case in which the first solid is a sphere of the radius a' , because both Lagrange and Ivory have used it to show that the reasonings of Laplace are incorrect. In this case, then, the surface described by the point p'' is that of a sphere whose radius is the difference between a and a' ; and the values of V , V' , V'' and A , are $\frac{4}{3}\pi a'^2$, $\frac{4}{3}\pi a^2$, $\frac{4}{3}\pi (a'-a)^2$, and $\frac{4}{3}\pi a'$, respectively.

Substituting these values in the equation $V + V' - V'' = 2aA$, and omitting the common factor $\frac{4}{3}\pi$, the resulting equation

$$a'^2 + a^2 - (a' - a)^2 = 2aa$$

ought to be identical;—and so it manifestly is.

L. *On a New Principle in Statics, called the PRINCIPLE OF LEAST PRESSURE. By the Rev. H. MOSELEY, B.A. Professor of Natural Philosophy in King's College, London*.*

LET there be conceived a system of forces, of which a certain number are given in magnitude and direction, and the rest are supplied by the resistances of as many fixed points.

Also let the points of application of all the forces of the system be supposed to be given.

Now to the complete determination of any force in magnitude and direction, its point of application being given, it is necessary that we know its resolved parts in the directions of three rectangular axes.

To ascertain, therefore, completely, the magnitudes and directions of the resistances on the different fixed points of the system we have supposed, we must have at least three times as many equations between the resolved parts of the forces and resistances which compose it, as there are points of resistance.

The known conditions of equilibrium supply, at most, but six such equations. If there be more than two points of resistance, these equations are therefore insufficient. And it remains to establish some other relation between the resistances, their several points of application, and the other forces of the system, as shall enable us to determine the former, in terms of the others, of which they are manifestly functions.

The following principle is sufficient for this determination. It is believed to constitute a new principle in statics. *If there be any number of forces in equilibrium among which there enters a system of resistances, then are these resistances such, that their sum is a MINIMUM; each being considered a function of the*

* Communicated by the Author.

coordinates of its point of application, taken with a positive sign, and subjected to the conditions imposed by the equilibrium of the whole.

Let A designate the *given* forces of the system, B the resistances, and C *any* other system of forces which being applied to the same points with the forces of the system B, would maintain the equilibrium. Also let the system C be supposed to replace the system B.

Now each force of the system C, under these circumstances, *just* sustains and is equivalent to the pressure propagated to its point of application by the forces of the system A; or it is equivalent to that pressure *together with* the pressure propagated to its point of application by the other forces of the system C.

In the former case, it is identical with the corresponding *resistance* of the system B. In the latter case it is *greater* than it.

The sum of the forces of the system B, each being considered *a function of the coordinates of its point of application, &c. &c.*, is therefore a minimum.

Let P represent any force of the system B; x, y, z its coordinates; and α, β, γ the angles it makes with the axes of these, respectively. Let M_1, M_2, M_3 represent the sums of the resolved parts of the forces of the system A, and N_1, N_2, N_3 , the sums of their moments;

$$\therefore \left. \begin{aligned} \Sigma P \cos \alpha &= M_1 \\ \Sigma P \cos \beta &= M_2 \\ \Sigma P \cos \gamma &= M_3 \end{aligned} \right\} \dots\dots\dots (1)$$

$$\left. \begin{aligned} \Sigma P (y \cos \alpha - x \cos \beta) &= N_1 \\ \Sigma P (x \cos \gamma - z \cos \alpha) &= N_2 \\ \Sigma P (z \cos \beta - y \cos \gamma) &= N_3 \end{aligned} \right\} \dots\dots\dots (2)$$

Also $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2, \&c.$ representing the values of α, β, γ , for different points of the system B, we have

$$\left. \begin{aligned} \cos^2 \alpha_1 + \cos^2 \beta_1 + \cos^2 \gamma_1 &= 1 \\ \cos^2 \alpha_2 + \cos^2 \beta_2 + \cos^2 \gamma_2 &= 1 \\ \&c. \qquad \qquad \&c. &= \end{aligned} \right\} \dots\dots (3)$$

Now the relations existing by reason of the nature of the system between the coordinates of the several points of application of the forces P may be expressed by the equations

$$\left. \begin{aligned} u_1 &= 0 \\ u_2 &= 0 \\ u_3 &= 0 \\ \&c. &= 0 \end{aligned} \right\} \dots\dots\dots (4)$$

$$\text{Also} \qquad \Sigma P = \text{minimum} = V \dots\dots\dots (5)$$

Differentiating the above equations, P being considered a function of x, y, z ; multiplying the differentials of equations (1) by the indeterminate quantities A_1, A_2, A_3 , respectively,

(2) by B_1, B_2, B_3 ,

(3) by $\mu_1, \mu_2, \mu_3, \mu_4$ &c. &c. &c.,

(4) by $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ &c. &c. &c.,

and adding the resulting equations (all whose right-hand members disappear) to the differential of equation (5), which is

$$\Sigma \left\{ \frac{dP}{dx} \delta x + \frac{dP}{dy} \delta y + \frac{dP}{dz} \delta z \right\} = \delta V,$$

we shall obtain an equation in which the coordinates of the several points of resistance $x_1, y_1, z_1, x_2, y_2, z_2$, &c. &c., and $\alpha_1, \beta_1, \gamma_1, \alpha_2, \beta_2, \gamma_2$, &c. &c., may be considered as independent variables; so that by the condition $V = \text{a minimum}$, the coefficients of $\delta x, \delta y, \delta z, \delta x_2, \delta y_2, \delta z_2$, &c. &c. $\delta \alpha_1, \delta \alpha_2$, &c. $\delta \beta_1, \delta \beta_2$, &c. $\delta \gamma_1, \delta \gamma_2$, &c. &c., respectively equal nothing. We thus obtain, in respect to each point of resistance, six equations, reduced to four, by the elimination of the indeterminate quantities λ and μ , which are different for the different points of resistance. Of these four equations, two, involving partial differential coefficients of P, from which one of the variables x, y, z may be eliminated by means of the equation $u = 0$, are equivalent to one complete differential equation between the other two variables, so that on the whole there are three equations determining the quantities P, α and β in terms of the coordinates x, y, z , and the indeterminate quantities $A_1, A_2, A_3, B_1, B_2, B_3$ (which are the same for all the different points of resistance), whilst these are in their turn determined, by substituting the resulting values of $P_1, \alpha_1, \beta_1, P_2, \alpha_2, \beta_2$, &c. in the equations (1) and (2).

Thus the values of P, α , and β , that is, the amount and direction of the pressure upon any fixed point of the system, are completely determined.

The various analytical operations indicated above, are in their nature too elaborate to be *here* brought under the eye of the reader. Those conversant with the methods of analysis will, however, readily supply the deficiency.

It may be observed that in the case in which the points of resistance are all in the same plane*, the pressure upon any

* This particular case of the general proposition has formed the subject of a paper by the illustrious Euler, entitled, "*De Pressione Ponderis in Planum cui incumbit*," Mem. Ac. Pet. Novi Commentarii, vol. xviii. His discussion of the question is most ingenious and elaborate. It is impossible, however, to admit the *hypothesis* on which he has grounded it.

point whose coordinates in that plane are x and y , is ascertained to come under the formula

$$\frac{C}{1 + Ax + By},$$

A , B and C being constants, dependent upon the number and positions of the points of support, and upon the forces impressed upon the system.

From this formula it follows, that there is a certain line, distant from the origin by a quantity equal to $\frac{1}{\sqrt{A^2 + B^2}}$, and

inclined to the axis of x at an angle whose tangent is $\frac{A}{B}$;

about which the moments of all the pressures are the same. This line may be properly called the *axis of least pressure*, as the principle whence its existence and properties have been deduced, may be designated the *principle of least pressure*.

Where the points of support are in the same line, this axis resolves itself into a point ; and it follows that the moments of the resistances about a certain point in the line in which they act, are all equal ;—a result which is verified by the known conditions of the pressure upon two points of support.

LI. Notice respecting certain Changes of Colour in the Choroid Coat of the Eyes of Animals. By Sir DAVID BREWSTER, LL.D. F.R.S.

IN the Number of this Journal for August last, (p. 87) Mr. Fielding has published some interesting experiments respecting certain changes of colour induced by chemical and other agents on the membrane lining the choroid coat of the eye ; and he has particularly described an experiment which shows that when the colours have disappeared by drying, they may be revived by simple immersion in water.

In the chapter on the colours of natural bodies in the article OPTICS in the Edinburgh Encyclopædia, I have brought forward this fact as illustrating and supporting Sir Isaac Newton's Theory of the Colours of Natural Bodies, and I am induced to mention this at present, not only for the purpose of giving to Dr. Drummond of Belfast, the merit of having first made this curious experiment, but also of making some further observations upon it in reference to its connexion with the theory of Newton. The following is the passage in which it is mentioned.

“ Dr. Drummond of Belfast observed, that the *membrane*