

from the mean of the levels at each end, the prismoidal rule can be at once applied.

In the case of "*Irregular cross-sections*," in which the inequalities of the surface of the ground have rendered it necessary to take more than these three levels, the rule will still apply after the following preparation. Conceive a series of vertical planes to pass through all the points on each cross-section, at which the transverse slope of the ground changes, and at which, therefore, levels have been taken, and to cut the other cross-section so as to divide the widths of the two *proportionally*.

Then the surfaces between these planes may be regarded as generated on our third hypothesis, and can therefore be calculated by the prismoidal rule; since it has been shown to apply to the surfaces of the first hypothesis, and these are known to be identical with those of the third. Thus, considering the ground on one side of a centre line, let one cross-section have depths of 6·00 in the centre, and 10·00 outside cutting. Let the other end be 8·00 in centre, 12·00 at four feet from centre, and 6·00 outside cutting. Let the half width of road bed be 10 feet, and side slopes 1 to 1. Then the vertical plane passing through the 12·00 level, at 4 feet, a quarter of the whole width (10+6), from centre, should cut the other section at one-quarter its width (10+10), or 6 feet, from centre. The depth at this point would be $6 + \frac{1}{4}(10-6) = 7·00$. This enables us to get a middle area; its depth being $\frac{1}{4}(8+6)$ at centre, $\frac{1}{2}(12+7)$ at $\frac{1}{2}(4+5)$ from centre, and $\frac{1}{4}(6+10)$ at the outside cutting.

The prismoidal rule can now be used. A similar preparation for calculation can be applied to cross-sections composed of any number of levels. The labor is much less in practice than it appears in description.

If the views here presented should meet with general acceptance, engineers would be enabled to economize much time and labor, since they would no longer feel themselves under the necessity of taking their cross-sections so near together that the ground between them should be approximately plane, but could take them as far apart as the ground varied uniformly, no matter how much or how far that might be.

The comparison of the results obtained on this principle, with those given by the usual methods, particularly that of "*Equivalent mean heights*," now usually employed when perfect accuracy is desired, will form the subject of another paper.

*On the Strength of Pillars of Cast Iron.** By EATON HODGKINSON, Esq., F. R. S., Professor of the Mechanical Principles of Engineering, University College, London.

In a previous paper on this subject (*Philosophical Transactions*, 1840), I had shown,—1st, that a long circular pillar, with its ends flat, was about three times as strong as a pillar of the same length and diameter with its ends rounded in such a manner that the pressure would pass through the axis, the ends being made to turn easily, but not so small as to be crushed by the weight; 2d, that if a pillar of the same length and dia-

* From the Lond., Edin., and Dub. Philos. Mag., August, 1837.

meter as the preceding had, one end rounded and one flat, the strength would be twice as great as that of one with both ends rounded; 3d, if, therefore, three pillars be taken, differing only in the form of their ends, the first having both ends rounded, the second one end rounded and one flat, and the third both ends flat, the strength of these pillars will be as 1—2—3 nearly.

The preceding properties having been arrived at experimentally, are here attempted to be demonstrated, at least approximately.

The pillars referred to in my former paper were cast from Low Moor iron No. 3; they were very numerous, but usually much smaller than those used in the present trials. I felt desirous, too, of using the Low Moor iron in the hollow pillars employed on this occasion, not on account of its superior strength, but its other good qualities. The pillars from this iron were cast 10 feet long, and from $2\frac{1}{2}$ to 4 inches diameter, approaching in some degree, as to size, to the smaller ones used in practice. The results from the breaking weights of these were moderately consistent with the formulæ in the former paper, with a slight alteration of the constants, rendered necessary by the castings being of a larger size, and therefore softer than before, a matter which will be adverted to further on.

The formulæ for the strength of a hollow pillar of Low Moor iron No. 2,—where w is the breaking weight, in tons, of a pillar whose length is l in feet, and the external and internal diameters D and d in inches, the ends being flat and well bedded—are as below:

$$w = 46.65 \times \frac{D^{3.55} - d^{3.55}}{l^{1.7}},$$

from formula in Phil. Trans. 1840;

$$w = 42.347 \times \frac{D^{3.5} - d^{3.5}}{l^{1.63}},$$

from formula in present paper.

To obtain some idea of the relative strength of different British irons, I applied, at Mr. Stephenson's suggestion, to Messrs. Easton and Amos, who procured for me twenty-two solid pillars, each 10 feet long and $2\frac{1}{2}$ inches diameter, cast out of eleven kinds of iron (nine simple irons and two mixtures). The pillars were all from the same model, and were cast vertically in dry sand, and turned flat at the ends, as the hollow ones had been; two being cast from the same kind of iron in each case. The simple unimixed irons tried were as below, and all of No. 1.

Mean breaking weight.		
Old Park iron,	Stourbridge,	29.50 tons.
Derwent iron,	Durham,	28.03 "
Portland iron,	Tovine, Scotland,	27.30 "
Calder iron,	Lanarkshire,	27.09 "
Level iron,	Staffordshire,	24.67 "
Coltness iron,	Edinburgh,	23.52 "
Carron iron,	Stirlingshire,	23.52 "
Blaenavon iron,	South Wales,	22.05 "
Old Hill iron,	Staffordshire,	20.05 "

The mean strength of the pillars from the irons above, varies from 20.05 to 29.50 tons; or as 2 to 3 nearly.

The pillars formed of mixed irons were found to be weaker than the three strongest of the unmixed series.

From many experiments, it was shown that the weight which would crush the pillars, if they were very short, would vary as 5 to 9 nearly.

The pillars in general were broken of four different lengths, 10 feet, 7 feet 6 inches, 6 feet 3 inches, and 5 feet, the ends of all being turned flat, and perpendicular to the axis. It was found that when the length was the same, the strength varied as the 3.5 power of the diameter; and when the diameter was the same and the length varied, the strength was inversely as the 1.63 power of the length. Both of these were obtained from the mean results of many experiments.

The formula for the strength of a solid pillar would therefore be

$$w = m \times \frac{d^{3.5}}{l^{1.63}},$$

where w is the breaking weight, d the diameter in inches, l the length in feet, and m a weight which varied from 49.94 tons in the strongest iron we tried, to 33.60 tons in the weakest.

The ultimate decreement of length, in pillars of various lengths but of the same diameter, varies inversely as the length nearly. Thus the ultimate decreements of pillars 10 feet, 7 feet 6 inches, 6 feet 3 inches, and 5 feet, vary as 2, 3, $3\frac{1}{2}$, and 4 nearly, according to the experiments, from which it appeared that the mean decreement of a 10-foot pillar was .176 inch.

Irregularity in Cast Iron.—The formulæ arrived at in this paper are on the supposition that the iron of which the pillars are composed is uniform throughout the whole section in every part; but this was not strictly the case in any of the solid pillars experimented upon. They were always found to be softer in the centre than in other parts. To ascertain the difference of strength in the sections of the pillars used, small cylinders $\frac{3}{4}$ -inch in diameter and $1\frac{1}{2}$ inches high, were cut from the centre, and from the part between the centre and the circumference, and there was always found to be a difference in the crushing strength of the metal from the two parts, amounting perhaps to about one-sixth. The thin rings of hollow cylinders resisted in a much higher degree than the iron from solid cylinders. As an example, the central part of a solid cylinder of Low Moor iron No. 2, was crushed with 29.65 tons per square inch, and the part nearer to the circumference required 34.59 tons per square inch; cylinders cut of a thin shell half an inch thick, of the same iron, required 39.06 tons per square inch; and other cylinders from still thinner shells of the same metal required 50 tons per square inch, or upwards, to crush them.

As these variations in cast iron have been little inquired into, except by myself, and have never, so far as I know, been subjected to computation, I have bestowed considerable trouble upon the matter, in an experimental point of view, and endeavored to introduce into the formulae previously given, changes which will in some degree include the irregularities observed.