree of approximation, with the original unadjusted results, more especially if no important systematic error can be pointed out in the formula. The applicability, therefore, of Mr. Makeham's formula can only be determined by trial in each individual case, and where it is found, as is the case, for example, in the H^M Table, that the monetary results based thereon are practically identical with those obtained by other methods of adjustment involving no assumption as to a law of mortality, there need be no hesitation in employing the formula in question, having regard to the immense advantage it affords, in many directions, in calculation of life contingencies.

In the tables we are now considering, the mortality of the assured lives (both sexes) is very well represented by the formula,

the differences between the actual and adjusted number of deaths, as shown in Table 5, not being greater than should be expected, and showing no tendency to a systematic departure of the graduated table from the original facts. In the Annuity Table, however, the agreement is not so satisfactory, as is, in fact, pointed out by its authors. It will be seen that from age 25 to 54 the actual deaths are largely in excess, while from 55-69 they are more than 6 per-cent below the adjusted numbers, being again in excess during the following 15 years of age, and about 8 per-cent below (on nearly 4,000 deaths) after age 85. The reasons for this want of agreement, which is, to a considerable extent, noticeable in other attempts to adjust the rates of mortality of annuitants by Makeham's formula, are, in the first place, that the mortality is disturbed for so large a portion of the table by new entrants, and, in the second place, that the male annuitants, in particular, under observation at the younger ages, are, on the whole, very inferior, in point of vitality, to assured lives of the same age, as may be at once seen by comparing the rates of mortality of male annuitants and assurers in Table 7.

Without stopping to consider particularly the cause of this phenomenon (the effect of which upon the character of the mortality table may, however, be fairly represented by employing Mr. Makeham's further development of Gompertz's formula) the question arises as to how far such a feature should be retained in a graduated mortality table intended to be used as a basis for the grant of annuities. The method employed by the French Actuaries in determining the constants for the annuitants' mortality table, in which the rate of mortality at age 25 was made identical with that for the same age in the table of assured lives, and the very limited experience between ages 25 and 40 was discarded, has had the effect of removing this feature of a heavy annuitant mortality at the younger ages, and keeping the rates throughout below those for assured lives of similar age. It appears to me that this is a perfectly sound proceeding, and that it is not easy to justify the use of rates of mortality, in a table upon which a Company grants annuities, higher than the rates it assumes in granting assurances.

The divergence in this respect between the Government Annuitants (male) and the H^M "Select" Tables, which are the Standard Tables for annuities and assurances in this country is, of

course, well known. It will be seen from the following values of annuities at 3 per-cent, according to the two tables, that up to age 30 it is equivalent to a difference of about five years in the age:

Age	Government Annuitants (Male) Select	H ^x "Select" (Sprague)	Excess of H ^x over Government Annuitants
20	20·560	21·342	·782
25	19·600	20·667	1·067
30	18·588	19·671	1·083
35	17·516	18·491	·975
40	16·376	17·132	·756
45	15·152	15·616	·464
50	13·813	13·998	·185

The lives upon whom annuities are granted at ages below, say 40–45, clearly form a mixed class, and include, from various causes which need not here be discussed, a large proportion of under-average lives, who, if proposed for assurance, would be accepted only at an extra premium. As, however, many annuitants at these ages will be quite equal, in point of vitality, to the average of assured lives, and as, moreover, a Company may, at any time, be called upon to grant both an annuity and an assurance upon the same life, it would, I consider, be desirable in practice boldly to substitute for the observed rates of mortality in the annuitant experience the rates representing the mortality of assured lives wherever the latter rates are the lower.

The value of Makeham's formula as an instrument for the adjustment of mortality tables must depend largely upon the facility with which it can be adapted to "Select" tables. That this is theoretically possible I have shown elsewhere.* This is, perhaps, scarcely the place to deal with the practical application of the theory, but the following result of a first rough attempt to represent Dr. Sprague's Select tables may be of interest:

Taking the value of

$$\mu_{[x]+t} = \{A + Hx + Bc^x\} - \phi(t)(A + Bc^x),$$

where the terms in the large bracket represent approximately the

* See *J.I.A.*, vol. xxxi, p. 359.

mortality of the $H^{M(5)}$ Table, and the value of $\phi(t)$ is taken arbitrarily as follows:

$t=0$		$\phi(t) = \frac{35}{70}$
1		$\frac{20}{70}$
2		$\frac{10}{70}$
3		$\frac{4}{70}$
4		$\frac{1}{70}$
5 and upwards	.	0

the resulting mortality table yields the following net annual premiums at 4 per-cent:

Age	Sprague's Select	Makeham's Graduation	Difference	
			+	-
20	1·391	1·370	...	·021
25	1·509	1·516	·007	...
30	1·714	1·719	·005	...
35	1·992	1·993	·001	...
40	2·361	2·362	·001	...
45	2·851	2·854	·003	...
50	3·488	3·510	·022	...
55	4·358	4·388	·030	...
60	5·541	5·561	·020	...
65	7·138	7·138	...	·000

The nature of these differences would indicate that the graduated results might easily be brought still closer to the "Select" values, but it will be conceded that the agreement is close enough for practical purposes.

RESULTS OF THE EXPERIENCE.

The main interest in the new table will, I apprehend, centre in a comparison of the results obtained in respect of both classes of lives with those obtained in this country. In the absence of select tables, which, however, it is to be hoped that the Committee of French Actuaries will see their way to publishing, it is only possible to make a comparison with aggregate tables, such

as the H^M and H^F , and with similar tables for the Government Annuitants. For this purpose, adjusted tables not having been prepared for the separate sexes, I have computed the mortality tables which follow by means of the unadjusted rates of mortality.

The number of annuitants observed below age 40 is too small to make the results of any value, but beyond that age, and generally throughout life so far as the assured lives are concerned, there is a sufficiency of data to bring out tables regular enough for purposes of comparison.

The following figures, extracted from the more complete tables following this paper, give a comparison of the expectation of life for each sex, in the case of the assured lives, with the H^M and H^F tables, and, in the case of annuities, with the Government Annuity Experiences for all ages at entry.

Expectation of Life of Assured Lives and Annuitants in France (unadjusted) as compared with the Institute of Actuaries and Government Annuity Experience.

Age	ASSURED LIVES				ANNUITANTS				Age
	Male		Female		Male		Female		
	French Experience	H ^M	French Experience	H ^F	French Experience	English Government Annuity	French Experience	English Government Annuity	
20	41.60	42.06	43.54	40.82	...	.	.	..	20
25	38.04	38.41	40.30	37.41	..	..	..	..	25
30	34.13	34.68	36.65	34.50	...	...	...	..	30
35	30.22	31.02	33.06	31.45	...	...	...	...	35
40	26.50	27.40	29.37	28.25	26.86	..	30.36	..	40
45	22.88	23.79	25.63	24.99	23.99	..	27.13	...	45
50	19.36	20.31	21.95	21.62	20.61	20.51	23.49	23.27	50
55	16.09	16.96	18.50	18.19	17.50	17.57	19.89	19.66	55
60	13.02	13.83	14.90	14.85	14.34	14.61	16.25	16.26	60
65	10.45	11.01	11.96	11.77	11.46	11.57	12.90	12.98	65
70	8.04	8.50	9.21	9.08	8.80	8.95	9.90	10.00	70
75	6.05	6.38	7.25	6.93	6.67	6.84	7.40	7.45	75
80	4.69	4.72	6.26	5.45	4.89	5.05	5.40	5.46	80

Dealing first of all with the assured male lives, it will be seen that the average duration of life is, throughout, below that of the H^M Table, the difference between the two tables running fairly regularly. After reaching a minimum about age 25 of .37 of a year, the difference rises to a maximum at age 50 of nearly one year, and thence diminishes almost uniformly to the end of life.

It is clear that these differences in the value of the "Expectation of life" must represent, at least at the older ages, substantial differences in the value of the net premiums, although in the absence of "select" tables, it is not possible to say accurately what these differences should be. A fairly trustworthy estimate may, however, be made if we assume the relation of the "select" to the "aggregate" tables to be similar (that, for example, the ratio of select to non-select premiums is the same in the French and in the H^M Experience). The following table exhibits the differences between the Institute 3 per-cent net premiums, and those indicated by the experience of French male assurants, and by the experience of the 23 German companies. At a time when so considerable a Continental business is done by English companies, I venture to think that the figures will not be without interest.

Three per-cent net Annual Premium per-cent, according to (1) H^M Table; (2) French Assured Lives (males); (3) Experience of 23 German Offices.

Age x	H^M 100P _x	FRENCH ASSURED LIVES—MALE			23 GERMAN OFFICES		
		100P _x	Extra per-cent		100P _x	Extra per-cent	
			On Sum Assured	On H^M Premium		On Sum Assured	On H^M Premium
20	1·427	1·450	·023	1½ per-cent	1·588	·161	11 per-cent
25	1·625	1·648	·023	1½ per-cent	1·776	·151	9 per-cent
30	1·880	1·923	·043	2 per-cent	2·040	·160	8½ per-cent
35	2·193	2·272	·079	3½ per-cent	2·379	·186	8½ per-cent
40	2·589	2·706	·117	4½ per-cent	2·817	·228	9 per-cent
45	3·114	3·274	·160	5 per-cent	3·384	·270	9 per-cent
50	3·801	4·032	·231	6 per-cent	4·145	·344	9 per-cent
55	4·725	5·030	·305	6½ per-cent	5·156	·431	9 per-cent
60	5·987	6·413	·426	7 per-cent	6·502	·515	9 per-cent

The premiums at the younger ages for the French Male Assurants would, of course, be somewhat increased if the correction suggested above (the deduction of one-eighth of the entrants at each age) were made in the numbers exposed to risk—and the effect of this would probably be to bring the extra at these ages to nearly 5 per-cent of the H^M premium.

On the other hand, the female assured lives show a distinctly lighter mortality, especially at the younger ages, than that shown by the H^F table, and, so far from demanding any extra premium for whole term assurances as compared with the male lives, they show

at all ages a considerably higher expectation of life. It will be seen, however, on reference to Table 5, where the "expected" deaths for each sex are computed by the same standard, that from age 10 to 35 the female assurants exhibit a somewhat higher rate of mortality than the males, although the difference is not very material, amounting only to about '001 in the value of q_x , equivalent, say, to about 2s. 6d. per-cent in the premiums for short term risks at those ages, unless this comparison is disturbed by a different effect of selection in the two tables.

The annuity experience, both in respect of males and females, agrees very closely with that of the English Government experience. For ages below 50 the English figures are not available, but the differences in the value of the function ℓ_x for that age onward, between the French and English Annuitants, whether male or female, is so small that the tables might almost be said to be identical. If we assume that the effect of selection upon the mortality in the case of annuitants is nearly the same in both countries, it would appear that the same annuity-rates may be employed in practice both here and in France.

After what has been said above, as to the relation of the French Experience to our own Standard Tables, it will be unnecessary to do more than refer to the comparison made in Tables 8 and 9 of the values of annuities, according to the various mortality tables there stated.

The French "Assured Lives" Table will be seen to occupy a position nearly midway between the English and German standards; and it is evident, from the run of the annuity-values, that while the premiums are higher than under the H^M Table, the valuation reserves will be also somewhat larger, the same being true of the German Table. The French Annuitant Table occupies a position intermediate between the Government Male and Female Tables, except at the ages below 40, to which reference has already been made above.

It is only necessary to add, in conclusion, that the data collected by the French Actuaries is of such interest and value, that it is to be hoped they will see their way to publish the materials in a form suitable for the construction of tables for each age at entry, and thus lay the actuarial profession under still further obligation.

TABLE 1.

FRENCH ASSURED LIVES, MALES.—*Unadjusted Mortality Table.*

Age	q_x	l_x	d_x	$\bar{e}_x$	Age	q_x	l_x	d_x	$\bar{e}_x$
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
0	·0329	10,000	329	51·94	50	·0167	6,305	105	19·36
1	·0335	9,671	324	...	51	·0180	6,200	112	...
2	·0214	9,347	200	...	52	·0195	6,088	119	...
3	·0141	9,147	129	...	53	·0212	5,969	127	...
4	·0132	9,018	119	...	54	·0217	5,842	127	...
5	·0080	8,899	71	53·12	55	·0232	5,715	133	16·09
6	·0067	8,828	59	...	56	·0241	5,582	134	...
7	·0054	8,769	47	...	57	·0248	5,448	135	...
8	·0049	8,722	43	...	58	·0287	5,313	152	...
9	·0049	8,679	43	...	59	·0312	5,161	161	...
10	·0032	8,636	28	49·67	60	·0361	5,000	181	13·02
11	·0028	8,608	24	...	61	·0387	4,819	187	...
12	·0032	8,584	27	...	62	·0390	4,632	181	...
13	·0028	8,557	24	...	63	·0432	4,451	192	...
14	·0042	8,533	36	...	64	·0465	4,259	198	...
15	·0049	8,497	42	45·44	65	·0460	4,061	187	10·45
16	·0052	8,455	44	...	66	·0507	3,874	196	...
17	·0053	8,411	45	...	67	·0513	3,678	189	...
18	·0065	8,366	54	...	68	·0599	3,489	209	...
19	·0062	8,312	52	...	69	·0695	3,280	228	...
20	·0093	8,260	77	41·60	70	·0690	3,052	210	8·04
21	·0065	8,183	53	...	71	·0785	2,842	223	...
22	·0069	8,130	56	...	72	·0898	2,619	235	...
23	·0050	8,074	40	...	73	·0855	2,384	204	...
24	·0065	8,034	52	...	74	·0891	2,180	194	...
25	·0057	7,982	46	38·04	75	·1250	1,988	248	6·05
26	·0056	7,936	44	...	76	·0863	1,738	150	...
27	·0062	7,892	49	...	77	·1330	1,588	211	...
28	·0060	7,843	47	...	78	·1520	1,377	209	...
29	·0066	7,796	51	...	79	·1520	1,168	178	...
30	·0056	7,745	43	34·13	80	·1380	990	137	4·69
31	·0072	7,702	55	...	81	·1330	853	113	...
32	·0069	7,647	53	...	82	·2330	740	172	...
33	·0069	7,594	52	...	83	·1850	568	105	...
34	·0075	7,542	57	...	84	·1390	463	64	...
35	·0083	7,485	62	30·22	85	·2440	399	97	3·33
36	·0088	7,423	65	...	86	·1850	302	56	...
37	·0093	7,358	68	...	87	·2110	246	52	...
38	·0092	7,290	67	...	88	·2860	194	55	...
39	·0095	7,223	69	...	89	·3330	139	46	...
40	·0103	7,154	74	26·50	90	·0000	93	...	2·17
41	·0110	7,080	78	...	91	·3330	93	31	...
42	·0108	7,002	76	...	92	1·0000	62	62	...
43	·0116	6,926	80	...	93	...	...	...	...
44	·0119	6,846	82	...	94	...	...	...	...
45	·0121	6,764	82	22·88	95	...	...	...	...
46	·0139	6,682	93	...					
47	·0148	6,589	98	...					
48	·0145	6,491	94	...					
49	·0144	6,397	92	...					

TABLE 2.

FRENCH ASSURED LIVES, FEMALES.—*Unadjusted Mortality Table.*

Age	q_x	l_x	d_x	$\bar{e}_x$	Age	q_x	l_x	d_x	$\bar{e}_x$
(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
0	·0323	10,000	323	53·54	50	·0166	6,364	106	21·95
1	·0285	9,677	276	...	51	·0139	6,258	87	...
2	·0201	9,401	189	...	52	·0127	6,171	78	...
3	·0153	9,212	141	...	53	·0161	6,093	98	...
4	·0120	9,071	109	...	54	·0169	5,995	101	...
5	·0100	8,962	90	54·51	55	·0161	5,894	95	18·50
6	·0078	8,872	69	...	56	·0146	5,799	85	...
7	·0053	8,803	47	...	57	·0161	5,714	92	...
8	·0033	8,756	29	...	58	·0163	5,622	92	...
9	·0028	8,727	24	...	59	·0204	5,530	113	...
10	·0051	8,703	44	51·08	60	·0248	5,417	134	14·90
11	·0045	8,659	39	...	61	·0256	5,283	135	...
12	·0040	8,620	34	...	62	·0283	5,148	146	...
13	·0042	8,586	36	...	63	·0308	5,002	154	...
14	·0039	8,550	33	...	64	·0441	4,848	214	...
15	·0069	8,517	59	47·14	65	·0351	4,634	163	11·96
16	·0068	8,458	58	...	66	·0405	4,471	181	...
17	·0063	8,400	53	...	67	·0312	4,290	134	...
18	·0065	8,347	54	...	68	·0581	4,156	242	...
19	·0040	8,293	33	...	69	·0452	3,914	177	...
20	·0088	8,260	73	43·54	70	·0691	3,737	258	9·21
21	·0081	8,187	66	...	71	·0930	3,479	324	...
22	·0113	8,121	92	...	72	·0640	3,155	202	...
23	·0068	8,029	55	...	73	·0706	2,953	208	...
24	·0065	7,974	52	...	74	·0500	2,745	137	...
25	·0056	7,922	44	40·30	75	·1360	2,608	355	7·25
26	·0085	7,878	67	...	76	·0850	2,253	191	...
27	·0070	7,811	55	...	77	·1110	2,062	229	...
28	·0077	7,756	60	...	78	·0761	1,833	139	...
29	·0063	7,696	48	...	79	·1480	1,694	250	...
30	·0084	7,648	64	36·65	80	·1080	1,444	156	6·26
31	·0079	7,584	60	...	81	·0741	1,288	96	...
32	·0085	7,524	64	...	82	·1440	1,192	172	...
33	·0076	7,460	57	...	83	·1580	1,020	161	...
34	·0078	7,403	58	...	84	·0313	859	27	...
35	·0090	7,345	66	33·06	85	·1724	832	143	4·26
36	·0087	7,279	63	...	86	·1000	689	69	...
37	·0077	7,216	56	...	87	·1714	620	106	...
38	·0075	7,160	54	...	88	·2140	514	110	...
39	·0088	7,106	63	...	89	·2110	404	85	...
40	·0081	7,043	57	29·37	90	·3330	319	106	3·24
41	·0096	6,986	67	...	91	·5000	213	107	...
42	·0106	6,919	73	...	92	·0000	106	...	...
43	·0077	6,846	53	...	93	·0000	106	...	...
44	·0095	6,793	65	...	94	·5000	106	53	...
45	·0108	6,728	73	25·63	95	...	53	...	...
46	·0089	6,655	59	...					
47	·0126	6,596	83	...					
48	·0118	6,513	77	...					
49	·0112	6,436	72	...					

TABLE 3.
FRENCH ANNUITANTS, MALES.—*Unadjusted Mortality Table.*

Age	q_x	l_x	d_x	e_x	Age
(1)	(2)	(3)	(4)	(5)	(6)
40	·0221	10,000	221	26·86	40
41	·0159	9,779	156	...	41
42	·0054	9,623	52	...	42
43	·0256	9,571	245	..	43
44	·0138	9,326	129	...	44
45	·0161	9,197	148	23·99	45
46	·0156	9,049	141	...	46
47	·0097	8,908	86	...	47
48	·0126	8,822	111	...	48
49	·0181	8,711	158	...	49
50	·0162	8,553	139	20·61	50
51	·0157	8,414	132	...	51
52	·0198	8,282	164	...	52
53	·0240	8,118	195	...	53
54	·0232	7,923	184	...	54
55	·0211	7,739	163	17·50	55
56	·0188	7,576	142	...	56
57	·0241	7,434	179	..	57
58	·0253	7,255	184	...	58
59	·0261	7,071	185	...	59
60	·0296	6,886	204	14·34	60
61	·0282	6,682	188	..	61
62	·0331	6,494	215	...	62
63	·0323	6,279	203	...	63
64	·3394	6,076	239	...	64
65	·0392	5,837	229	11·46	65
66	·0422	5,608	237	...	66
67	·0446	5,371	240	...	67
68	·0501	5,131	257	...	68
69	·0521	4,874	254	...	69
70	·0596	4,620	275	8·80	70
71	·0704	4,345	306	..	71
72	·0700	4,039	283	...	72
73	·0768	3,756	288	...	73
74	·0849	3,468	294	...	74
75	·0906	3,174	288	6·67	75
76	·0985	2,886	284	...	76
77	·1031	2,602	268	...	77
78	·1150	2,334	268	...	78
79	·1286	2,066	266	...	79
80	·1419	1,800	255	4·89	80
81	·1500	1,545	232	...	81
82	·1675	1,313	220	...	82
83	·1777	1,093	193	...	83
84	·1865	900	168	...	84
85	·2140	732	157	3·66	85
86	·2170	575	125	...	86
87	·2150	450	99	...	87
88	·2440	351	86	...	88
89	·2570	265	68	...	89
90	·2780	197	55	2·91	90
91	·2590	142	37	...	91
92	·2800	105	29	..	92
93	·2500	76	19	...	93
94	·3260	57	19	...	94
95	·3490	38	13	2·0	95
96	·2000	25	13	...	96
97	·3640	13	4	...	97
98	·2300	8	2	...	98
99	·3750	6	2	...	99
100	·4000	4	2	...	100
101	1·0000	2	2	...	101

TABLE 4.
FRENCH ANNUITANTS, FEMALES.—*Unadjusted Mortality Table.*

Age	q_x	l_x	d_x	e_x	Age
(1)	(2)	(3)	(4)	(5)	(6)
40	·0081	10,000	81	30·36	40
41	·0149	9,919	148	...	41
42	·0131	9,771	128	...	42
43	·0113	9,643	109	...	43
44	·0141	9,534	134	...	44
45	·0097	9,400	91	27·13	45
46	·0078	9,309	73	...	46
47	·0102	9,236	94	...	47
48	·0108	9,142	99	...	48
49	·0145	9,043	131	...	49
50	·0094	8,912	84	23·49	50
51	·0140	8,828	124	...	51
52	·0119	8,704	104	...	52
53	·0153	8,600	132	...	53
54	·0145	8,468	123	...	54
55	·0120	8,345	100	19·89	55
56	·0143	8,245	118	...	56
57	·0137	8,127	111	...	57
58	·0173	8,016	139	...	58
59	·0186	7,877	146	...	59
60	·0195	7,731	151	16·25	60
61	·0200	7,580	152	...	61
62	·0219	7,428	163	...	62
63	·0257	7,265	187	...	63
64	·0255	7,078	181	...	64
65	·0263	6,897	181	12·90	65
66	·0326	6,716	219	...	66
67	·0363	6,497	236	...	67
68	·0392	6,261	245	...	68
69	·0403	6,016	242	...	69
70	·0481	5,774	278	9·90	70
71	·0520	5,496	286	...	71
72	·0572	5,210	298	...	72
73	·0624	4,912	306	...	73
74	·0666	4,616	307	...	74
75	·0739	4,309	318	7·40	75
76	·0835	3,991	333	...	76
77	·0885	3,658	324	...	77
78	·0982	3,334	327	...	78
79	·1130	3,007	340	...	79
80	·1259	2,667	336	5·40	80
81	·1373	2,331	319	...	81
82	·1490	2,012	300	...	82
83	·1547	1,712	265	...	83
84	·1640	1,447	237	...	84
85	·1720	1,210	208	4·11	85
86	·1860	1,002	186	...	86
87	·2100	816	171	...	87
88	·2120	645	137	...	88
89	·2490	508	126	...	89
90	·2460	382	94	3·14	90
91	·2610	288	75	...	91
92	·2860	213	61	...	92
93	·2600	152	40	...	93
94	·3360	112	38	...	94
95	·2060	74	15	2·81	95
96	·2210	59	13	...	96
97	·2500	46	12	...	97
98	·6130	34	21	...	98
99	·4450	13	6	...	99
100	·5000	7	3	2·21	100
101	·5000	4	2	...	101
102	·0000	2	...	...	102
103	·0000	2	...	...	103
104	·0000	2	...	...	104

TABLE 5.

FRENCH ASSURED LIVES EXPERIENCE.

*Comparison of Actual with Expected Deaths according to the
Adjusted Mortality Table.*

Ages	MALES			FEMALES			BOTH SEXES		
	Expected Deaths	Actual Deaths	Actual percentage of Expected	Expected Deaths	Actual Deaths	Actual percentage of Expected	Expected Deaths	Actual Deaths	Actual percentage of Expected
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0-4	246.3	256	104	242.8	238	98	489.1	494	101
5-9	157.2	162	103	164.0	159	97	321.2	321	100
10-14	147.7	119	80	156.2	166	106	303.9	285	94
15-19	198.4	191	96	191.3	203	106	389.7	394	100
20-24	181.4	131	100	127.4	154	121	258.8	285	110
25-29	578.8	535	92	241.8	259	107	820.6	794	96
30-34	1,472.6	1,397	95	364.5	395	108	1,837.1	1,792	98
35-39	2,270.9	2,358	104	444.4	426	96	2,715.3	2,784	103
40-44	2,681.5	2,787	104	451.0	385	85	3,132.5	3,172	101
45-49	2,696.4	2,725	101	409.5	329	80	3,105.9	3,054	98
50-54	2,447.3	2,543	104	349.8	289	83	2,797.1	2,832	101
55-59	2,062.7	2,086	101	289.3	186	64	2,352.0	2,272	97
60-64	1,515.6	1,652	109	223.6	180	81	1,739.2	1,832	105
65-69	983.1	986	100	157.2	120	76	1,140.3	1,106	97
70-74	560.2	572	102	103.3	93	90	663.5	665	100
75-79	277.3	297	107	67.2	64	95	344.5	361	105
80-84	104.4	99	95	41.3	25	60	145.7	124	85
85-89	28.5	27	96	22.6	15	70	51.1	42	83
90-95	2.7	3		6.1	5		8.8	8	
All ages	18,563.0	18,926	102	4,053.3	3,691	91	22,616.3	22,617	100

TABLE 6.
FRENCH ANNUITANTS' EXPERIENCE.
*Comparison of Actual with Expected Deaths according to the
Adjusted Mortality Table.*

Ages	MALES			FEMALES			BOTH SEXES		
	Expected Deaths	Actual Deaths	Actual percentage of Expected	Expected Deaths	Actual Deaths	Actual percentage of Expected	Expected Deaths	Actual Deaths	Actual percentage of Expected
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0-4	1.3	4	...	1.1	...	...	2.4	4	...
5-9	.7	1	103	.7	...	100	1.4	1	101
10-14	.8	...		.5	2		1.3	2	
15-19	1.0	2		1.0	1		2.0	3	
20-24	1.4	1	260	1.8	1	175	3.2	2	205
25-29	1.9	3		3.1	6		5.0	9	
30-34	3.5	11		6.6	11		10.1	22	
35-39	6.7	7	158	15.2	10	118	21.9	17	131
40-44	17.2	31		38.8	54		56.0	85	
45-49	47.3	62		100.8	99		148.1	161	
50-54	156.7	220	140	300.9	276	92	457.6	496	108
55-59	437.3	514	118	788.6	614	78	1,225.9	1,128	92
60-69	1,130.3	1,305	116	1,884.7	1,508	80	3,015.0	2,813	93
65-69	2,183.6	2,388	109	3,445.3	2,889	84	5,628.9	5,277	94
70-74	3,205.4	3,674	115	4,743.3	4,303	91	7,948.7	7,977	100
75-79	3,399.9	3,773	111	4,989.0	4,704	94	8,388.9	8,477	101
80-84	2,530.7	2,829	112	3,746.2	3,726	99	6,276.9	6,555	104
85-89	1,195.8	1,245	104	1,911.1	1,743	91	3,106.9	2,988	96
90-94	339.1	294	87	580.9	488	84	920.0	782	85
95-99	60.8	41	70	108.2	67	62	169.0	108	65
100-106	4.7	5		7.0	4		11.7	9	
All ages	14,726.1	16,410	115	22,674.7	20,506	90	37,400.8	36,916	99

TABLE 7.
COMPARISON OF THE MORTALITY OF ANNUITANTS AND
ASSURED LIVES.

Ages	SUM OF THE UNADJUSTED VALUES OF q_x FOR AGES GIVEN IN COL. 1							
	Male Lives				Female Lives			
	Government Annuitants	H ^M	French Annuitants	French Assurants	Government Annuitants	H ^F	French Annuitants	French Assurants
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
25-29	.133	.035	.050	.030	.077	.058	.060	.035
30-34	.061	.041	.113	.034	.047	.059	.051	.040
35-39	.078	.047	.044	.045	.047	.060	.025	.042
40-44	.087	.054	.083	.056	.055	.065	.062	.046
45-49	.088	.068	.072	.070	.055	.069	.053	.055
50-54	.109	.089	.099	.097	.064	.079	.065	.076
55-59	.125	.121	.115	.132	.090	.100	.076	.084

TABLE 8.
COMPARISON OF ANNUITY-VALUES AT 3 PER-CENT INTEREST.
Assured Lives.

Age	American 30 Offices	H ^m	French Experience 1892	German 23 Offices	Age
(1)	(2)	(3)	(4)	(5)	(6)
15	23·088	23·153	22·954	...	15
20	22·290	22·043	22·046	21·222	20
25	21·360	21·038	21·053	20·329	25
30	20·283	19·867	19·836	19·198	30
35	19·049	18·587	18·470	17·896	35
40	17·652	17·176	16·957	16·452	40
45	16·096	15·594	15·311	14·883	45
50	14·396	13·896	13·556	13·172	50
55	12·582	12·094	11·731	11·394	55
60	10·702	10·236	9·889	9·622	60
65	8·820	8·418	8·091	7·897	65
70	7·009	6·657	6·402	6·256	70
75	5·343	5·061	4·880	4·796	75
80	3·881	3·742	3·570	3·520	80
85	2·646	2·739	2·493	2·549	85
90	1·560	1·740	1·651	1·516	90
95	0·800	0·415	1·023	0·599	95

Annuityants.

Age	Government Annuityants— Males, Select	Government Annuityants— Females, Select	French Experience 1892	Carlisle	Age
(1)	(2)	(3)	(4)	(5)	(6)
15	21·471	23·093	23·497	22·582	15
20	20·561	22·292	22·695	21·695	20
25	19·601	21·415	21·831	20·666	25
30	18·588	20·451	20·751	19·557	30
35	17·515	19·380	19·514	18·434	35
40	16·376	18·180	18·114	17·142	40
45	15·152	16·820	16·553	15·863	45
50	13·813	15·270	14·844	14·302	50
55	12·309	13·607	13·017	12·409	55
60	10·601	11·791	11·116	10·491	60
65	8·902	9·909	9·204	8·917	65
70	7·299	8·000	7·355	7·123	70
75	5·809	6·366	5·645	5·512	75
80	4·553	4·937	4·140	4·365	80
85	...	...	2·885	3·229	85
90	...	...	1·894	2·499	90
95	...	...	1·157	2·757	95

DISCUSSION.

The PRESIDENT (Mr. T. E. Young) said the original and exhaustive character of Mr. Hardy's writings was so fully recognized, that but few preliminary observations from him were requisite; for in whatever field of knowledge Mr. Hardy was engaged, he always gleaned so vigorously and vigilantly, that the work of his successors was largely confined to the very pleasing duty of simply attributing praise. He (the President) need not stay to point out the special opportuneness of the subject which Mr. Hardy had introduced in connection with the important, indeed almost national, investigation in which they themselves were engaged. He had now to call upon the referees to discuss the paper, in pursuance of the practice adopted, and to ask other members to favour the meeting with their views.

Mr. A. B. ADLARD said that at the beginning of the paper the author directed attention to the obligation under which the actuarial profession is placed to the Committee of French Actuaries for collecting and publishing so valuable an addition to the statistics of mortality among assured lives and annuitants; and he was sure that the expression of such an obligation was one in which they would all most heartily concur. At the same time, he was sure that all who were present would also concur in expressing their thanks and obligation to Mr. Hardy for the trouble he had taken in preparing his paper, in which the general results of the investigations of their French *confrères* were presented in so concise and interesting a form. As the paper partook greatly of the nature of a review of the volume which was published in Paris in 1895, he did not think there was very much to be said by way of criticism—that is, criticism of Mr. Hardy's paper—but at the same time there were many points of interest which arose in connection with the subject. Under the heading "Mode of Collection", Mr. Hardy was not quite accurate in his enumeration of the different classes of assurances as to which the particulars were collected for separate tabulation if thought desirable. They were as follows: (1) Assurances for the whole of life; and for these *white* cards were used. (2) Endowment assurances, called by the French actuaries "*Assurances mixtes*", because they belong to both of the two classes into which their transactions are primarily divided—i.e., "*Assurances en cas de décès*", and "*Assurances en cas de vie*", and for these endowment assurances *blue* cards were used. Then (3) there were what are called "*Assurances à terme fixe*", under which the sum assured is payable at a fixed date, whether the assured be dead or alive, while the payment of the premiums depends upon his continued existence; and for these *rose-coloured* cards were used. Then (4) there were ordinary temporary assurances, for which they used *grey* cards; and lastly, there were what the French call "*Les contre-assurances*", for which the colour of the cards was *vert d'eau*—a very pale green. These "*contre-assurances*" were not contingent survivorships, but were temporary increasing assurances, and he believed were usually effected in connection with an endowment, or a deferred annuity, so as to secure a return of the premiums paid in the event of death occurring before the benefit was entered upon. The

distinction of sex, as regards assured lives, was indicated by a double bar being placed across the cards relating to female lives. As regards annuitants, *violet* cards were used for male lives, and *dark green* for female lives. He was indebted to the courtesy of Mons. Léon Marie (the actuary of Le Phénix) for the specimen cards, which, he thought, might be of some interest to those present. On examining them, one could not but be struck with the variety of colours made use of, necessitating, one would almost think, a preliminary examination of the clerks to be employed, similar to that which was adopted in the case of signalmen on railways, to make quite sure that they were not afflicted with colour blindness. As regards the calculation of the number *exposed to risk*, he had very carefully followed the description of the course adopted in the volume itself (and beyond that they could not go, because the figures were not given); and he quite agreed with Mr. Hardy that the effect of the method appeared to be to over-estimate the numbers exposed to risk, by approximately one-eighth of the number of entrants, at each age. As regards the mode in which the tables were finally presented, it was much to be regretted, he thought, that, although the ungraduated results were given separately for each sex, the graduated tables, upon which all the monetary results were based, were only given for males and females combined. They knew it had always been the practice of French companies to charge the same rates of premium for assurances, and to grant the same rates of annuities, for males and females; and when these rates were based upon a combined experience such as those now under consideration, an assumption was practically involved that the relative numbers of the sexes would always coincide with those in the observations upon which the rates were based—an assumption which, of course, might or might not in the main be realized. In the case of annuities, if any considerable increase in the proportion of female lives should occur, the result must of course be very detrimental to the company granting the annuities. As to the method of graduation adopted, they all knew that Makeham's formula invariably produced a curve of absolute smoothness, whatever might be the irregularities, either characteristic or accidental, of the initial series, and on that account it was not suitable for universal adoption. With regard to the experience of the French *assured* lives, both the sexes combined, the table of comparison of *actual* with *expected* deaths given by Mr. Hardy, shows that by use of the adjusted values of q_x the actual deaths were practically reproduced throughout; and there could therefore be no doubt whatever as to the suitability of Mr. Makeham's formula for the table. As regards the *annuitants'* experience, where the characteristic irregularities of the original series were of a marked nature, as might readily be seen by reference to the table giving the sum of the unadjusted values of q_x in quinquennial groups, the method was clearly not so suitable, as admitted by the authors, and as shown in Mr. Hardy's table immediately above the one last referred to. Still, when they considered that a table of this nature was only likely to be used for annuity calculations and not for benefits of temporary duration, he thought that they would all concede that the agreement of the adjusted table with the original facts was quite close enough to admit of the adoption of the method,

particularly when the great advantages which it affords in the calculation of benefits involving two or more lives were taken into account. In connection with this point it was, he thought, a little surprising that, after having adopted Makeham's formula, and given the values of annuities upon 1, 2, 3, 4, &c., up to 9 lives of equal ages, the French actuaries should have gone to the trouble of calculating complete tables of the values of annuities on two lives, at no less than *five* rates of interest—a monument, certainly, to their industry and perseverance. It was, also, more than they would have expected to find, that complete tables, both of *temporary* and *deferred* annuities, at all the five rates of interest and on both tables of mortality, are given. When referring to the mode in which the French actuaries have dealt with the annuitants' mortality table at the younger ages, Mr. Hardy remarked that it was "not easy to justify the use of higher rates of mortality in a table upon which a company grants annuities, than the rates it assumes in granting assurances." That was a somewhat stronger statement than he (Mr. Adlard) would be prepared to follow, *so far as regarded lives under 40 or 45*, where, as it appeared to him, the medical examination of lives proposed for assurance would be far more effective in creating a select class than any so-called self-selection of the annuitants. Many of the annuities that were purchased at such ages would, he thought, be purchased by executors or trustees to facilitate the winding-up of estates, without any regard to the state of health of the nominees; and others would probably be purchased by persons upon whom the cares of life had pressed somewhat heavily in their earlier years, and from which they were desirous of freeing themselves; and such would be hardly likely to form a select class. At the older ages at which annuities were usually purchased, the case was, of course, quite different; but at the *younger ages, to which those remarks exclusively applied*, he himself did not see any objection to adopting Mr. Sprague's H^M Select Table as the basis for premiums for assurances, and at the same time the Government Annuitants' Male Select Table as the basis for annuities on male lives. He could not leave that division of the paper without remarking on the very great success of Mr. Hardy's first rough attempt (as he called it) to represent Mr. Sprague's Select Table by means of Makeham's formula. With regard to the results of the experience, they had had their attention drawn to the fact that among French *male* assured lives the mortality was heavier than that shown by the H^M Table; and this, he thought, might to some extent be accounted for by the circumstance that the French assurance companies have no rated-up lives. With them the practice had been either to accept a proposal in accordance with their ordinary tariff, or to reject it. So it might happen, one would think, on some occasions, that lives which were not exactly what would be called first-class, and which would, according to the practice in England, be accepted with an addition of perhaps 3, 5, or even 7 years to the age, might, in the case of the French companies, be accepted at the ordinary rates, as there might not appear to be sufficient reason to justify their rejection altogether. In considering the subject, it at first occurred to him that the comparatively high rate of mortality among the

male assured lives might, perhaps, have been partly attributable to the wars in which France had been engaged; but that idea was abandoned on finding that at the annual meeting of La Générale (one of the four companies) in July 1871, immediately after the Franco-German War, the manager said in his speech: "I do not mention the claims occasioned by the war, because they are not numerous enough to exercise a sensible influence on our accounts"; and it also appeared, from another speech made at the same meeting, that death in war, or from wounds received in war, was not covered by their guarantees; and that, although they had given a liberal interpretation to their policies in favour of those required to serve in the National Guard, even when they had volunteered, the adoption of that course entailed upon them no loss of importance. The wars, therefore, could not have had any material effect on the mortality experience; and he could not help thinking that the circumstance to which he had alluded, of the French having no rated-up lives, lay at the root of the comparatively high rate of mortality shown among the French male assured lives. With regard to the female assured lives, among whom the mortality was comparatively light, he had been trying to find out some solution, but he could not say that he had met with any definite success. At first he thought that the birth-rate being so much lower in France than in England—22·8 per thousand as compared with 31·2 on an average of 5 years—might have had something to do with it; but, on making a further comparison with the birth rates in other European countries, and finding that the ratios in some of those countries were also lower than in England, he felt obliged to give up that idea. Then, there was also the fact that the mortality amongst infants in France was considerably higher than in England; and he had thought that perhaps this might have some favourable influence upon the mortality in after life, on the principle of what had been called the survival of the fittest; but on finding that the mortality among infants under 5 years of age was almost as high in Germany as in France, and at the same time that the mortality among German female lives, as shown by the experience of the twenty-three German offices, was much heavier than in the case of the French female assured lives, he could not do otherwise than conclude that the rate of infantile mortality could not have much to do with the matter. The only possible solution which he could suggest was, that there might be some difference in the measure of severity with which the proposals for assurances on female lives were scrutinized by the respective assurance companies of France and England. As far as his own knowledge went, the medical examination of female lives in England had been *less* severe than in the case of male lives; and it was, of course, within the limits of possibility that there might be a difference in this respect in the practice of French companies. Should that be so, such a difference in practice might be quite sufficient to account for the comparatively light rate of mortality among female assured lives shown in the French tables. He joined very heartily with Mr. Hardy in expressing a hope that their French *confrères*, notwithstanding the fact that they had already received *un grand prix* at the International Exhibition, would not consider

that their labours were completely ended, but that, acting on the principle "*nil actum reputans dum quid superesset agendum*", they would, before long, publish the necessary materials for the construction of select tables, and thus (as Mr. Hardy had put it) place the actuarial profession under a still further obligation.

Mr. GEOFFREY MARKS echoed very heartily the President's statement as to the welcome always accorded to any paper by Mr. Hardy. He believed he was speaking for many present in saying that he viewed any contribution from that gentleman with particular pleasure, coming as it did not only from so distinguished a member of the profession, but from so kindly and able a teacher. They were, as the President said, accustomed to original work from Mr. Hardy, and anything coming from him was sure to bear very strongly the stamp of his own views. Mr. Adlard's comments had been so full that very little was left for other speakers. There were, however, one or two points as to the practical part of the paper to which he should like to refer. No doubt the information which their French colleagues had been at the pains to collect was very valuable, from the fact that it was practically the first thing of its kind in France. Of course, independently of their theoretical value, the investigations had a strong bearing on questions in which they were themselves interested as practical men. The large extent to which, of late years, business on the Continent had developed, must have very often forced on the attention of gentlemen responsible for the management of offices the enquiry whether or not the rates they were charging were adequate to meet the requirements of Continental risks. He believed it was the practice for most English offices to charge the same rates for Continental lives as for home lives. There was, no doubt, some additional risk; and the conclusions drawn by Mr. Hardy from the data collected by the French actuaries confirmed that view. Mr. Adlard had said that the war risk had very little influence on the result. He (Mr. Marks) was not quite sure whether that was so, or whether it could be permanently excluded from the view of English offices, although the Continental offices, on whose experience the existing tables were founded, did not, he thought, undertake war risks as part of the ordinary contract. He believed that that was so in Germany, and perhaps the President might be able to say something on that point. It seemed very clear that the English offices should, on Mr. Hardy's results, charge something in the nature of an extra premium to cover the difference in the vitality; and he thought they should also charge something to cover the war risk, because he did not think it was the practice of English offices to exclude that risk from their contracts, however the Continental offices might treat it. An addition to the premium, as Mr. Hardy had suggested, of 5 per-cent for French lives, and 10 per-cent for German lives, seemed desirable. What the rate should be on the borderland he did not know. The question as to the exposed-to-risk involved rather a curious point. In reading Mr. Hardy's paper, he was at first disposed to differ from him; but, on talking the matter over with the author, he was bound to say that he thought he was perfectly correct. Certainly the numbers exposed to risk were over-estimated. He could not quite see how it arose, but he thought it must be susceptible of some explanation, and

that it must be a slip, from the simple fact that no explanation was offered by the French actuaries in their preface to the tables. They stated it as if it were quite an ordinary thing, which would be obvious to the reader, but it was not so. Indeed, there seemed to be no reason whatever for it, and he was at a loss to understand it. One thing that had struck him was, why so much importance should appear to be attached to the annuity experience as compared with that derived from the life assurance risks. In all the French investigations, the investigation of the annuity experience was preceded by that of the assurance experience; a course which did not seem warranted by the comparative volume of business. Mr. Hardy's note as to the applicability of Makeham's formula to the graduation of the tables was very interesting. On setting out the unadjusted rates of mortality graphically, it was evident that the ages, say from 25 to 75, would be very suitably treated by Makeham's formula. It would have been better, and probably would have produced more satisfactory results, if, instead of a first adjustment by Woolhouse, the French actuaries had adopted some graphic adjustment, at least of the extremities of the tables, as a preliminary to the use of Makeham's formula. It seemed very clear that Makeham's formula did not apply to annuitant experience, either at home or abroad. He thought—it might be a heresy—that there was a strong probability that the very remarkable vitality which was experienced, both in the United Kingdom and abroad, among annuitants was to some extent due to fraud. It was, perhaps, not a right thing to say, but they knew in their experience that the means employed to identify the claimants of annuities were somewhat lax, especially in the offices which had a large business. It would be impossible to keep a proper watch on every claimant for an annuity. They did their best, but the best might be better. They knew that in the early part of the century, very considerable frauds were practised on the Government, and he thought it would pay offices if they looked a little more closely after the older annuitants when they made their claims.

Mr. T. G. ACKLAND said that Mr. Hardy had taken up a subject which was somewhat out of the ordinary course—and for that they were the more indebted to him—in examining the interesting volume which had been published by the four French companies. Those who were at all familiar with the practice of the French companies in the past, would be aware that the tables generally employed, until this recent publication was available—those of Deparcieux and Duvillard—were extremely unsatisfactory; and that the valuation methods followed approximated closely to those which were known in England as the hypothetical or re-assurance method, with certain modifications. The extent of the data was stated by Mr. Hardy, and he referred to it as “rather more extensive than that of the Institute.” Perhaps it might be well to modify that, by explaining that Mr. Hardy referred to the H^M Institute experience published in 1869. He thought they were already in a position to say that, in connection with the present investigation, their data would very largely exceed the figures of the earlier experience. Mr. Adlard had referred to the special classes of assurance granted by the French assurance companies. The *assurances à terme fixe* were of a special character. He supposed

Mr. Hardy did not include them in his list, because they were practically unknown to English companies, except to those transacting a Continental business. The *contre-assurance*, he had no doubt, as Mr. Adlard had stated, referred to what were sometimes spoken of among us as counter-assurances, against the return of premium in the case mainly of pure endowments, and deferred annuities and assurances. It was somewhat interesting to note that the female lives, of which so much had been said, were in the French experience about as 1 to 5 of the whole experience, while in the Institute Tables they were about as 1 to 9—that was, there were relatively about twice as many females in the French experience as in the Institute experience. Turning to the table of expectations of assured lives and annuitants in France, as compared with the Institute of Actuaries and Government annuity experience, they would note that the female vitality of assured lives in France was throughout superior to the Institute H^F table, and he was quite unable to suggest any explanation of that remarkable fact. They were prepared to see that the male assured experience showed a higher mortality in France; but that the females should throughout at every age show a superior vitality, was certainly a singular fact. In the case of male lives, it might be that the effects of war, to which Mr. Adlard referred, were not fully seen at the date to which he referred, July 1871; and that although actually death was not to any great extent caused by war, yet a material deterioration might have subsequently affected the mortality. It was also, he believed, a fact that although the French companies did not as a rule rate-up their lives, they did occasionally transfer proposals from the whole-term class to the endowment assurances class, when they were below the standard of health; and that might also have a material effect upon the table. The table given by the author of “3 per-cent net annual premium according to the H^M Table, the French assured lives, and the experience of 23 German offices”, would, he thought, have been of more value if it had been expressed in the form of select premiums. The premiums in the table being throughout based on aggregate tables, the comparisons could hardly be fully relied upon, although they were no doubt the best obtainable under the circumstances. It would also be very interesting if a corresponding table could be given for female lives, but he did not know whether the data would permit of that.

The paper divided itself into two separate parts; the one relating to the French mortality tables, and the other—which he thought of at least equal interest—to the special application of the method of graduation by the formula of Makeham. He thought they had not had before them for a long time so interesting a communication as that which Mr. Hardy made to them in deducing the select premiums on the basis of the H^M Table by the formula of Makeham, and he wished that the author, instead of devoting a portion only of his paper to this interesting subject, had more fully gone into this question for their benefit. So far as he could follow the formula which Mr. Hardy gave for the application of Makeham's formula, he had adopted the second modification of Makeham; so that the expression within brackets, representing the mortality of the $H^{M(5)}$ Table, was deduced from that “further modification” which Makeham

published only a short time before his death. Apparently, the second portion of Mr. Hardy's expression was based upon the earlier of Makeham's formulas, stated in the familiar form which represented the force of mortality as made up of a constant and an expression increasing geometrically; the effect of selection being ingeniously arrived at by the adoption of arbitrary fractions, the numerators of which represented the series of figurate numbers of the fourth order. He should like to ask Mr. Hardy whether it was not a fact, assuming that he had adopted the second modification of Makeham, that the application to joint lives was somewhat more cumbersome than if the simpler first modification had been adopted throughout. He believed that Makeham had shown, in his paper of 1890 above referred to, that they were able in this case still to avail themselves of the law of uniform seniority, but under somewhat modified circumstances, as to the rate of interest involved. Turning now to table 5 appended to the paper, it seemed to be a remarkable fact that there should be the excess of the actual deaths over the expected in the case of female lives between ages 10 and 35; and this did not appear to be altogether consistent with what one would expect from the general comparisons of the rates of mortality; but no doubt the facts were as stated. The table 7, "Comparison of the mortality prevailing among annuitants and assured lives", showed on the unadjusted facts, when summed in groups of five, a very irregular progression; and it looked as if the original data were unusually irregular, or else that at some ages there were hardly sufficient data to produce a regular or accurate curve. Finally, in one small point of criticism, he would venture to ask whether the word "assurant" was a word fully admitted into the English language, and if Mr. Hardy was responsible for it, or whether it had received a prior sponsor?*

Mr. A. H. BAILEY thought the word "assurant" was a convenient word, and a very good suggestion, as obviating the difficulty now found in discriminating between the *assured* and the *assurer*, terms which are now frequently confounded.

Mr. H. W. MANLY said they were very much indebted to Mr. Hardy for the paper which he had read. Those members who were, like himself, unfamiliar with the French language, could now read, mark, learn and inwardly digest the important results which the French actuaries had produced in their great work, without the trouble of translating it. He hardly liked to raise the question of the addition of the one quarter of the entrants in the last half of the year to the numbers at risk during the first year of observation, after others, better qualified than himself, had said they were quite certain it was wrong; but he thought there might be a possible explanation found in the want of more complete information as to how the data was manipulated. For instance, it was important to know what was done with the deaths that occurred in the latter half of the year amongst those who entered during that period; and also how the withdrawals were treated in the first year of exposure. If a large number of those who entered in the latter half of the year did not renew their policies, and the withdrawals were treated as being under

* The word "assurant" was often used by the late Dr. Farr.—ED. J.I.A.

observation for half a year only, and if the deaths which occurred among the new entrants in the latter half of the year were brought into account, then they would produce a rate of mortality which was too high. It might be, that to correct this, a rough adjustment, by adding to the exposed to risk in the first year a quarter of the numbers brought forward, was made. He only threw out that as a possible explanation of what otherwise seemed to be an unintelligible proceeding. With regard to graduation, he thought Mr. Hardy had thrown out a hint which might be extremely useful in future graduations. A good graduation was at no time a very easy thing to accomplish. It required a great deal of labour, of time, and of thought, if it was to be done well. He felt sure they might be able to get an excellent graduation, if Makeham's formula were applied to a table already well graduated by the graphic method. He did not think there was any mode of graduation worth considering, but the graphic, and Makeham's method. He quite agreed with the author, that the ease with which, by Makeham's formula, the functions which they constantly required in practice could be obtained, far outweighed any slight deviation from the original facts exhibited by the application of Makeham's method; and that there need be no hesitation in employing that formula, provided the values of the annuities by the adjusted table were practically identical with those obtained by other methods of adjustment involving no assumption as to the law of mortality. He had himself recently succeeded in making a graphic graduation which would very closely comply with the theory of errors; and if any of the young actuaries would like to try their hand at making a graduation by Makeham's method from those figures, he should be very pleased indeed to give them the opportunity. While on the question of graduation, he would like to point out what their friend, Mr. Bailey, had so often insisted upon—that there was a striking increase in the rate of mortality between the ages of 20 and 25. In the unadjusted mortality table—males—it would be seen that before the age of 20 and after the age of 22 the rates of mortality ran fairly well, but curiously enough, at the age of 20 there was a very considerable increase; and amongst the female lives it was even more marked where there was a very large increase from age 21 to age 24. A graphic curve drawn to represent the mortality prevailing amongst both those classes would show what might be called a "hump" at those ages, and this was true of almost every table constructed from actual observation.

Mr. F. T. M. BYERS asked whether the assured lives in these experiences were mainly French lives? It occurred to him that it was possible that the French companies had got a large number of lives from Switzerland, particularly from French Switzerland, because the Swiss were a provident and an assuring community. The mortality amongst the Swiss was said to be high, and he believed it was attributed to phthisis and pulmonary diseases in the valleys of Switzerland, those parts being very densely populated. If the experience of the French companies included a large number of Swiss lives, it was possible the rate of mortality among the French lives assured might be more favourable than that which appeared in the table Mr. Hardy had brought under their notice.

The PRESIDENT, in conveying to Mr. Hardy their cordial acknowledgments of his very valuable and thoughtful paper, said he could not avoid expressing the gratification they naturally felt that the investigations of their French colleagues had been prompted by the example set by the Institute in past years; and the gratification was enhanced by the fact that the two methods of graduation which they had adopted were those which had been devised by two of their most distinguished mathematicians, formerly connected with the Institute, Mr. Woolhouse and Mr. Makeham, the former in connection with the preliminary observations, and the latter with those results in their revised and completed form. Having had an opportunity of perusing the introduction to the French volume, he was very forcibly reminded of the crusade which was started by Sir David Brewster a considerable number of years ago, with the object of securing what was termed "the endowment of research." It was very interesting to observe that when those preliminary results were submitted at the great Exhibition in Paris in 1889, the jury not merely awarded a Grand Prix to the work, but presented, in addition, a Gold Medal apiece to the four actuaries concerned in its preparation. He thought it was obvious from this fact, that the opinion expressed in the immortal work of "Tristram Shandy", "that they managed these things better in France", still held good. He should like to add that Mr. Finlaison, whose judgment on that subject was extremely impressive, had written to him as follows: "I think the general confirmation of our standard English Tables is remarkable, particularly as to the relative rates of mortality of the two sexes. What induced the French actuaries to adopt one annuity rate for males and females after the superiority of female life was proved by their own observations, and confirmed by what has been long known here, it is difficult to understand. I should have thought that the preponderance of the number of female annuitants might have convinced them of the risk of using an amalgamated table."

He (the President) thought it might not be unfitting on his part to take this opportunity of calling to their remembrance the fact that, within the last few days, the most distinguished member of the Institute as a pure mathematician had passed away, in the person of Professor Sylvester. Mr. Sylvester's connection with the profession lay in the fact that he was the first actuary of the Equity and Law, from 1844 to 1855; and they had the pleasure that evening of seeing their friend Mr. Bailey, who was his immediate successor, and Mr. Burridge, who became his successor at a remoter period. Professor Sylvester's researches lay quite apart from the routine of their work, and proved of so recondite a character that, as in the case of all original minds, many years must necessarily elapse before the value and meaning of his results could be appreciated and applied; but with regard to his native force and power of intellect, they might very fairly say, if they adopted Professor Tyndall's method of graphic comparison, that Professor Sylvester, in the region of pure mathematics, ranked at no very considerable distance from the master mind of Newton himself.

He congratulated Mr. Hardy, in begging him to respond, that, owing to the exhaustiveness and insight displayed in the paper, he

occupied the happy position of having mainly to reply to approbation and praise.

Mr. HARDY, in replying, thanked the speakers and the members generally for the appreciative way in which they had received his somewhat short and perhaps rather sketchy paper. Mr. Adlard, he said, threw a considerable amount of light on the subject, and he was much obliged to him for pointing out that he had not accurately described the classes of assurance into which the experience had been divided. With respect to the question of the relative mortality of annuitants and assured lives, Mr. Adlard thought that there was sufficient reason in the observed higher rates of annuitants to retain those rates in constructing monetary tables. But they had to meet the fact that frequently now-a-days they were asked to grant a double contract on the same life. He had known several cases of somewhat large magnitude, in which offices had been asked to grant an annuity and an assurance at the same time, on the same life, and it was quite clear, in a case of that kind, it would not only be an anomaly to use an annuity table which gave a higher rate of mortality than the assurance table, but it would also involve the office in certain loss. The extra rate of mortality observed amongst the French lives had been commented on by various speakers, and various suggestions had been made as to how it could be accounted for. He did not attempt in his paper any explanation, because none of those explanations appeared to him to quite meet the case. War mortality, for example, scarcely seemed to him an adequate explanation, as the extra mortality was observable not specially at the younger ages, but in a more marked degree as they reached the older periods of life. Then as to the suggestion that the extra mortality is due to the fact that rated-up lives were not taken by the French companies, that at first sight seemed to be a satisfactory, or at least a partial, explanation; but exactly the same thing took place in America—the offices there did not take rated-up lives, and the American Offices' Experience was very much better than their own. Mr. Marks had referred to the effect upon the recorded annuitant mortality of possible frauds. Of course, it was difficult to say to what extent fraud might affect the recorded deaths in the case of annuitants, but he should not think himself that it would be to any serious extent. He should be inclined to say that offices were more liable to be content with insufficient proof of age in the case of annuitants, especially the older annuitants, than to be lax in respect of proof of existence. The older the life the more difficult it was to produce absolutely satisfactory proof of age, and the consequence was that they were often asked to accept as proof of age evidence which was not absolutely satisfactory and conclusive; and if, in those cases, they got certain over-statements of age, then, of course, their experience would suffer. He was glad to find that Mr. Bailey approved of the word "assurant." It was not a word of his own invention, although he was not prepared to say at the moment where he found it. Mr. Ackland had referred to the comparison of the premiums, and had rightly said that they could not properly compare those premiums unless they had select tables to compare them by. That was strictly true. He thought they might,

however, make a fairly approximate comparison, and when they bore in mind that the average duration of the assured lives on which the French Table was based was about 8 years, against their $9\frac{1}{4}$ in the H^M Table, they would see that the effect of selection there would probably be greater proportionately than in their own tables, and therefore the French experience would have the benefit of that larger proportionate number of selected lives. If they had select tables, instead of the aggregate tables they had to be content with at present, they would no doubt find that there was a still greater difference between the French experience and their own. Referring to the question of the graduation by Makeham's formula of select tables, the example given was, as Mr. Ackland pointed out, according to Makeham's second modification of Gompertz, because he wished to bring the tables as near to the original $H^{M(5)}$ as possible, although he did not attach much importance to the twists in that table at young ages. Of course it would be quite possible, by carefully selecting constants, to get a fairly correct agreement between the adjusted and unadjusted table without using his second modification. Mr. Byers had made some interesting remarks as to the mortality of Swiss lives, which, he said, were subject to a much higher rate than the average, and that fact might possibly to some extent account for the higher mortality amongst the French offices. But he did not think it would go far in that direction, because they could scarcely suppose that the Swiss lives would form a very large percentage of the whole lives assured by these four French companies. He was very glad to find that both his referees were in accord with him as to the procedure of the French actuaries in calculating the "exposed to risk." Mr. Manly did not appear to be quite convinced on the point. He thought, however, that a reference to the specimen schedule and the description of the process given in the work would bear out the opinion he had expressed in the paper. He was glad to say, owing to the suggestion kindly made by Mr. Young, there were several copies of the French experience now in the library, so that the members of the Institute would have an opportunity of going to the fountain head, and of forming their own judgment, not only on the point under discussion, but on all other points connected with it.