ART. XXVII.—*Note on a Method in Teaching Optical Mineralogy*; by F. W. MCNAIR.

THE writer has for some years given a short non-mathematical course in polarized light as related to crystals, for the purpose of preparing students to take up optical mineralogy, the latter being applied in turn to the study of rocks. The course has been necessarily very brief, non-mathematical, and confined to the bare essentials needed to develop its applications. Under these conditions an attempt has been made to give the student a logical basis for his conclusions, to develop in him some ability to reason about observed phenomena, and so to render him as far as may be independent of mere rules of procedure.

In the effort to condense the course, and to base its entire structure on the smallest possible number of new ideas, the form of the wave shell and the deductions therefrom have been rested as directly as possible upon the so-called reciprocal ellipsoid, as introduced by McCullagh. It will be remembered that this ellipsoid, whose equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

is constructed on semi-axes proportional to the square roots of the elasticities in the three principal directions. In the usual notation, these axes are named so that $a > b > c$ and their directions are labeled respectively A, B and C. This ellipsoid gives the wave shell, or ray surface, by the following construction: if through the center perpendicular to a given ray direction a plane is passed, the section is an ellipse, and if on the ray direction, distances be laid off proportional to the semi-axes of the ellipse the locus of all points so determined is the wave shell. Furthermore, the vibration directions belonging to the given ray lie in planes determined by the ray and the axes of the elliptical section.

If one may judge by the text books, the ellipsoid, whether that of Fresnel or this of McCullagh, is used in the non-mathematical presentations of this subject to obtain the wave shell, or rather its three principal sections, and is then immediately abandoned. The device which occurred to me some years since, and which I have found useful in obtaining results with my students, is to carry the use of the ellipsoid into a considerable number of the applications of the theory to the properties of crystals. The "section cut from the ellipsoid"

is used at once in accounting for the phenomena exhibited by thin crystal plates between crossed nicols. It is applied next to the simple cases of superposition of plates of even thickness, then to those cases in which the plates are not of even thickness. By its use the average student gains some facility in predicting the phenomena which will be observed in a given case, such, say, as that of an even plate over one shaped like a convex lens. Afterward the method is applied to crystal plates in convergent plane polarized light and perhaps the most conspicuous example of its usefulness lies in its application to the distinction between positive and negative crystals in the convergent beam of plane polarized light.

FIG. 1.

Let fig. 1 represent a central section through the convergent cone which traverses a plate cut perpendicular to the acute bisectrix. If the crystal is positive, this acute bisectrix is C , the direction of minimum elasticity. If for each ray direction the ellipsoid is constructed, and the section perpendicular to the ray is cut, the elliptical sections will appear in this figure only as traces marked E_1 , E_2 , etc. If now a diagrammatic plan be made on the assumption that the above section coincides with the axial plane, and, for brevity, with that plane in "the 45° position", and if the ellipses be diagrammed on this in their full area, that is, if each be tipped into the plane of the diagram before being drawn, we shall obtain something like fig. 2. On this figure lines representing the position of the hyperbola have been drawn to enable it to be referred to the biaxial interference picture, and the

ellipses have been numbered to correspond with those shown in fig. 1. The ellipse for the central ray of the cone being drawn perpendicular to the C direction has for its semi-axes a and b .

If now we consider the series of ellipses belonging to rays in the A-C, or axial, plane, represented in the section above, it will be seen that as we leave the central ray, passing outward to rays making a continually increasing angle with the central one, the axis perpendicular to the A-C plane remains constant

FIG. 2.

and of the value $2b$, while the axis in the axial plane decreases from its maximum, $2a$, toward the value $2c$. Furthermore, when the ray has the direction of the ray axis of the crystal the ellipse obtained for it is a circle. Beyond this ray the axis in the A-C plane is the minor axis of the ellipse. Thus it appears that the ellipses on one side lie with the major axes at right angles to the major axes of those on the other side, the

change occurring at the ray axis, which is close to the nose of the hyperbola of the picture.

Proceeding now from the center of the figure at right angles to the axial plane, the ellipses undergo no such change, the major axis now remains constant at $2a$ while the minor axis grows smaller, reducing from $2b$ toward $2c$. The major and minor axes of these ellipses represent the speeds of the two disturbances vibrating respectively in the planes of these axes. If we call the difference of these speeds Δ , as is convenient, it is easily seen that Δ near the nose of the hyperbola is zero, and that from this spot it increases toward the center of the figure, while at right angles to the trace of the axial plane it increases away from the center. Outside of the hyperbola and away from the center it increases in numerical value but is negative.

Suppose now that the quartz wedge is pushed over the plate, thin edge first, with its vibration directions as indicated by the ellipse drawn in its corner. It will readily be seen that this ellipse lies parallel to that for the central ray of the cone, and to all others inside of the hyperbola. The faster ray in the plate will also be the faster in the wedge, and the slower in the plate will be slower in the wedge. The effects of plate and wedge will be added. Now the location of the color bands depends directly on the value of Δ , hence for a given position of the wedge a particular band will be found where the combined Δ of wedge and plate equals the Δ of the plate alone at the original location of the band before the superposition of the quartz wedge. As the wedge is advanced the combined Δ at any point of the plate is increased continuously, while any particular value of the combined Δ necessarily shifts toward that spot in the plate where Δ is least. Therefore, as the wedge is pushed over, the color fringes will travel from the center toward the nose of the hyperbola. Likewise, on the line at right angles to the axial plane they must travel in toward the center.

Outside of the hyperbola, however, the ellipses are in crossed position, and the slower ray in the plate is now the faster in the wedge. A difference of effects results, and a given color fringe will now travel toward a position in the plate where Δ is numerically greater. Therefore, outside of the hyperbola the color fringes will travel away from the nose. Of course, a reversal of the quartz wedge, either by turning it over to reverse the position of its ellipse or pushing it thick end first instead of thin, reverses the travel of the fringes. For a negative crystal, the travel would be in each case in the opposite direction.

There are approximations in the foregoing which have not

been pointed out, but which are sufficiently close to avoid the possibility of error in any practical application. I have used this device with many students, and its justification rests in the readiness with which a student who once comprehends the meaning of the ellipsoid becomes independent in his application of this test of the quartz wedge, applying the wedge in either position, and reasoning out his results with an assurance of correctness. If there were no other application I should consider that the results have justified my continued use of the scheme. It applies of course to the uniaxial crystal and the quarter wave-plate, or the first order red, as readily as to the biaxial case.

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