

Terrestrial Magnetism *and* *Atmospheric Electricity*

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ON THE DISTRIBUTION OF MAGNETISM OVER THE EARTH'S SURFACE.—III.¹

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Chapter III. Reduction of Magnetic Observations to an Epoch—
Concluded. [Pp. 114-132.]

Theory of the Magnetic Variations in Anomalous Regions.
[Pp. 114-127.] Both the secular and the diurnal variations, under normal conditions, change comparatively slowly from point to point on the Earth's surface, and on this consideration is based the reduction of the elements to an epoch. Hence the quantities, ΔX , ΔY , ΔZ , which must be added to the rectangular magnetic components X (N), Y (W), Z (vertical, + down), for a given moment, are taken the same for a considerable area. The diurnal variation corrections, as computed for a given hour, local time, are considered the same for entire parallels. But if at two points close to one another the mean declination is very different, then these corrections for declination and for horizontal intensity, as von Bezold² showed, may be of an entirely different type.

He investigates the diurnal change in the following way: Plotting in the XY -plane, for each moment, the points whose co-ordinates are equal to the values, ΔX , ΔY , and connecting them, there results what we call the vector diagram (fig. 1). If MM be the direction of the magnetic meridian, the perpendicular from t (corre-

¹ For previous installments see this Journal, Vol. XIII, p. 105 and p. 161.

² W. VON BEZOLD. Zur Theorie des Erdmagnetismus. Sitzber. Berl. Akad, 1897, pp. 404-449.



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Independently and almost simultaneously Pilchikoff⁴ pointed out that if, at a given locality, an anomalous force be added to that of normal or undisturbed magnetism, the variations of declination, inclination, and horizontal intensity may be different from those pertaining without the presence of the anomaly. The author first called attention to the fact that the usual methods of reduction are not applicable to surveys in strongly anomalous regions, and gave formulae for the reduction of the horizontal component and of the declination.

Now the question arises, cannot there be cases where the hypothesis, which lies at the foundation of von Bezold's and Pilchikoff's investigations, fails, *i. e.*, cannot local disturbances show an influence on the variations ΔX , ΔY , ΔZ ? If an anomaly exist, due to masses below the surface possessing magnetic permeability, then, as the total force of terrestrial magnetism, so also its variations must produce a corresponding induction in the magnetic masses and a change in their magnetic density.

Let us consider the case of one isotropic homogeneous mass, and find out how the changes of the rectangular components for the locality, and the normal and anomalous distributions of magnetism are related, supposing that there is no hysteresis. [The following equations are developed on pp. 117-121.]

$$\begin{aligned}\Delta X_a &= a_{00} \Delta X_n + a_{01} \Delta Y_n + a_{02} \Delta Z_n, \\ \Delta Y_a &= a_{10} \Delta X_n + a_{11} \Delta Y_n + a_{12} \Delta Z_n, \\ \Delta Z_a &= a_{20} \Delta X_n + a_{21} \Delta Y_n + a_{22} \Delta Z_n.\end{aligned}$$

Here the quantities with the subscripts n denote the variations of the normal magnetic elements, whereas, those with subscripts a apply to the variations due to the anomalies, so that at a point P in an anomalous region we have :

$$\Delta X = \Delta X_n + \Delta X_a; \Delta Y = \Delta Y_n + \Delta Y_a; \Delta Z = \Delta Z_n + \Delta Z_a.$$

For the determination of ΔX_a , ΔY_a , ΔZ_a , it is necessary to derive the coefficients a by the method of least squares, simultaneous observations having been made at two points, one in the disturbed region, and the other in the region beyond.

These equations evidently hold for several homogenous isotropic masses, on condition, however, that the field produced within the volume of each mass by the other masses, may be considered homogeneous. [To judge of the value of the coefficients, $a_{00} \dots a_{22}$, the preceding theory is applied to the simplest case,

⁴M. PILTSCHIKOFF: Zur les variations périodiques des éléments du magnetisme terrestre dans les régions anomaes. Académie de Toulouse, 1900.

that of a sphere, pp. 122-125. It is found that the changes ΔX_a , etc., may reach considerable values and far exceed ΔX_n .]

In the above equations we see that in the case $Y=Z=0$ and $\Delta X=\Delta Z=0$, i. e., if we are concerned with a single component and its variations:

$$X_a = a_{00} X; \Delta X_a = a_{00} \Delta X_n.$$

By X_a we had denoted that part of the anomalous component in the direction of the X axis, which is produced by magnetism induced in a mass. The equations show that the larger the anomalous component is, the greater becomes the anomalous variation, ΔX_a , and conversely.

The same may be said with regard to the component Y_a and Z_a . Generally speaking, therefore, the greater the anomalous force at a given point, the greater is its variation, yet instances may occur for which some of the terms in the right-hand members of the equations cancel each other so that the changes ΔX_a , ΔY_a , ΔZ_a do not correspond to the values of the components X_a , Y_a , Z_a .

Turning to my variations at the points A and B, the following table⁵ is obtained:

Hour	POINT A		POINT B	
	ΔX_a	ΔY_a	ΔX_a	ΔY_a
h		γ		γ
8 a	+	17	—	17
9	+	15	—	22
10	+	11	—	12
11	—	7	+	21
Noon	—	20	+	48
1	—	12	+	52
2	—	14	—	7
3	—	1	+	2
4	—	4	+	10
5 p	+	11	—	10
Range	37	41	155	18

⁵ This table gives the results of the author's variation observations at two stations, three kilometers apart, in the anomalous region, Kriwoi-Rog north of Odessa, Russia. The observations extended over ten hours. At the point A, the horizontal intensity was 0.043 C. G. S., and at B, 0.464, or about eleven times more. The observed ranges were as follows:

STATION A		STATION B.	
Declination,	29.0 (in Odessa 7.6)	5.0 (in Odessa 8.3)	
Hor'l Intensity,	217 (in Odessa 357)	1797 (in Odessa 387)	

It will be recalled that Leyst also made extensive variation observations in the disturbed region, Kursk. The author's declination variations confirm Leyst's, however, the large range in H at station B, for large value of H was an unexpected phenomenon, and the author was thus induced to take up his theoretical investigation. Cf. T. M. vol. XIII, pp. 173-174.—ED.

This table shows that the variations $\Delta X_a, \Delta Y_a$, may attain to large values and far exceed the normal variations, $\Delta X_n, \Delta Y_n$. It will be noted that the range in ΔX_a for B is about four times that for A . [$1\gamma = 0.00001$ C. G. S.]

Hence we may consider it as established that in anomalous regions the variations considerably depart from the normal ones, and can not be computed in advance unless the coefficients $a_{00} \dots a_{22}$ have been determined from observations. The study of their geographical distribution in anomalous regions, where they may change within wide limits, deserves great attention. If we go further, since there are not many points with normal values of the elements (and, besides, the definition of "normal magnetism" can not be considered as well-established), we may speak in general of a purely local character of variation; whence the question arises, what should be considered the *normal* changes of terrestrial magnetism at a given point?

We should further note that the normal and anomalous changes may be connected by a more complicated relation than that established by the above equations, if the fundamental suppositions can not be considered as correct within known limits (*e. g.*, the substance may not be homogeneous, there may be an aggregate of masses acting differently, hysteresis may play an essential part, etc.).

We must now recognize that the reduction to an epoch of observations taken in strongly disturbed regions can not give close approximations to the truth, and it remains to subject such observations to a further discussion.

Scheme of reduction of observations. [Pp. 128-132.] In conducting and reducing a survey the object is to obtain, for each point, the most characteristic values of the elements, free from casual changes.

What, in particular, should be considered the value of any element at the central station (the observatory) for the epoch of the survey? If all observations are said to be reduced to the 1st of Jan., 1895, then by "value of the element at that epoch" we do not mean the quantity actually observed at midnight between Dec. 31, 1894, and Jan. 1, 1895, but the average of hourly values for several days before Jan. 1st and as many days after. In France the average is taken for the whole of December and January, and,

still better, the average may be taken for the two contiguous years. Such averages may be considered free from casual disturbances. Therefore it is desirable to obtain similar averages for each point.

In the reduction several assumptions are made, which are only partly correct, as we have seen. These assumptions are:

1. The diurnal variation in the whole region of the survey, is the same as the central observatory.

2. Magnetic storms occur by absolute time, simultaneously for the region.

Suppose some element, $e_{t, \tau}$, has been observed in the year t , at the hour T (local time). Its value e_0 at midnight, and for the epoch t_0 is required. Usually, during the survey, a magnetograph is continuously in operation at the central observatory, and its registerings are available. Suppose that the diurnal variation of the element (*i. e.*, its departure from the mean) has been represented by a curve for the month when $e_{t, \tau}$ was observed.

We first reduce the observed $e_{t, \tau}$ to the mean of the day of observation. To this end we measure on the curve, the deviation from the mean for the hour T , which we denote by ΔE_T . The mean for the day becomes $e_{t, \tau} - \Delta E_T$. Now this value is to be corrected on account of possible disturbances. For this purpose we measure on the magnetograph curve the element E for the absolute moment, when the observation $e_{t, \tau}$ was taken, and on the curve of mean diurnal variation, the value E' for the hour $T + \lambda$, λ being the longitude of the point with reference to the observatory. Assuming—which is generally nearly correct—that the mean diurnal variation for the month corresponds to the variation of the element on quiet days, the quantity $E - E' = \Delta E'$ expresses the disturbance at the time of observation, and should be subtracted from $e_{t, \tau} - \Delta E_T$. Thus the mean value of the element for the day, free from the effect of storms, becomes $e_{t, \tau} - \Delta E - \Delta E'$.

This quantity remains to be corrected for the secular variation during the interval $t - t_0$. On a chart or table of secular variation we find for the latitude and longitude of the point, its secular variation per annum. Then the required value of the element for the epoch t_0 will be (supposing secular variation to be proportional to time): $e_0 = e_{t, \tau} - \Delta E - \Delta E' - x (t - t_0)$.

This method of reduction has to be applied when there is an observatory available for a large district of the survey. If, however, the district is not large, and it may be assumed that within its extent all changes take place simultaneously, it is better and more convenient to apply a different method.

Let e be the observed value of any element; E its value at the observatory at the same time, E_0 its value at the same place for the epoch t_0 (in the sense pointed out above). If e_0 is the required value for the point of observation, then $e_0 - e = E_0 - E$, and therefore: $e_0 = e + E_0 - E$.

This method reduces the observation e to an epoch in a much simpler manner than the preceding one. It was used during the survey of France.

Now the question arises, when is this latter method applicable, *i. e.*, for what limitation of area are the assumptions true? This question has to be solved for each individual case, *i. e.*, for each observatory the case is determined, within which, in an interval of time equal to the greatest distance in longitude (E or W .) from the observatory, the change of the element as to diurnal variation is less than the error to be expected in the reduction. Thereupon it must be determined how far N . and S . the area should extend, in order that, within the limits of the desired accuracy, the same diurnal variation may be assumed. Evidently the second question must be solved for the first method of reduction too, but unfortunately it can be answered only after an accurate investigation of the dependence of diurnal variation on latitude and longitude. At present we only know the qualitative side of the matter, *viz.*, that the diurnal variation of the horizontal elements increases with approach to the pole, but we have no accurate knowledge of the law of increase.

The third problem (regarding the secular variation) may be solved, if there are at least a few points in the region for which the secular variation is known.

The first of the above assumptions respecting the diurnal variation is answered very simply with the aid of the hourly record of the change in the elements kept at the central observatory. Thus, let us take, *e. g.*, the month of greatest diurnal variation for Odessa—July. In the following table of mean diurnal variation we find the change of the elements from hour to hour (only for the day time) without reference to sign :

Hourly Changes in the Magnetic Elements for the Month of July at Odessa, Russia.

HOURLY INTERVAL	CHANGES IN		
	Declination	Horizontal Intensity	Inclination
h h	'	γ	'
5—6 a	1.35	3	0.2
6—7	0.42	6	0.5
7—8	0.02	9	0.5
8—9	0.69	7	0.4
9—10	1.79	2	0.1
10—11	1.96	4	0.4
11—12	2.14	5	0.3
12—1	1.67	5	0.4
1—2	0.95	1	0.1
2—3	0.53	2	0.0
3—4	1.34	1	0.1
4—5	1.72	0	0.1
5—6	0.86	0	0.0
6—7	0.42	4	0.2
7—8 p	0.20	2	0.2

Hence we see that the greatest hourly change of declination takes place from 11 to 12 o'clock (2'.1), of inclination and horizontal intensity, from 7 to 8 a. m. (0'.5; 9γ). Taking as accuracy of reduction $\pm 1'$ for declination and inclination, $\pm 5\gamma$ for horizontal intensity, we find that the difference in longitude may amount to half an hour, or 7 to 8 degrees, *i. e.*, the second method is applicable in the whole area of S. W. Russia with reference to the one observatory at Odessa.

Turning to A. A. Tillo's secular variation charts, we see that in European Russia the declination for the middle of the nineteenth century changed from $-7'$ to $-9'$ for one year, but in the Black Sea only from $-5\frac{1}{2}'$ to $-6\frac{1}{2}'$; the inclination changed in all Russia from $-2'$ to $+0.5$, but in the Black Sea only from $-1\frac{1}{2}'$ to $-2\frac{1}{2}'$. Finally the change of horizontal intensity was from $+15\gamma$ to -10γ for all European Russia, and from $+15\gamma$ to $+9\gamma$ in the area of the Black Sea.

This shows that for the determination of secular variation one observatory is sufficient for S. W. Russia. But it cannot be shown that another, or even several other observatories, would be useless in that region, as we know nothing concerning the variability of storms, nor can it be averred that in this area no purely local influences make themselves felt on secular as well as on diurnal variation of the elements.

Taking into consideration that a survey is not only to furnish charts of the distribution of magnetism, but also material for the study of secular variation, it will be necessary, also, when the second method of reduction is applied to visit some stations twice or oftener. Especially deserving of attention in this connection are the anomalies, where the secular variation, as yet, has not been studied at all.⁶

Chapter IV. [Pp. 133-182.]

REDUCTION OF MAGNETIC OBSERVATIONS TO THE SAME LEVEL.

Influence of Altitude on Observed Magnetic Elements. It has been remarked long ago that the magnetic elements may change with ascent to mountain tops. Thus Humboldt found that on ascending the Santa Fè (Guadeloupe) and Silla (Caracas) the horizontal intensity had diminished, in the ascent; however, for the volcano Antisana it had increased. Biot, too, found decreasing intensity on the ascent of Mt. Superga.

The ascent of Gay-Lussac and Biot (up to 6884 meters) did not show an influence of height upon the range of oscillation of the magnetic needle.

Of more modern investigations we shall name the observations of Koristka on four points close to Schemnitz. He found:

Height	Horizontal Intensity
3400 feet	0.1862 C. G. S.
2800	.1927
2000	.2032
1500	.2041

whence it follows that the intensity decreases noticeably. However, it is necessary to refer to these observations with care on account of the primitiveness of the methods—thus a seconds' pendulum served for the determination of the time of vibration, instead of a chronometer, etc.

Kreil investigated the change of the total intensity for seven mountainous regions, taking for the intensity on the surface means

⁶ The experience encountered in recent work is that the secular variation corrections may be such as to make it inadvisable to rely solely upon the secular changes found at but one place in the region covered. Also the outcome of land survey work makes it questionable whether it is worth while to apply elaborate reductions on account of diurnal variations, especially so with respect to horizontal intensity and inclination. A much better general result may be obtained by spending the time instead in multiplying stations, and thus reducing the far more serious error, viz., the "station error".—ED.

from various points in a circle. Thus it follows from his observations that the entire intensity decreases, and that this diminution is equal to 0.0047 C. G. S. for 1000 meters. At the same time Kreil draws erroneous conclusions from his observations on account of a simple error in the computations already noticed by Liznar.

Lamont inferred from his observations, that altitude, apparently, has no influence on the magnetic elements.

Hartl found, that the difference in inclination at the two regions, Oldach and Zirbitzkogl (the difference in height being 1523 meters) was 0'.2, and of the horizontal intensity, 0.00016 C. G. S.; for Kals and Adlersruhe (difference in height 2163 m.) the change in inclination was 1'.6, horizontal intensity, —0.00098, and finally for Nikolsdorf and Ziehenkopf (difference in height 1770 m.), the inclination changed 1'.8, and the horizontal intensity 0.0008.

O. E. Meyer, having made observations on the hills of Saxony, found, that the horizontal and vertical intensity increased in the ascent, and the inclination did not considerably diminish. Thus, if at the foot of the Schneekoppe he puts $H = 1$, then on the summit H is = 1.020.

I. B. Messerschmidt took observations by means of the mountain magnetometer of Meyer, and determined the relation of the entire force in Swiss mountain regions with the force in certain central places. He arrived at the conclusion, that the total force diminished with altitude.

Alfonso Sella took observations on Monte Rosa (Glacier "Grenz" at 4300 m., and Glacier "Garstelet"), in Biella (high plain on the left bank of the brook Cervo), and at Rima (Farnesina). Eliminating the variation of horizontal intensity, we find:

$$H_B : H_R = 1.0067;$$

where H_B and H_R denote, respectively, the intensity at Biella and on Mt. Rosa. Thus a diminution of 0.001 per kilometer of altitude is obtained for the horizontal intensity. The author calls attention to the magnetic character of the outcropping rock.

Preston made observations on the Hawaiian Islands at different altitudes. The results obtained by him are compiled on p. 42, Vol. II, *Journal Terrestrial Magnetism*.⁷

⁷ The highest station was 3981 meters above sea-level. Without a more careful study no safe conclusion could be drawn though it appeared that the total intensity, as well as the components with the exception of the eastward ones (I 's), decreased when the high altitudes were reached. However, it should be borne in mind that these observations were made in volcanic regions known to be locally disturbed.—ED.

Van Rijckevorsel and van Bemmelen undertook measurements of the elements during an ascent of Mt. Rigi and arrived at the conclusion, that the horizontal intensity diminished, but the vertical one increased at a somewhat higher rate. The latter, apparently increased by 0.00020 C. G. S. per kilometer. This quantity is very small, if we take into consideration the unavoidable errors of observation. The authors call attention to the circumstance, that the mountain proved a weak center of attraction.

Liznar discussed the observations for Austria-Hungary by the method of least squares, and found the following values for the increase of the elements with altitude (the intensities are given in units of the fourth decimal C. G. S., the heights in meters):

$$\begin{aligned}\delta X &= -0.00344 \text{ } h; \quad \delta Y = +0.00295 \text{ } h; \quad \delta Z = -0.00636 \text{ } h; \\ \delta H &= -0.00290 \text{ } h; \quad \delta F = -0.00685 \text{ } h; \quad \delta D = +0.00503 \text{ } h; \\ \delta I &= 0.00065 \text{ } h.\end{aligned}$$

Whence it follows that the west component and the declination increase with altitude, the remaining elements, however, diminish.

Pochettino determined the difference in horizontal intensity for two regions, one situated above the other by 2100 m. There was taken into consideration the variation in horizontal direction. His observations show, that this component diminishes by 0.0005 C. G. S. per kilometer of ascent.

Larger diminutions were found by Moureaux in the Pyrenees:

Locality	Altitude	Total Intensity	Difference from Bagnères Altitude	Bagnères Intensity
Bagnères-de-Bigorre,	540 m	0.4516	—	—
Campan,	668	.4500	128 m	—0.0016
Col de Sencours,	2366	.4492	1826	—0.0024
Pu du Midi,	2858	.4477	2316	—0.0093

From the results of the observations above detailed, the general law of diminution with altitude becomes quite apparent. The discrepancies, as we recognized before, are explained by local influences so that a definite answer to this interesting question cannot at present be drawn from observations on mountains. It is necessary to have recourse to theoretical considerations based on the properties of the potential.

Laws of the changes of magnetism with altitude. [Pp. 138]. As known, Gauss gives a method of computing the value of each of the elements by means of converging series [spherical har-

monics] from which it is possible to find the diminution with altitude of that part of the magnetic force represented by the series. Liznar was the first to apply this method, but, as was seen above, he failed to obtain true results on account of neglecting certain terms.

For that part of the Earth's magnetism, which is not represented by Gauss's formulæ, further laws are not at hand. Therefore we must have recourse to other methods.

[Pp. 139-155 are chiefly mathematical and can be followed readily in the original. Starting with the well-known Gaussian formulæ and after various transformations and introducing Schmidt's functions R_m^n , the author finally derives equations (A).]

$$\left. \begin{aligned} X_h &= X_o - \frac{3 X_o - a'}{R + h} \cdot h, \\ Y_h &= Y_o - \frac{3 Y_o - a''}{R + h} \cdot h, \\ Z_h &= Z_o - \frac{3 Z_o - a'''}{R + h} \cdot h. \end{aligned} \right\} \quad (A)$$

Here X_o , Y_o , Z_o are the magnetic components at sea level and those with subscripts h are the values at altitude h , R is the Earth's mean radius and a' , a'' , a''' represent expressions derived from the potential formulæ. In Liznar's formulæ the latter terms were considered negligible.

Differentiating above expressions we get:

$$-\frac{dX_h}{dh} = \frac{R(3X_o - a')}{(R + h)^2} = \frac{3X_o - a'}{R + h} \left(1 - \frac{h}{R}\right). \quad (B)$$

From which it is seen that the rate of diminution varies with altitude, h , being less with increase of altitude and greatest at the surface. If ΔX be the average diminution for the distance between sea-level and height h , i. e., $X_h = X_o - h \Delta X$, then

$$\Delta X = \frac{3X_o - a'}{R + h}, \quad (C)$$

which differs from the previous expression only by the factor

$$\left(1 - \frac{h}{R}\right).$$

Resolving [p. 155], in accordance with Leyst's proposal, the Earth's magnetism into two parts—one due simply to the latitude

terms in the Gaussian expression, called the "normal" part, and the other due to all the remaining terms, the "abnormal" part:

$$X_o = X_\phi + X_a \quad , \quad (D)$$

$$\Delta X = \Delta X_\phi + \Delta X_a \quad , \quad (E)$$

$$\Delta X_\phi = \frac{3 X_\phi - a'_\phi}{R + h} \quad , \quad (F)$$

$$\Delta X_a = \frac{3 X_a - a'_a}{R + h} \quad . \quad (G)$$

With the aid of Leyst's values of X_ϕ and X_a , the following tables are derived giving the average diminutions ΔX_ϕ , ΔX_a , ΔX per 1000 meters for the interval 0—10000 meters, taking $R = 6371$ kms. The unit is $\frac{1}{10}$ γ or 0.000001 C. G. S. unit.

Values of 1000 ΔX_ϕ

Parallel	60° N	40° N	20° N	Equator	20° S	40° S	60° S
1000 ΔX_ϕ	47	111	161	163	129	98	90

Long. E. of Gr.	1000 ΔX_a							1000 ΔX						
	60°	40°	20°	0°	20°	40°	60°	60°	40°	20°	0°	20°	40°	60°
	N	N	N	Equa- tor	S	S	S	N	N	N	Equa- tor	S	S	S
0°	+13	-8	-23	-33	-48	-32	+16	60	103	138	130	81	66	106
30	+21	+11	+5	-18	-49	-48	-9	68	122	166	145	80	50	81
60	+11	+20	+27	+1	-31	-41	-23	58	131	188	164	98	57	67
90	0	+20	+37	+25	+1	-24	-39	47	131	198	188	130	74	51
120	+5	+17	+24	+35	+31	-4	-46	52	128	185	198	160	94	44
150	+23	+16	0	+19	+42	+14	-36	70	127	161	182	171	112	54
180	+30	+4	-20	+3	+38	+36	-11	77	115	141	166	167	134	79
210	+12	+2	-15	0	+29	+37	+6	59	113	146	163	158	135	96
240	-22	-2	+10	+13	+18	+23	+13	25	109	171	176	147	121	103
270	-47	-15	+13	+12	+6	+18	+27	0	96	174	175	135	116	117
300	-37	-33	-17	-17	-10	+20	+52	10	78	144	146	119	118	142
330	-11	-34	-41	-35	-28	+3	+46	36	77	120	128	101	101	136

Hence it appears that the diminution of the component X with altitude, on the assumption of the Gaussian potential distribution, is, in general, not large. [For a difference of altitude of 1000 meters, the table shows that in order to reduce an observed X to the lower level, a positive correction would have to be applied which, at its maximum, would be about 0.0002 C. G. S. The

author describes briefly, on pp. 158-159, the chief features presented by the chart of the value of $1000 \Delta X_a$, which is found to resemble largely Leyst's chart of the component X_a .]

The following F values have been reduced to zero meridian:

Parallel	60° N	40° N	20° N	Equator	20° S	40° S	60° S
$1000 \frac{\Delta X}{X}$	0.00045	0.00046	0.00048	0.00046	0.00038	0.00036	0.00056
$1000 \frac{\Delta X_a}{X_a}$	0.00130	0.00044	0.00061	0.00060	0.00060	0.00089	0.00050

Liznar had found from his formulae $\frac{\Delta X}{X} = \frac{3h}{R} = 0.00047$ (constant); for the zero meridian it is seen that the ratio may vary from this value by 19 per cent. It will be noticed also that the ratio $\frac{\Delta X_a}{X_a}$ varies within wider limits—meaning that the abnormal part diminishes more rapidly with altitude, and, hence, that the distribution becomes more uniform with ascent.

[Precisely similar formulae to those for X are next established on pp. 160-171 for the Y and the Z components, and the following tables derived in a similar manner as before. For the Y there is of course no corresponding Y_ϕ term, being zero for the assumed "normal" portion of the Earth's magnetism. The designations and units are the same as before; thus the table $1000 \Delta Y$ gives the average diminution per kilometer between the surface and the altitude 10 kms., expressed in γ .]

Values of $1000 \Delta Y$

Long. E. of G.	60° N	40° N	20° N	0° Equator	20° S	40° S	60° S
0	+ 22	+ 29	+ 41	+ 57	+ 61	+ 53	+ 37
30	— 13	— 3	+ 12	+ 25	+ 37	+ 49	+ 52
60	— 33	— 26	— 11	0	+ 16	+ 42	+ 61
90	— 20	— 19	— 15	— 9	+ 12	+ 43	+ 64
120	+ 13	+ 19	+ 5	— 7	— 2	+ 17	+ 33
150	+ 18	+ 19	+ 1	— 19	— 30	— 23	— 4
180	— 17	— 21	— 22	— 27	— 33	— 35	— 27
210	— 44	— 43	— 23	— 7	— 13	— 22	— 32
240	— 37	— 43	— 28	— 10	— 15	— 29	— 51
270	+ 8	— 8	— 18	— 31	— 43	— 60	— 78
300	+ 49	+ 38	+ 11	— 11	— 25	— 40	— 58
330	+ 52	+ 50	+ 50	+ 47	+ 37	+ 21	— 3

For zero meridian we have again :

Parallel	60° N	40° N	20° N	Equator	20° S	40° S	60° S
1000 $\frac{\Delta Y}{Y}$	0.00043	0.00048	0.00054	0.00058	0.00059	0.00059	0.00050

These quantities differ from Liznar's constant ratio, for 20° S, for example, by 26 %.

Parallel	60° N	40° N	20° N	Equator	20° S	40° S	60° S
1000 ΔZ_ϕ	+249	+214	+112	-19	-119	-181	-243

Long. of G.C.	1000 ΔZ_a							1000 ΔZ						
	60°	40°	20°	0°	20°	40°	60°	60°	40°	20°	0°	20°	40°	60°
	N	N	N	Equa- tor	S	S	S	N	N	N	Equa- tor	S	S	S
0°	-56	-47	-41	-20	+31	+44	+36	+193	+167	+71	-39	-88	-137	-207
30	-41	-50	-65	-75	35	+1	+19	+208	+164	+47	-9	-154	-180	-224
60	-5	-11	-42	-64	56	-26	+8	+244	+203	+70	-83	-175	-207	-235
90	+24	+26	-9	-48	66	-48	-22	+273	+240	+103	-67	-185	-229	-265
120	+20	+22	+3	27	81	-83	-38	+269	+236	+115	-46	-200	-264	-281
150	-12	-36	-27	-17	61	-65	-11	+237	+178	+85	-36	-180	-246	-254
180	-27	-59	-28	+19	1	-20	-35	+220	+155	+84	0	-120	-201	-278
210	+11	-18	+2	+27	18	-3	-21	+260	+196	+114	+8	-101	-184	-264
240	+57	+49	+32	+16	+7	-2	-28	+306	+263	+144	-3	-112	-183	-271
270	+38	+89	+61	+37	+35	+22	-4	+287	+303	+173	+18	-84	-159	-247
300	+23	+53	+70	+79	+100	+91	+51	+272	+267	+182	+60	-19	-90	-192
330	-26	-16	+37	+74	+109	+94	+59	+223	+198	+149	+55	-10	-87	-184

This table shows that the Z component diminishes the most rapidly of the rectangular components X , Y , Z —in the maximum 0.0003 C. G. S. for 1000 meter ascent, so that a reduction correction to sea level might have to be applied. In the general features the chart of ΔZ_a resembles Leyst's of the component Z_a . Finding the ratios for the zero meridians we have again :

Parallel	60° N	40° N	20° N	Equator	20° S	40° S	60° S
1000 $\frac{\Delta Z}{Z}$	0.00042	0.00044	0.00042	0.00091	0.00052	0.00054	0.00056
1000 $\frac{\Delta Z_a}{Z_a}$	0.00074	0.00090	0.00069	0.00095	0.00044	0.00032	0.00023

Here again it is seen that the ratio is not constant and may for the equator (zero longitude) exceed the constant ratio of 0.00047 by 94%. It will again be noted also that for the "abnormal" portion the ratio varies between wider limits.

Thus it is found that the diminution of the components X, Y, Z of the portion of the Earth's magnetism represented by the Gaussian potential expression, does not follow Liznar's approximate relation:

$$\frac{\Delta X}{X} = \frac{\Delta Y}{Y} = \frac{\Delta Z}{Z} = \frac{3h}{R}.$$

The expressions deduced and given above show that the relation is a more complicated one, hence the value of the computed results given in the tables. For the reduction of the components measured at an altitude h_1 , formulae similar to the following will serve:

$$X_1 = X + \Delta X (h - h_1). \quad (H)$$

Now we pass [pp. 172-182] to the case in which the magnetism is not supposed to be represented by the Gaussian analysis. Then, of course, we do not adopt the foregoing methods of determination of the change of the magnetic elements. *A priori* exceptionally large changes of the force components are to be expected in places of magnetic anomaly. The question is, whether it is not possible with a given distribution of observations of the magnetic elements on the surface, to find their change with altitude without assumptions concerning the causes of the local disturbance. The only assumption which we shall make is that the forces that cause a given magnetic field have a potential.

In small regions it may be assumed that at all points the directions towards the North, West, and downwards are parallel. We shall denote these directions by X, Y, Z . Say, moreover, that at a point x, y, z , some element has the value E , and that at the consecutive point x_1, y_1, z_1 , the value of this same element is E_1 . Evidently:

$$E_1 = E + (x_1 - x) \frac{\partial E}{\partial x} + \dots \quad (I)$$

If the point x_1, y_1, z_1 lies on the same vertical line with x, y, z , then $x_1 = x, y_1 = y, z_1 - z = h$, and then:

$$E_1 = E + h \frac{\partial E}{\partial z} \times \frac{h^2}{1.2} \frac{\partial^2 E}{\partial z^2} \times \dots$$

Similar equations hold for all the components X, Y, Z .

The derivatives of the elements, with regard to the Z axis standing on the right side of these equations, are unknown and cannot be determined from the observations. Therefore, in order to make the formulae applicable, these quantities have to be replaced by others. To this end we recall the fundamental properties of the derivatives of the potential :

$$\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} + \frac{\partial Z}{\partial z} = 0; \quad \frac{\partial X}{\partial y} = \frac{\partial Y}{\partial x}; \quad \frac{\partial X}{\partial z} = \frac{\partial Z}{\partial x}; \quad \frac{\partial Y}{\partial z} = \frac{\partial Z}{\partial y}. \quad (J)$$

In the matter of change of the components X, Y, Z in the immediate neighborhood of the horizontal surface, where the determination of the quantities can be procured, (*viz.*, of the derivatives $\frac{\partial X}{\partial z}, \frac{\partial Y}{\partial z}, \frac{\partial Z}{\partial z}$), the latter equations give at once the desired results. From them it follows that :

$$-\frac{\partial X}{\partial z} = -\frac{\partial Z}{\partial x}; \quad -\frac{\partial Y}{\partial z} = -\frac{\partial Z}{\partial y}; \quad -\frac{\partial Z}{\partial z} = \frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}. \quad (K)$$

Hence the change of the X -component for increased altitude, is equal to that of Z for displacement towards the south. The change of Y is equal to that of Z for displacement east, and the change of Z to the sum of the changes of X for displacement north, and of Y for displacement west. The determination of the changes in a horizontal plane (the plane of the observations) presents no difficulties, the second members of the latter equations may thus be found from the observations. These equations confirm the opinion expressed above, that if in regions of anomaly the changes in the horizontal direction are large, then the changes in a vertical direction are likewise large. If we may limit ourselves to two terms only, *i. e.*, if the reduction has to be made to a level close to the plane of observations, the latter equations at once afford the possibility of effecting the reduction solely on the knowledge of the distribution of the elements in a horizontal plane. But in the case of greater differences of height, we may not stop at the first differential, and must find higher derivatives of X, Y, Z with regard to z .

[The analysis is continued on pp. 174-176 and the formulæ (L) finally derived containing derivatives now with respect to x and y instead of z . Hence if the observations in the horizontal plane are sufficiently close to admit of accurate determination of the horizontal gradients and their differentials, the variations with altitude may be computed.

$$\left. \begin{aligned}
 X_1 &= X + h \frac{\partial Z}{\partial x} - \frac{h^2}{1.2} \left(\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 Y}{\partial x \partial y} \right) - \frac{h^3}{1.2.3} \left(\frac{\partial^3 Z}{\partial x^3} + \frac{\partial^3 Z}{\partial x \partial y^2} \right) + \dots \\
 Y_1 &= Y + h \frac{\partial Y}{\partial y} - \frac{h^2}{1.2} \left(\frac{\partial^2 X}{\partial x \partial y} + \frac{\partial^2 Y}{\partial y^2} \right) - \frac{h^3}{1.2.3} \left(\frac{\partial^3 Z}{\partial x^2 \partial y} + \frac{\partial^3 Z}{\partial y^3} \right) + \dots \\
 Z_1 &= Z - h \left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y} \right) - \frac{h^2}{1.2} \left(\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2} \right) + \frac{h^3}{1.2.3} \left(\frac{\partial^3 X}{\partial x^3} + \right. \\
 &\quad \left. \frac{\partial^3 Y}{\partial x^2 \partial y} + \frac{\partial^3 X}{\partial x \partial y^2} + \frac{\partial^3 Y}{\partial y^3} \right) + \dots
 \end{aligned} \right\} (L)$$

In case the region considered is of such extent that its curvature must be taken into account so that the observations are no longer all in the same horizontal plane or if the plane be inclined, then

$$z = \phi(x, y). \quad (M)$$

The previous equations for X_1 etc., still hold good, only the determination of the required derivatives is rendered a little more difficult. However, if it is possible to express any observed element in the following form:

$$E = f(x, y, z), \quad (N)$$

$$\text{then} \quad \frac{\partial E}{\partial x} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial \phi} \frac{\partial \phi}{\partial x}; \quad \frac{\partial E}{\partial y} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial \phi} \frac{\partial \phi}{\partial y}. \quad (O)$$

In a similar manner the other derivatives are found.

[Pp. 177-182 contain a critical discussion of the results derived by Liznar⁸ already referred to above. Liznar divides the observations of Austria-Hungary into three groups: the first embracing 77 points ranging in altitude from 0 m to 200 m, the second consisting of 72 points from 201 to 400 m high, and the third of 56 points, 401 m and over in altitude. By a least square treatment he finds per kilometer:

$$\frac{\delta X}{X} = +0.00163; \quad \frac{\delta Y}{Y} = -0.00890 \text{ and } \frac{\delta Z}{Z} = +0.00158,$$

whereas theoretically he had derived, by the neglect of terms above shown to be of appreciable magnitude, the constant value 0.00047.

Liznar sought to explain the differences between observed and theoretical value by a system of atmospheric electric currents. Passalskij shows, however, that this hypothesis was not as yet justified, chiefly because of the not strict fulfillment of the assumptions made in the derivation of both the computed and the theoretical ratios.

There next follows a brief summary of the main conclusions derived in the chapter and the author finally expresses his belief that the true laws of change of the magnetic elements with altitude can only be obtained from magnetic observations in the free atmosphere—for instance in balloons—and not from observations on mountains (or in mines) regarding which there will always be the suspicion of the presence of local disturbing influence.]

⁸ Die Vertheilung der Erdmagnetischen Kraft in Oesterreich—Ungarn. II Theil, p. 7, Wien, 1895.