A SECONDARY SCHOOL MATHEMATICS CLUB.

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The club whose workings are here described was organized in 1903 in Shattuck School, Faribault, Minn. So far as is known, this is the first attempt to conduct a club of this sort in a secondary school. That the experiment has been a success, the increasing enthusiasm for the work shown by the members of the club during the last eight years abundantly proves.

While Shattuck is a private school, which may make the matter of organization somewhat simpler, the course in mathematics is no higher than that in any good public high school, and the boys are the same age.

With some modifications, work could be done along similar lines, I believe, in any public or private high school. For that reason and because there have been several requests for information as to the workings of our club, our experience is here set down briefly for the benefit of secondary school teachers who may be interested.

The object of the club is to study certain interesting matters connected with mathematics which do not properly find a place in the usual classroom work. Such questions as the history of mathematics, its famous problems, unusual applications, mathematical puzzles, fallacies, and tricks are considered proper subjects for study, anything in fact that is capable of a mathematical solution or explanation and that promises to be entertaining. This is the main criterion, a subject must be interesting to find a place on our program; a club of boys will not thrive if the meetings are dull.

The club is made up of those members of the senior class who are taking the college preparatory course in mathematics, higher algebra, solid geometry and trigonometry, and numbers about fifteen. The instructor of the class is the leader, and chairman, *ex-officio*, and the only teacher usually present at the meetings. The club meets in their classroom on the evening of the weekly holiday. To make it easier for the boys to spare the time for these meetings it is understood that no preparation will be expected from them for the next day's regular lesson.

Each evening is devoted to some general subject, and reports are prepared on three or four special topics by as many members. This requires that each one of the members shall make

such a report about once in five or six weeks. At each meeting the program for the following meeting is announced, and thus those who are to take part have a week's time in which to prepare.

We have a reference library, freely accessible, in our recitation room, containing quite a number of books which will yield some information on the subjects in which we are interested. A list of the most useful of these books is given below. Besides the books, we have collected perhaps forty or fifty popular magazine articles on special topics in our field, as for example, one by Professor Newcomb on the "Fairyland of Geometry," others on "The Fourth Dimension," etc.

As boys of high school age are new to investigations of this kind it is necessary to help them in the preparation of their reports. They require rather explicit directions and suggestions as to the treatment of a given topic, what to include, and more especially what to leave out. They are given this help in the form of brief outlines of their topics together with references, often by pages, to two or three of the books available.

These reports are variously presented by the different members. Some of them talk extemporaneously from notes, making use of illustrations previously placed on the blackboard. Others prefer to mark certain pages or paragraphs, and to read directly from the books what they wish to tell us. Still others, on certain subjects, read carefully written papers, which they are not long in discovering may be submitted to other instructors in lieu of their monthly themes, or even elaborated into graduation theses.

The plan of our meetings is that each of these reports shall consume about fifteen minutes, and be followed by an informal discussion in which anyone may ask questions or raise objections to be met by the one presenting the report. If the leader can add anything of interest that has not been brought out he does so briefly, but the central idea of the whole scheme is to have the boys do most of the talking. The leader's function is to preside at the meetings, to keep the interest alive, and try to preserve that happy mien of informality which shall put the boys entirely at their ease, and yet maintain a certain dignity in the discussions.

Of course we can consider only such subjects as will be intelligible to secondary school pupils, and these only in an elementary and somewhat superficial way. To give an idea of the various subjects considered in this mathematics club, the program for last year is appended which may be regarded as a typ-

ical year's work. I may be pardoned perhaps if I describe the work of one or two of these meetings somewhat in detail in order to show the scope of our investigations.

On the evening devoted to the History of Geometry, for example, the principal topics are (1) The Beginnings of Geometry, (2) Early Greek Geometry, (3) The Golden Age of Greek Geometry, (4) Recent Developments in Geometry. In the first paper is considered the extent of geometrical knowledge among the early nations, the knowledge of astronomy among the Babylonians, and of mensuration, and surveying among the Egyptians, the Ahmes value of π , and his incorrect statements for the area of the trapezoid and isosceles triangle, the orientation of their temples, the "rope stretchers," and so on. In the consideration of the Early Greek Geometry the marked tendency of the Greek mind toward logic and the appreciation of form are referred to as accounting for this early development among them of geometry as a science. Thales is mentioned, and the theorems attributed to him, with their probable proofs. There is also a brief biography of Pythagoras, a description of his school, and the brotherhood of the Pythagoreans. In the third paper on the Golden Age of Greek Geometry, Euclid and his work are similarly discussed, his famous Elements described, and its influence on the modern study of geometry pointed out. The fourth paper considers recent contributions to Euclidean geometry and some of the elementary notions of the modern geometries.

When puzzles and fallacies are to be considered, the one presenting the report first proposes a fallacy or puzzle to the other members for solution; if they fail to solve the puzzle or point out the fallacy within a reasonable time, they are given a hint or the solution is shown them. Similarly in the case of Mechanical Puzzles, Card Tricks, Problems on a Chess Board, etc. In each case the conjurer after mystifying his audience, is required to give away his trick, and to explain the mathematical principle involved.

In introducing subjects like Our Number System or the Foundations of Geometry, it is necessary for the leader to do a larger part of the work, but even here it is possible for the students under direction to report on certain phases or details of the general topic. Again, in subjects like Non-Euclidean Geometry, Infinity, and the Fourth Dimension, we are on unfamiliar, and at the same time on rather dangerous ground, though it is still possible for the leader to point out the way, and with some care to

steer a safe course. Even if the students do not acquire perfectly clear notions about these rather vague subjects, it does no harm at least to set them thinking.

This is perhaps the most valuable result of the work of our club. It sets the boys to thinking and stimulates their imagination. They find that there is more of interest in the dry subject of mathematics than they had dreamed. Our investigations are not very profound to be sure, and will result in no contributions to the sum of mathematical knowledge, but the boys enjoy the meetings, and I think profit by them, and as these are the two distinct objects of this mathematics club, I feel that it has proved itself worth while.

As was said above, the treatment of the topics is suggested to the members of the club, by outlines which are prepared for them in advance. In order to show the extent of the help thus furnished, and as an example of how fully they are able to treat a subject, there is given below the outlines for two meetings.

THE HISTORY OF ARITHMETIC AND ALGEBRA.

I. Among the Ancient Nations.

- a. The Egyptians: Ahmes' papyrus, its antiquity, some of its problems, Egyptian fractions.
- b. Greek arithmetic and algebra: not so highly developed as geometry, Greeks interested in theory of numbers rather than calculation (examples). Computations on dust board (explanation). Diophantus and equations.
- c. Roman arithmetic: no algebra, arithmetic, only enough for commercial purposes. Use of abacus (illustrations).

II. Among the Hindus and Arabs.

- a. The Hindus (sixth and seventh century): Great computers, arithmetic and algebra their specialty as geometry for Greeks. Extent of their mathematical knowledge. Negative numbers, irrational numbers, zero. Their methods in arithmetic, false position, inversion. Examples of their poetically stated problems.
- b. The Arabs (eighth to the tenth century): brief history, great scholars of their time. Arabs as teachers and preservers of Greek and Roman culture, text-book writers. The name algebra, syncopated algebra.

III. Early European Arithmetic.

General education in the Middle Ages, schools of church

and monastery, universities (quadrivium, trivium),
Rechenschulen of Hanseatic League.

Examples of four fundamental operations as performed in
the Middle Ages; multiplication by Napier's rods, di-
vision by scratch method.

IV. Development of Symbolic Algebra.

Algebra originally rhetorical; work expressed fully in
words, no symbols.

Syncopated algebra: abbreviations for the words, still few
symbols, Diophantus (330 A.D.), the Arabs.

Symbolic Algebra: took modern form with Vieta (1600).

Examples of solution of an equation in:

1. Old rhetorical form.
2. Syncopated form.
3. As stated by Vieta.
4. In twentieth century symbols.

Advantage of our symbols.

V. History of Common Symbols of Operation.

Various devices used at different times to express powers,
roots, equality, the four fundamental operations, etc.,
with their origin and dates.

First use of the present symbols.

NUMBER SYSTEMS AND NUMERALS.

I. Primitive Systems of Numeration.

First notions of numbers, counting, etc., among animals,
among savages.

Gesture symbols, finger reckoning. Early written symbols.

Systems in use among primitive races:

Decimal, Egyptian, Greek, Roman.

Quinary (scale of 5), certain aborigines of America.

Vicigesimal (scale of 20), South American tribes, etc.

Binary (scale of 2), New Zealand, Patagonian. Traces
of other systems.

II. Development of our Decimal Positional System.

Due to ten fingers, how? Value of idea of position, of zero,
discoveries of the Hindus (date). Advantages of our
system (illustrated by multiplying two numbers in
Roman notation).

Disadvantages of decimal as compared with duodecimal
system.

- III. *History of Arabic Numerals and Other Number Symbols.*
 Babylonian, Egyptian, Greek symbols (illustrate). Awkwardness of Roman system.
 History of development of so-called Arabic Numerals (Hindu origin).
- IV. *Problems in Other Systems.*
 Numbers written in other systems. Multiplication tables in other systems.
 Problems in addition, division, etc., with numbers expressed in scale of 9, scale of 5, scale of 2, etc.
 Problem in non-positional system like Greek or Roman, hence necessity for use of abacus.
- V. *Number Systems of Algebra.*
 Fractions: Egyptian unit fractions; Babylonian fractions, denominator always 60; Greek and Roman fractions; decimal fractions (recent date).
 Irrational Numbers: first use, brief history, etc.
 Negative Numbers: the same.
 Imaginary Numbers: the same.

APPENDIX A.

THE MATHEMATICS CLUB OF SHATTUCK SCHOOL.

*Program for 1909-10.***FIRST MEETING:** *Algebra Fallacies.*

An informal consideration of certain proofs (?) that 2 equals 1, 1 equals 0, 1 equals -1 , etc.

SECOND MEETING: *Our Number System.*

1. First notions of numbers.
2. Primitive numeration.
3. Development of decimal system.
4. The positional idea.

THIRD MEETING: *Number Systems and Symbols.*

1. History of our Arabic symbols.
2. Number symbols of other systems.
3. Nondecimal systems.
4. Some problems in a nondecimal system.
5. The duodecimal vs. the decimal.

FOURTH MEETING: *History of Arithmetic and Algebra.*

1. Among the ancient nations.
2. Among the Greeks and Romans.
3. Among the Hindus and Arabs.
4. In mediæval Europe.
5. The development of algebraic symbolism.

FIFTH MEETING: *Numerical Curiosities.*

1. Mystic properties of numbers.
2. Prime numbers, triangular numbers, squares, cubes, etc.
3. Magic squares.
4. Large numbers.
5. Number forms.

SIXTH MEETING: Numerical Curiosities Continued.

1. The number 9 and its properties.
2. Other curious numbers.
3. Mathematical short cuts.
4. Mental calculations.

SEVENTH MEETING: Numerical Tricks and Puzzles.

1. Numerical tricks.
2. Numerical puzzles and catch questions.
3. To discover a number thought of.

EIGHTH MEETING: Geometrical Tricks and Puzzles and Mathematical Games.

Informal consideration, no formal reports.

NINTH MEETING: Card Tricks Involving Some Mathematical Principle of Number or Position.

Problems on a chess board.

TENTH MEETING: Foundations of Geometry.

1. The assumptions.
2. Nature of space.
3. Definitions.
4. Logic of geometry.

ELEVENTH MEETING: History of Geometry.

1. Beginnings of geometry.
2. Early Greek geometry.
3. Euclid and his immortal elements.
4. Recent developments in geometry.

TWELFTH MEETING: Famous Problems of Geometry.

1. Squaring the circle.
2. The duplication of the cube.
3. Regular polygons and polyhedrons.
4. Famous problems of solid geometry.

THIRTEENTH MEETING: Foundations of Geometry.

1. Geometric assumptions.
2. The straight line, and how to draw one.
3. Non-Euclidean geometry.
4. Some criticisms of Wells geometry.

FOURTEENTH MEETING: The Mathematics of Common Things.

1. The mathematical principles of maps.
2. Optical illusions.
3. The carpenter's square.
4. Weighing and measuring.
5. Mathematical symmetry in nature.

FIFTEENTH MEETING: The Fairyland of Geometry.

1. The fourth dimension.
2. A visit to flatland.
3. A trip to infinity.
4. Curved space.

SIXTEENTH MEETING: Higher Mathematics.

1. History of trigonometry.
2. History of logarithms.
3. Calculus, and other pleasures to come.

APPENDIX B.

Below are appended the titles of a few books and pamphlets selected from among the many available; they may be of interest

in connection with this account to teachers of secondary mathematics. Only publications in the English language are mentioned. Additional references will be found in Smith's *Teaching of Elementary Mathematics*, Young's *Teaching of Mathematics*, Withers' *Parallel Postulate*, White's *Scrap Book of Elementary Mathematics*, Ahren's *Unterhaltungen und Spiele*.

This list does not include text-books, nor articles in encyclopedias or magazines. Among magazines frequently containing such articles of interest are *SCHOOL SCIENCE AND MATHEMATICS*, *The Open Court*, *The Monist*, *Science*, *The Popular Science Monthly*, and *The Scientific American*.

I. Foundations and Criticisms.

Common Sense of the Exact Sciences	
Clifford	Appleton
Grammar of Science	
Pearson	Black (London)
Foundations of Geometry	
Hilbert	Open Court
Foundations of Mathematics	
Carus	Open Court
Space and Geometry	
Mach	Open Court
Euclid's Parallel Postulate	
Withers	Open Court
Foundations of Mathematics	
Russell	Macmillan
Euclid	
Frankland	Wessel & Co.
Non-Euclidean Geometry for Teachers	
Halsted	Halsted
Non-Euclidean Geometry	
Manning	Ginn & Co.
Geometric Axioms (Popular Science Lectures, Second Series)	
Helmholtz	Appleton
Rational Geometry	
Halsted	Wiley & Son
The Thirteen Books of Euclid's Elements with Introduction and Commentary	
Heath	Cambridge University Press
Mathematical Monographs	
Young	Longmans
Lectures on Elementary Mathematics	
Lagrange	Open Court
Foundations of Mathematics	
Compto	Harpers (out of print)
Theories of Parallelism	
Frankland	Cambridge Press

II. History.

History of Mathematics	
Cajori	Macmillan

- History of Elementary Mathematics
 Cajori Macmillan
- History of Mathematics in the United States
 Cajori Government Printing Office, Washington
- Short History of Mathematics
 Ball Macmillan
- Primer of the History of Mathematics
 Ball Macmillan
- Brief History of Mathematics
 Fink Open Court
- Greek Geometry from Thales to Euclid
 Allman Dublin University Press
- History of Greek Mathematics
 Gow Cambridge University Press
- Euclid
 Smith Scribner
- The Story of Euclid
 Frankland Wessel & Co.
- Famous Problems of Geometry
 Klein Ginn & Co.
- Mathematical Monographs
 Heath & Co.
- History of Modern Mathematics
 Smith Ginn & Co.
- History of Teaching of Elementary Geometry
 Stamper Teachers College, Columbia University
- History of the Inductive Sciences
 Whewell Appleton
- Portraits of Mathematicians
 Open Court
- The Teaching of Elementary Mathematics
 Smith Macmillan
- Rara Arithmetic
 Smith Ginn & Co.
- Sixteenth Century Arithmetic
 Jackson Teachers' College, Columbia University
- The Hindu Arabic Numerals
 Smith Ginn & Co.
- The Teaching of Geometry
 Smith Ginn & Co.

III. *Recreations.*

- Scrap Book of Elementary Mathematics
 White Open Court
- Mathematical Recreations
 Ball Macmillan
- Mathematical Essay
 Schubert Open Court
- The Canterbury Puzzles
 Dudeney Dutton & Co.
- Paradoxes of Nature and Science
 Harpson Dutton & Co.
- The Number Concept
 Conant Macmillan
- Philosophy of Arithmetic
 Brooks Normal Publishing Company, Philadelphia

Recreations in Science and Mathematics

Ozanam

(Translated by Hutton; out of print; only second-hand copies obtainable)

Scientific Romances

HintonSwan, Sonnenschein & Co.

Fourth Dimension

HintonSwan, Sonnenschein & Co.

Flatland

Anon.Little, Brown & Co.

Geometric Exercises in Paper Folding

Row Open Court

How to Draw a Straight Line

Kempe Macmillan

PROVISIONAL REPORT OF THE NATIONAL COMMITTEE OF
FIFTEEN ON GEOMETRY SYLLABUS.¹

PROBLEMS INVOLVING LOCI.

(a) **Phraseology.** While the committee does not wish to prescribe the exact phraseology of any definition, it would recommend greater care in the formulation of the definitions underlying the subject of loci. It is suggested that any definition used should be substantially equivalent to the following:

The locus of a point (or the locus of points) satisfying given conditions is a configuration such that:

- (1) All points lying on the configuration satisfy the conditions;
- (2) All points satisfying the conditions lie on the configuration.

It would seem desirable to make all proofs on loci conform to this definition. It is of course understood that the teacher will lead the pupil up to such a definition through varied forms of concrete description, such as "path of a point in motion," etc.

(b) **Motion in geometry.** It seems well to give some consideration to the place of motion in a well-rounded course in elementary geometry, and to bear in mind that this course is all the geometry to be studied by the majority of high school pupils. It has recently been urged by prominent European mathematicians that motion should be given a more prominent place at this stage. We may well recall that the space concepts dealt with in our usual courses in geometry are almost entirely to be described as static. There is in theorems and problems on loci a dynamic element that is of importance. The pupil is pretty fa-

¹NOTE. The Geometry report is completed in this issue. It has already been reprinted in pamphlet form and sent to two hundred selected teachers for the purpose of securing further suggestions and criticisms prior to its final presentation at the meeting of the National Educational Association at San Francisco in July. The Committee would invite suggestions and criticisms from any reader of SCHOOL SCIENCE AND MATHEMATICS who may be interested. Such communications may be sent to any member of the Committee or directly to the Chairman, H. E. Slaught, 5535 Monroe Ave., Chicago, Ill.