A Formula for Calculating approximately the Self-induction of a Coil

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at the end of these notes were made after the meeting of the Physical Society at which the paper was read, and that some other additions have been made to the fourth part of these notes.

II. *A Formula for Calculating approximately the Self-induction of a Coil.* By PROFESSOR JOHN PERRY, D.Sc., F.R.S.*

AN easy empirical formula for enabling the coefficient of self-induction (*the inductance*) of any coil to be calculated with approximate correctness is very much wanted. I have been led to take up the matter through having to design an instrument with various cylindric coils whose inductances should bear certain relations to one another, and it was necessary for me to see at a glance with some exactness the effect of altering their dimensions. It is to be observed that for practical purposes extreme accuracy of calculation is of no importance, because the mechanical inaccuracies in constructing coils are considerable. It will be seen that my method of arriving at what may almost be called a rational formula has been suggested by Prof. James Thomson's ingenious method of considering the flow of water over a rectangular gauge-notch.

I consider only hollow cylindric coils. Thus fig. 1 shows the section of a coil whose dimensions are c centimetres measured radially, b parallel to the axis, mean radius of coil a , oo' being the axis. The supposition I make is that all the coils considered are so long (b so great) that, $\frac{c}{a}$ being the same in all, the field near the ends is the same in all. Without much straining of the meaning of words, we may say that the inductance of a coil is the total induction when unit current passes in the coil; and if there is only one convolution, it is proportional to the reciprocal of what may be called the magnetic resistance. My supposition comes to this: that if the dimension b is great enough, increase of the magnetic resistance produced by increasing b is proportional to the increase of b . Let MNM' (fig. 2) show the limiting length beyond which increase of resistance will be proportional to increase of

* Read June 20, 1890.

length. Evidently MM' must be a function of c and a , and if written in the shape $2sc$ it may be expected that s will be constant for small values of $\frac{c}{a}$. The magnetic resistance due, then, to the end parts, AB , of the coil (fig. 1) will be the same as that of the coil, fig. 2. Now the magnetic resistances of all similar coils are inversely proportional to their dimensions, and for all coils similar to that shown in fig. 2 for any value of a or c but for the same values of $\frac{c}{a}$, the magnetic resistance is inversely proportional to a , say that it is $\frac{p}{a}$,

p being a function of $\frac{c}{a}$. Now the remaining part of the coil (fig. 1) may be considered made up of coils of square section $c \times c$ and of mean radius a ; and the additional magnetic resistance introduced by each of these is inversely proportional to a if $\frac{c}{a}$ is constant. There are $\frac{b-2sc}{c}$ of them, and the total magnetic resistance, irrespective of the ends, is proportional to $\frac{b-2sc}{ca}$ if $\frac{c}{a}$ remains constant. Say that it is $(b-2sc) \frac{q}{a^2}$, where q is a function of $\frac{c}{a}$. The whole magnetic resistance is then

$$\frac{p}{a} + \frac{q}{a^2} (b-2sc).$$

And the inductance L for one winding is the reciprocal of this. If there are n windings,

$$L = \frac{n^2 a^2}{pa + qb - 2sqc} \quad \dots \quad (1)$$

As sc is comparable with the axial distance inside a coil from the end, at which the lines of force are no longer perceptibly curved, s may be supposed to be a proper fraction. In finding the values of p , q , and s , p and q being functions of $\frac{c}{a}$, and s being nearly constant, it is evident that if we, experimentally or by calculation, find L for various values of a , b , and c , it is enough if one of these remain constant. Mr. C. E. Holland has worked out for me the following values of $\frac{L}{4\pi n^2}$ in C.G.S. units, keeping $c=1$ centimetre, from the series published as Appendix III., page 321, vol. ii. of the second edition of Maxwell's 'Electricity and Magnetism.' Unfortunately, unless the ratios of $\frac{c}{a}$ and $\frac{b}{a}$ are small, this series does not enable L to be calculated. I propose later to amend the formula when I have, either by calculation or experiment, found L for values of $\frac{c}{a}$ and $\frac{b}{a}$ greater than $\frac{1}{2}$; and I regret that the difficulty of obtaining values of L in these cases is so great as it is. Calling Mr. Holland's calculated $\frac{L}{4\pi n^2}$ by the letter l , the following Table shows the results of his calculations:—

$a.$	$b.$	$c.$	$l.$
4	1	1	9.060
4	2	1	7.303
4	3	1	5.993
2	1	1	3.094
2	2	1	1.985
2	3	1	1.034
4	1	2	7.460
3	1	1	5.914
3	2	1	4.510
3	3	1	3.420
6	1	1	16.0436
6	2	1	13.527
6	3	1	11.704
10	1	1	31.8655
10	2	1	27.763
10	3	1	24.834

Now it is evident that if of any coil the dimensions are a , b , and c , then L for this coil is c times the L for a coil whose dimensions are $\frac{a}{c}$, $\frac{b}{c}$, 1. Hence Mr. Holland's numbers enable us to study the general case, so long as $\frac{c}{a}$ and $\frac{b}{a}$ are less than $\frac{1}{2}$. I find that with more than enough accuracy for practical purposes we may take, when $c=1$ centim.,

$$\frac{L}{4\pi n^2} \text{ or } l = \frac{a^2}{.2317a + .39 + .44b} \quad \dots \quad (2)$$

Hence generally, from the law as to similar coils, l is

$$\frac{c \frac{a^2}{c^2}}{.2317 \frac{c}{a} + .39 + .44 \frac{b}{c}}, \text{ or } \frac{a^2}{.2317a + .39c + .44b}.$$

So that generally

$$L \text{ in centimetres} = \frac{4\pi n^2 a^2}{.2317a + .39c + .44b} \quad \dots \quad (3)$$

I give it in this form to enable its accuracy to be tested. And it is obvious that q of (1) is a constant, whereas p is a linear function of $\frac{c}{a}$ including a constant.

$$L \text{ in centimetres} = \frac{n^2 a^2}{.01844a + .031c + .035b} \quad \dots \quad (4)$$

Or taking the ohm as 10^9 centimetres per second,

$$L \text{ in secohms} = \frac{n^2 a^2 \div 10^7}{1.844a + 3.1c + 3.5b} \quad \dots \quad (5)$$

If α is the cross sectional area pertaining to one winding, whether made up partly of insulation or not,

$$n = \frac{bc}{\alpha};$$

and if R is the resistance of the coil in ohms, as $R = \frac{2\pi a \rho bc}{\alpha^2}$,

$$\frac{L}{R} = \frac{V \div 10^9 \pi \rho}{.2317a + .39c + .44b}; \quad \dots \quad (6)$$

where V is the volume of the coil in cubic centimetres, ρ = resistance in ohms of a copper wire 1 centim. long, of smaller section than 1 centim., just as much smaller as the actual wire is than the covered wire.

Let ρ_0 = specific resistance of copper in ohms.

Taking it that if there is no insulation, $\rho = \rho_0 = 1.62 \times 10^{-6}$.

Let W_1 = weight of copper in the coil (disregarding the weight of the insulating material, this is the weight of coil),

W_2 = weight of volume of the coil of copper,

$$\frac{W_2}{W_1} = \frac{\alpha}{\text{section of copper}} = \frac{\rho}{\rho_0}.$$

It is evident that if for any coil whatever we calculate $\frac{L}{R}$ on the assumption that there is no space occupied by insulation, we have only to multiply this by $\frac{W_1}{W_2}$ to obtain $\frac{L}{R}$, taking insulation into account. Another way of putting our result is

$$\frac{L}{R} = \frac{V' \div 1000}{.728a + 1.33c + 1.5b}, \quad (7)$$

where V' is the volume, in centimetres, of *copper* in the coil. For a coil of given volume wound with given wire, to diminish the ratio of $\frac{L}{R}$, increasing c is nearly of the same importance as increasing b ; whereas these are about twice as important as increasing a . In fact $\frac{L}{R}$ is nearly proportional to $\frac{abc}{a + 2(b + c)}$; and of course it is less as the wire is finer, because the volume of the insulation is relatively greater.

It may be noted that the empirical formula given in this paper, although seemingly truer when $\frac{b}{a}$ and $\frac{c}{a}$ are smaller, is not true when $\frac{b}{a}$ and $\frac{c}{a}$ are zero. The inductance of one spire of wire whose section is infinitely small is infinite; and it would have been possible to get an empirical formula, fairly correct for ordinary values of a , b , and c , which would give an infinite value for L when $\frac{b}{a}$ and $\frac{c}{a}$ are zero, based on the expression given by Maxwell,

$$\frac{L}{L\pi n^2} = a \left(\log \frac{8a}{R} - 2 \right),$$

where R is a function of b and c . Such a formula would, however, be much less useful for ordinary purposes than the one I have given.