68. In a Spherical Quadrangle the Arcs Joining the Middle Points of the Three Pairs of Opposite Sides Are Concurrent

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but would be too complicated for use in drawing the curve, which is probably most easily obtained from the intersections of the equiangular spirals

$$\rho = kae^{-\mu\theta}, \quad \rho' = kce^{-\mu\theta'},$$

drawn for different values of k , which is easily done after one has been drawn.

To save labour the values of k should proceed geometrically, and c should be equal to one of the values of ka . The two systems of curves are then merely differently placed repetitions of each other, one system having O as pole, the pole of the other system being A . Indeed, they are all repetitions of one equiangular spiral, its various positions round O (or A) being obtained by successive rotations round O (or A) through a definite angle each time if k increases geometrically as suggested.

The curves required pass through the intersections of consecutive spirals, and are themselves spiral curves, each coiling an infinite number of times round O in gradually widening circuits, and then coiling round A an infinite number of times in gradually decreasing circuits (or, of course, *vice versa*). One of the curves (viz. when $c=a$) goes off to infinity after coiling round O , and comes back from the opposite direction of infinity to coil round A , its direction at infinity being parallel to the line $\theta=a$.

If the coaxial circles have real limiting points instead of real points of intersection, O and A will be the limiting points, and μ will then = $\tan a$ instead of $\cot a$.

If A coincides with O , the curves become another similar system of circles through O , the common tangents at O to the two systems cutting each other at the required angle.

A. LODGE.

[The general curve of the whole system for all values of the constant angle of section is a spiral with two poles O, A , which does not go to infinity, and which takes roughly the shape of the letter S. The whole system, however, includes three systems which form exceptions to this statement. One is the system which goes to infinity; and the other two are the system of circles through O, A , and a second system of circles cutting the first or given system orthogonally.

It is clear that any one of the spirals not only cuts the first system of circles at a constant angle, but also the second system. This second system is also a coaxial system, but with imaginary common points, having the poles of the spirals for limiting points. The spirals may be considered as being given by either system of circles.

The whole system of spirals when inverted with respect to any point becomes an exactly similar system, the new poles being the inverses of the old ones. This follows from the fact that angles are unaltered by inversion, and circles invert into circles.

If one pole goes to infinity, the spirals become equiangular spirals, with the remaining pole as pole. Hence, by regarding an equiangular spiral as having in reality two poles, one of which is at infinity, we may regard the system of spirals with two finite poles as being a very simple generalisation of equiangular spirals. From this aspect their properties with respect to inversion and their relation to infinity gain in interest.

A further generalization of an equiangular spiral is the curve cutting at a constant angle the system of circles which touch two given circles; and this includes, as a special case, the curve which cuts the tangents to a given circle at a constant angle.

F. S. MACAULAY.]

68. In a spherical quadrangle the arcs joining the middle points of the three pairs of opposite sides are concurrent. (Question 90, p. 16. Taken from Casey's *Spherical Trigonometry*.)

To solve this problem by using the relations between the trigonometrical ratios of the arcs involved would appear to be a very laborious operation.

The fact of the problem being given in a work on trigonometry does not appear to render a geometrical solution inadmissible. This seems to be sufficiently indicated by the free use of purely geometrical methods in establishing some of the important propositions of spherical trigonometry. What, from its extreme simplicity, appears to me to be emphatically the solution of the problem, is as follows :

Let A, B, C, D be the four vertices of the quadrangle ; L, M, N, L', M', N' the middle points of the arcs BC, CA, AB, AD, BD, CD ; and l, m, n, l', m', n' the middle points of the corresponding chords. Then if O is the centre of the sphere, OL is a straight line ; similarly for $OmM, Ol'L$, etc. Then the straight line lm is parallel to the chord AB and half of it, since l, m bisect the chords CB, CA . Similarly for $l'm'$. Thus $lml'm'$ is a parallelogram, and ll', mm' bisect one another. Thus ll', mm', nn' meet in a common point k ; the planes OLL', Omm', Onn' (that is, the planes OLL', OMM', ONN') meet in a common line Ok ; and the arcs LL', MM', NN' meet in a common point K , viz. the point where Ok cuts the surface of the sphere. J. C. PALMER.

EXAMINATION QUESTIONS AND PROBLEMS.

Our readers are earnestly asked to help in making this section of the GAZETTE attractive by sending either original or selected problems.

Solutions should be sent within three months of the date of publication. They should be written clearly on one side of the paper. Contractions not intended for printing should be avoided. Figures should be drawn with the greatest care on as small a scale as possible, and on a separate sheet.

The question need not be re-written, but the number should precede every solution.

The source of problems when not otherwise indicated is shown by—C. (Cambridge), O. (Oxford), D. (Dublin), W. (Woolwich), Sc. (Science and Art Department).

260. The tangent to an ellipse at P meets the equiconjugate diameters at T, T' . The tangent at P to the circle $T'CP$ meets CT in L . Show that $PL^2 = CL \cdot TL$. W. F. BEARD.

261. If opposite sides of a hexagon $ABCDEF$ be parallel, and another hexagon $PQRSTV$ be formed by intersections of AC and BD , BD and CE , CE and DF , etc., prove that Pascal's theorem holds true for all the hexagons (or hexagrams) formed by the points A, B, C, D, E, F , and that Brianchon's theorem holds for $PQRSTV$. W. S. COONEY.

262. P is a point on an ellipse, foci S, S' . PR , the symmedian through P of the triangle SPS' , cuts the circumcircle in R . RO perpendicular to PR cuts the normal at P in O the centre of curvature at P . R. F. DAVIS.

263. $ABCD$ is a quadrilateral with given sides, on a fixed base AB . A triangle is inscribed in a fixed circle, sides parallel to BC, CD, DA respectively. Show that each side of the triangle envelopes a circle, and that the four circles are coaxial. A. C. DIXON.

264. The circles in No. 248 cut on PQR . W. J. GREENSTREET.