wards. I found the application of 5% solutions of gelatine to be as effective and less deleterious than any other agent. But it is obvious that the differences in environment—from a normal medium to one to which such gelatine has been added—when repeated more than once especially within a short period for purposes of measurement must have some influence upon later growth even when the greatest care is taken, and it is highly probable that this influence is seen in the difference in index correlation to which Professor Pearson calls attention in his note below.

Note on Dr Simpson's Memoir on *Paramœcium caudatum*.

(1) DR SIMPSON'S results are clearly not purely homotypic as far as the correlations between pairs of members are concerned, but largely affected by the growth factor, the measurements of pairs having been made at all sorts of intervals after division. They cannot therefore be used in their present condition as a means of appreciating homotypy in the results of linear division. In order to reach pure homotypy we must obtain measurements of growth in length and breadth during a variety of periods after division. The labour, however, of obtaining measurements on pairs of members at a definite interval after division being very great, Dr Simpson was only able to provide the set of measurements in the accompanying Table IV. These involve eleven pairs measured twice and three of them measured thrice. It will be seen at once that the growth is very irregular, and even more so in breadth

TABLE IV.

	Divided	Measured	Length	Breadth	Measured	Length	Breadth	Measured	Length	Breadth
1st pair	May 22, 11 a.m.	5 p.m.	210	70	May 23, noon	225	75	May 24, 11.30 a.m.	225	80
			230	55		245	65		245	70
2nd pair	May 22, 11 a.m.	4.30 p.m.	230	80	May 23, 1 p.m.	240	90			
			245	70		260	90			
3rd pair	May 22, 10 a.m.	4 p.m.	225	60	May 23, 1.30 p.m.	230	75			
			220	60		225	70			
4th pair	May 22, 5 a.m.	10 a.m.	210	75	May 23, 12.30 p.m.	220	75	May 24, noon	225	75
			220	75		230	75		235	80
5th pair	May 22, noon	5.30 p.m.	240	85	May 23, 1.15 p.m.	245	85	May 24, 1.30 p.m.	245	85
			230	80		235	80		235	80
6th pair	May 22, 6 a.m.	3 p.m.	220	65	May 23, 4 p.m.	230	70			
			215	70		220	75			
7th pair	May 23, 5.30 p.m.	6.30 p.m.	175	65	May 24, 1 p.m.	230	85			
			180	60		240	80			
8th pair	May 23, 4.30 a.m.	6 p.m.	195	60	May 24, 1.30 p.m.	230	80			
			220	70		245	90			
9th pair	May 23, 4 a.m.	6.15 p.m.	230	60	May 24, 12.30 p.m.	250	70			
			215	75		240	90			
10th pair	May 23, 4 a.m.	6.45 p.m.	225	70	May 24, noon	260	95			
			235	65		270	85			
11th pair	May 23, 1 p.m.	5.45 p.m.	205	60	May 24, 2 p.m.	220	75			
			230	70		235	90			

than in length. Thus of the six individuals measured thrice, four exhibited no change at all in length, three no change at all in breadth during the last 24 hours. It seems therefore that individuals either cease growing at a certain interval after division or else have periods of rest. The unit of 5μ does not seem sufficiently small to obtain accurate results for breadth measurements. For, anything between 72.5 and 77.5 would be registered as 75. But anything just below 72.5 or just above 77.5 would be registered as 70 or 80 respectively. Thus a real growth just less than 5μ might not appear at all, and a real growth just greater than 5μ might appear as a growth of 10μ . As the standard deviation in breadth for the whole series is less than twice 5μ , this cannot fail to have serious influence in dealing with such a short series as 11 pairs. Hence the conclusions reached below are purely tentative, but they will suffice to show the importance of allowing for the growth factor in dealing with like cases of homotyposis.

(2) In order that we may safely apply the results deduced for growth from the present series to Dr Simpson's earlier series, we must determine whether the former may, as far as its statistical constants are concerned, be considered a random sample of the latter. Accordingly the new measurements were taken as if they had been 44* original measurements of length, breadth, etc., on separate individuals.

TABLE V.
Constants of Two Series.

Organ	Mean		Standard Deviation		Coefficient of Variation	
	100 series	44 series	100 series	44 series	100 series	44 series
Length	229.050	227.954	19.152	18.099	8.361	7.940
Breadth	68.125	74.205	9.155	9.997	13.439	13.472
Index	29.913	32.557	4.029	3.470	—	—
Interval from division in hrs.	—	18.023	—	11.153	—	—

TABLE VI.
Correlations of Two Series.

Organs	100 series	44 series
Lengths of 1st and 2nd Members	.914 ± .011	.698 ± .074
Breadths of 1st and 2nd Members	.782 ± .028	.574 ± .097
Breadth of 1st and Length of 2nd Member†	.447 ± .054	.641 ± .060
Indices of 1st and 2nd Members	.684 ± .038	.114 ± .100
Length and Breadth of 1st Member† ...	.421 ± .055	.594 ± .066

* In order not to weight individuals and to leave the series as homogeneous in age-variation as possible the three pairs measured thrice were only included as far as their first two measurements are concerned.

† It is a remarkable result that breadth of second member is more highly correlated than breadth of first member with length of first member in both Dr Simpson's series. How far is some element of "compensation in division" indicated?

The least examination of these two series shows that one cannot be considered as a random sample of the other. Not only are the means of breadth and index sensibly different, but there has been a general lowering of the direct and a raising of the cross-correlations, which is far beyond the limits suggested by even the large probable errors. As for the index-correlation, its immense change is very mysterious*. There has been either considerable change in environment, in average interval from division at which measurements were taken, or, as suggested in (1), the breadth to 5μ is really not close enough to get consistent results.

It appears therefore that, as the second series cannot be looked upon as a random sample of the first, it would be idle to attempt to use results for growth deduced from it to correct for the growth factor the values of the correlations obtained in the first series. Accordingly all that seemed possible was to apply the growth results obtained for the short series to the correlations obtained for the short series and so reach some appreciation of the screening influence of growth on pure homotyposis.

(3) The interval after division was accordingly correlated with length, breadth and index for the 44 cases, with the following results:

TABLE VII.

Interval and Size.

Organ	Correlation
Length and Interval ...	$\cdot547 \pm \cdot071$
Breadth and Interval...	$\cdot538 \pm \cdot072$
Index and Interval ...	$\cdot273 \pm \cdot094$

It will be seen at once that growth-correlation is a most sensible factor in the resemblance between individuals when pairs of them are measured at the same interval after division. It is least for the index, but is quite sensible even in this case. In other words, the index is a more reliable character than absolute length or breadth to deduce *pure* homotyposis from†, but clearly the average percentage growths in length and in breadth are not equal, and accordingly the index itself alters with the interval from division.

The lengths between 3 and 38 hours interval from division were plotted to the intervals; the results were, of course, very irregular, but a straight line certainly would represent them as effectively as any curve. For the data available accordingly the regression line of age and size was taken as the average curve of growth. In other words, for an organ x the growth in time τ was taken to be $\frac{r_{x\tau}\sigma_x}{\sigma_t}\tau$, where $r_{x\tau}$ was the correlation for interval from division and size tabled above in VII, σ_x the standard deviation of the organ x and σ_t that of the intervals of time given in Table V. Thus if x be the observed size of an organ at time t its most probable size, at the mean interval $\bar{t}$ after division, would be

$$x' = x - \frac{r_{x\tau}\sigma_x}{\sigma_t}(t - \bar{t}).$$

* The value of the index-correlation was also determined for the second series indirectly from the formula *R. S. Proc.* Vol. 60, p. 493. The value found was $\cdot180$, which agrees as well as could be expected with the direct value $\cdot114$ given above.

† Used by K. Pearson for gills of mushrooms and by E. Warren for ivy leaves for this reason: see *Phil. Trans.* Vol. 197, A, pp. 240, 888.

Let y be the observed size of any other organ at the same interval after the mean time, then its probable size at the mean time is given by

$$y' = y - \frac{r_{xy}\sigma_y}{\sigma_x}(t - \bar{t}).$$

Clearly means of x' and y' will be equal to means of x and y , and we easily find*

$$\sigma_{x'}^2 = \sigma_x^2(1 - r_{xt}^2), \quad \sigma_{y'}^2 = \sigma_y^2(1 - r_{yt}^2), \dots \dots \dots (i)$$

$$R_{x'y'} = \frac{\rho_{xy} - r_{xt}r_{yt}}{\sqrt{1 - r_{xt}^2}\sqrt{1 - r_{yt}^2}} \dots \dots \dots (ii)$$

where $R_{x'y'}$ is the correlation of the x' and y' characters allowing for growth and ρ_{xy} of x and y , or without allowance for growth.

If x and y are the same organ for the pair, we have

$$R = \frac{\rho - r^2}{1 - r^2} \dots \dots \dots (iii)$$

where r is the correlation of growth and interval of time for the organ in question.

If we apply these formulae to the values of ρ given in Table VI and to the values of r_{xt} given in Table VII, we deduce the values of the homotypic correlations, when allowance is made for growth. We find

TABLE VIII.

Homotyposis allowing for Growth †.

Lengths of 1st and 2nd Members	·569 ± ·074
Breadths of 1st and 2nd Members	·400 ± ·151
Length of 1st and Breadth of 2nd Member ...	·491 ± ?
Indices of 1st and 2nd Members	·043 ± ?
Length of 1st and Breadth of 1st Member ...	·425 ± ?

The first two of these results are quite as satisfactory as we could expect from such data. They fit in fairly well with the homotypic correlation distribution clustering about ·5. The fourth is very improbable. But the drop from ·664 to ·114 in index correlation between Dr Simpson's first and second series of measurements is so inexplicable that we cannot possibly expect anything from the index figures. The third result clearly does not satisfy the theorem I have given for cross-homotyposis‡. But the lowness of the index correlation, and the fact that the length of the 1st member is more highly correlated with the breadth of the 2nd member than with that of the 1st would undoubtedly be explicable, if, as I have suggested, there be some principle of "compensation in division" at work. We can only hope that at some future time we may have available a much more extensive system of growth measurements, made on the *same* long series as the homotypic measurements. Without this correspondence it is impossible to distinguish how much of the degree of resemblance between members resulting from the same division is due to growth and how much to pure homotyposis. But the results for growth in Table VII combined with those for pure homotyposis in Table VIII suffice to show that great care must be used in not treating what are really coefficients of gross resemblance as in Table V—due to pure homotyposis+growth+individual environment of the pairs of members—as in any way a proper measure of the first factor, i.e. of pure homotyposis only.

* These formulae may be obtained by straightforward algebra from the above results, but they are really simple cases of a theory developed at length in an unpublished memoir on "Selection allowing for Growth"; (ii) is clearly the partial correlation coefficient for the time-factor constant.

† Probable errors in first two cases calculated from unpublished formulae.

‡ *Phil. Trans.*, Vol. 197, A, p. 287.