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XXVII. Essay upon machines in general

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consequence to the pleasure of the gentleman, as well as to the profit of the gardener.

Old as I am, I certainly intend this year to commence experiments on the myrtle and the laurel: I trust, therefore, it will not be thought presumptuous in me to invite those of my brethren of this most useful Society, who are younger than I am, and who of course will see the effect of more generations than I shall do, to take measures for bringing to the test of experiment the theory I have ventured to bring forward, I hope not without some prospect of success.

The settlement lately made at New Holland gives a large scope to these experiments: many plants have been brought from thence which endure our climate with very little protection; and some of these arrive at puberty at an early period; we have already three from the south point of Van Diemen's Island, where the climate cannot be wholly without frost; *mimosa verticillata*, *eucalyptus hirsuta*, and *obliqua*. The first of these appears to have produced flowers within eight years of its first introduction; but as a settlement is now made very near the spot where the seeds of these shrubs were collected, we may reasonably hope to receive further supplies, and, among them, the *Winterana aromatica*, an inhabitant of the inhospitable shore of Terra del Fuego, which Mr. Brown has discovered on the south part of Van Diemen's Island also.

XXVII. *Essay upon Machines in General.* By M. CARNOT,
Member of the French Institute, &c. &c.

[Continued from p. 36.]

Part II. [*Of Machines properly so called**.]

DEFINITIONS.

XXXI. **A**MONG the forces applied to a machine in motion, some are of such a nature that each of them forms an acute angle with the velocity of the point at which it is

* Vide p. 36 of the present volume.

applied,

applied, while others form obtuse angles with their points. This being granted, I shall call the former *moving* or *soliciting forces*; and the others *resisting forces*: for instance, if a person raises a weight by means of a lever, a pulley, a screw, &c., it is clear that the weight and the velocity of the weight necessarily form by their concurrence an obtuse angle; otherwise it is evident that the weight would descend in place of ascending; but the *vis motrix* and its velocity form an acute angle: thus, according to our definition, the weight will be the *resisting force*, and the effort of the person will be the *soliciting force*: it is evident, in short, that the latter tends to favour the actual movement of the machine, while the other opposes it.

We shall observe that the soliciting forces may be directed in the same ratio with their velocities, since then the angle formed by their concurrence is null, and consequently acute, and the resisting forces may act in the direction precisely opposite to that of their velocities, since then the angle formed by their concurrence is 180° , and consequently obtuse.

It is also to be remarked, that any force which is soliciting might become resisting if the movement should change; that any force which is resisting at a certain instant, may become soliciting at another instant; and lastly, in order to judge of it at each instant, we must consider the angle which it makes with the velocity of the point where we suppose it applied: if this angle be acute, the force will be soliciting; and if it be obtuse, it will be resisting, until the angle in question changes. We see from this, that if we make any system of power assume a geometrical movement, each of them will be soliciting or resisting in respect of this geometrical movement, accordingly as the angle formed by this force and by its geometrical velocity shall be acute or obtuse.

XXXII. If a force P be moved with the velocity u , and the angle formed by the concurrence of u and P be z , the quantity $P \cos z u dt$, in which dt expresses the element of time, will be named *momentum of activity*, consumed by the force P during dt ; i. e. the momentum of activity consumed

consumed by a force P , in a time infinitely short, is the produce of this force estimated in the ratio of its velocity, by the path described in this infinitely short time by the point to which it is applied.

I shall call the *momentum of activity*, consumed by this force, in a given time, the sum of the *momenta of activity* consumed by it at each instant, in such a manner that $sP \cos \alpha \, u \, dt$ is the *momentum of activity*, consumed by it in an indeterminate time: for instance, if P be a weight, the *momentum of activity* consumed in an indeterminate time t will be $Ps \, u \, dt \cos \alpha$; let us suppose, therefore, that after the time t , the weight P has descended from the quantity H , we shall clearly have $dH = u \, dt \cos \alpha$; therefore the *momentum of activity* consumed during dt will be $Ps \, dH = PH$.

XXXIII. When we are speaking of a system of forces applied to a machine in movement, I shall call *momentum of activity*, consumed by all the forces of the system, the sum of the *momenta of activity* consumed at the same time by each of the forces which compose it: thus, the *momentum of activity* consumed by the soliciting forces, will be the sum of the *momenta of activity* consumed at the same time by each of them: and the *momentum of activity* consumed by the resisting forces will be the sum of the *momenta of activity* consumed by each of these forces: and as each resisting force makes an obtuse angle with the direction of its velocity, the cosine of this angle is negative; the *momentum of activity* consumed by the resisting forces is therefore also a negative quantity; and therefore the *momentum of activity* consumed by all the forces of the system, is the same thing as the difference between the *momentum of activity* consumed by the soliciting forces, and the *momentum of activity* consumed at the same time by the resisting forces considered as a positive quantity.

A force estimated in a sense directly opposite to that of its velocity, and multiplied by the path described in an infinitely short time by the point where it is applied, will be called the *momentum of activity produced* by this force in
this

this infinitely short time: in such a manner *that the momentum of activity consumed, and the momentum of activity produced*, are two equal quantities, but of contrary signs; and there is a difference between them analogous to that which we find (XXI) between the *momenta of the quantity of movement gained and lost*, by a body, in respect of any geometrical movement.

I shall also give the name of *momentum of activity exercised by a force*, to what I have called its momentum of activity consumed, if it be soliciting, and to what I have called its *momentum of activity produced*, if it be resisting: thus, the *momentum of activity exercised* by any given force in an infinitely short time is in general the produce of this force, by the path which it describes in this infinitely short time, and by the cosine of the smallest of the two angles formed by the directions of this force and of its velocity; whence it clearly follows, that this *momentum of activity exercised* is always a positive quantity.

We shall make, with respect to the quantities which we call *momenta of activity produced* and *momenta of activity exercised*, the same remarks with those we have made above, upon the subject of *momentum of activity consumed* by a force or system of powers in a given time.

These definitions being admitted, I shall proceed to the general principle of equilibrium and of movement, in machines properly so called; and the inquiry into which has been the principal object of this essay.

FUNDAMENTAL THEOREM.

General Principle of Equilibrium and of Movement in Machines.

XXXIV. *Whatever is the state of repose or of movement in which any given system of forces applied to a machine exists, if we make it all at once assume any given geometrical movement, without changing these forces in any respect, the sum of the products of each of them, by the velocity which the point at which it is applied will have in the first instant, estimated in the direction of this force, will be equal to zero.*

That

That is to say, by calling F each of these forces*, u the velocity which the point where it is applied will have at the first instant, if we make the machine assume a geometrical movement, and z the angle comprehended between the directions of F and of u , it must prove that we shall have for the whole system $\sum F u \cos z = 0$. Now this equation is precisely the equation (AA) found (XXX), which is nothing else in the end but the same fundamental equation (F), presented under another form.

It is easy to perceive that this general principle is, properly speaking, nothing else than that of Descartes, to which a sufficient extension is to be given, in order that it may contain not only all the conditions of the equilibrium between

* It will not perhaps be useless to anticipate an objection which might occur to those who have not paid sufficient attention to what has been said (XXX) upon the true meaning we ought to attach to the word *force*: Let us imagine, for instance, they will say, a wheel and axle to the cylinder of which weights are suspended by means of cords; if there be equilibrium, or if the movement be uniform, the weight attached to the wheel will be to that of the cylinder as the radius of the cylinder is to the radius of the wheel; which is conformable to the proposition. But the case is not the same when the machine assumes an accelerated or a retarded movement: it seems, therefore, that here the forces are not in reciprocal ratio of their velocities estimated in the direction of these forces, as would follow from the proposition. The answer to that is, that in the case where this movement is not uniform, the weights in question are not the only forces exercised in the system; for the movement of each body changing continually, it also opposes at each instant, by its *vis inertiae*, a resistance to this change of state: we must, therefore, keep an account of this resistance. We have already said (XXX. see the note,) how this force should be estimated, and we shall see further on (XLI), how we should make it enter into the calculation. In the mean time it is sufficient to remark, that the forces applied to the machine in question are not the weights, but the quantities of movement lost by these weights (XXX), which should be estimated by the tensions of the cords to which they are suspended: now whether the machine be at rest or in motion, whether this motion be uniform or not, the tension of the cord attached to the wheel is to that of the cord attached to the cylinder, as the radius of the cylinder is to the radius of the wheel, *i. e.* these tensions are always in reciprocal ratio of the velocities of the weights they support: this agrees with the proposition. But these tensions are not equal to the weights; they are (XXX. see the note) the results of these weights and of their *vis inertiae*, which are themselves (XXX. see the note) the results of the actual movements of these bodies, and of the movements equal and directly opposed to those which they will really assume the instant afterwards.

FIRST

two forces, but also all those of equilibrium and of movement, in a system composed of any number of powers: thus the first consequence of this theorem will be the principle of Descartes, rendered complete by the conditions which we have seen were wanting in it (V).

FIRST COROLLARY.

General Principle of Equilibrium between two Powers.

XXXV. *When any two agents applied to a machine form a mutual equilibrium; if we make this machine assume any arbitrary geometrical movement: 1st, The forces exercised by the agents will be in a reciprocal ratio to their velocities estimated in the direction of these forces: 2d, One of these powers will make an acute angle with the direction of its velocity, and the other an obtuse angle with its velocity.*

For if the forces exercised by the agents are named F and F' ; their velocities u and u' , the angles formed by these powers and their velocities z and z' , we shall have by the preceding theorem, $F u \cos z + F' u' \cos z' = 0$: therefore $F : F' :: -u' \cos z' : u \cos z$, which is the proportion announced by the first part of this corollary, and by which we see at the same time that the relation of $\cos z$ to $\cos z'$ is negative; whence it follows that one of these angles is necessarily acute, and the other obtuse.

SECOND COROLLARY.

General Principle of Equilibrium in Weighing Machines.

XXXVI. *When several weights applied to any given machine mutually form an equilibrium, if we make this machine assume any geometrical movement, the velocity of the centre of gravity of the system, estimated in the vertical direction, will be null at the first instant.*

For if we call M the total mass of the system, m that of each of the bodies which compose it, u the absolute velocity of m , V the velocity of the centre of gravity estimated in the vertical ratio, g the gravity, z the angle formed by u and by the direction of the weight, we shall have, according to the theorem, $s m g u \cos z = 0$; but by the geometrical properties of the centre of gravity we have $s m u d t \cos z$
sine

sine $z = M V d t$, or $s m g u$ cosine $z = M V g$; therefore, since the first member of this equation is equal to zero, the second is so also: therefore $V = 0$. Q. E. D.

In order to have all the conditions of the equilibrium in a weighing machine, it is only necessary to make the machine successively assume different geometrical movements, and to equal in each of these cases the vertical velocity of the centre of gravity at zero.

THIRD COROLLARY.

General Principle of Equilibrium between two Weights.

XXXVII. *When two weights form a mutual equilibrium, if we make the machine assume any geometrical movement: 1st, The velocities of these bodies, estimated in the vertical ratio, will be in a reciprocal ratio to their weights: 2d, One of these bodies will necessarily ascend, while the other will descend.*

This proposition is a manifest consequence of the preceding corollary, and is still more evidently deduced from the first corollary.

We may remark by the way, how essential it is for the precision of all these propositions, that the movements impressed upon the machine should be geometrical, and not simply possible; for the slightest attention will show by some particular example, that without this condition all these propositions would be absurd.

Remark.

XXXVIII. We generally take the principle of equilibrium in weighing machines when the centre of gravity of the system is at the lowest possible point; but we know that this principle is not generally true; for besides that this point would be in certain cases at the highest point, there is an infinity of others where it is neither at the highest nor at the lowest point: for instance, if the whole system be reduced to a weighing body, and this moveable article be placed upon a curve which has a point of inflexion, the tangent of which is horizontal, it will remain visibly in equilibrium, if we place it upon this point of inflexion, which nevertheless

nevertheless is not the lowest weight, nor the highest point possible.

We may also take for the principle of equilibrium in a weighing machine the proposition which we have already given (II), and which we shall repeat, in order to give a rigorous demonstration of it.

In order to ascertain that several weights applied to any given machine should mutually form an equilibrium, it is sufficient to prove, that if we abandon this machine to itself, the centre of gravity of the system will not descend.

In order to prove it, let us name M the total mass of the system, m that of each of the weights which compose it, g the gravity; and suppose that if the machine did not remain in equilibrium, as I assert that it should, the velocity of m after the time t would be V , the height from which the centre of gravity would have descended at the end of the same time H , and that from which the body would have descended mh ; we shall then have (XXIV) $s m g d h - s m V d V = 0$: therefore by integrating $M g H = \frac{1}{2} s m V^2$; but by hypothesis $H = 0$, therefore $s m V^2 = 0$; besides, V^2 is necessarily positive, as is evident: therefore the equation $s m V^2 = 0$ cannot take place, unless we have $V = 0$, *i. e.* unless there be equilibrium. Q. E. D.

Hence it follows, as we have said (III), that there is necessarily equilibrium in a system of weights, the centre of gravity of which is at the lowest possible point; but we have seen (XXXVIII) that the inverse is not always true, *i. e.* that every time there is equilibrium in a system of weight, it does not always follow that the centre of gravity is at the lowest point possible.

FOURTH COROLLARY.

Particular Laws of Equilibrium in Machines.

XXXIX. *If there be equilibrium between several powers applied to a machine, and having decomposed all the forces of the system, as well those which are applied to the machine as those which are exercised by the obstacles or fixed points which form part of it; if we decompose them, I say, each*

each into three others parallel to any three axes perpendicular to each other :

1st. The sum of the component forces, which are parallel to one and the same axis, and conspiring towards one and the same side, is equal to the sum of those which, being parallel to this same axis, conspire towards the opposite side :

2d. The sum of the momenta of the component forces which tend to turn around one and the same axis, and which conspire in one and the same ratio, is equal to the sum of the momenta of those which tend to turn around the same axis, but in a contrary direction.

In order to demonstrate this proposition, let us begin by imagining, that in place of each of the forces exercised by the resistance of obstacles, we substitute an active force equal to this resistance, and directed in the same ratio: this change does not alter the state of equilibrium, and makes of the machine a system of powers perfectly free, *i. e.* freed from every obstacle. This being granted, if we make this system assume any geometrical movement, we shall have by the fundamental theorem $\sum F u \cos \alpha = 0$, by calling F each of these forces, u its velocity, and α the angle comprehended between F and u : thus,

1st. If we suppose that u is the same with respect to all the points of the system, and parallel to any one of the axes, the movement will be geometrical, and the equation, on account of u constant, will be reduced to $\sum F \cos \alpha = 0$: *i. e.* the sum of the forces of the system estimated in the ratio of the velocity u , impressed parallel to this axis, will be null; which evidently reverts to the first part of the proposition.

2d. If we make the whole system turn round any one of the axes, without changing in any respect the respective position of the parts which compose it, this movement will still be geometrical; u will be proportional to the distance of each power from the axis; and therefore might be expressed by AR , R expressing this distance, and A a constant: thus the equation will be reduced to $\sum FR \cos \alpha = 0$; which, as may easily be seen, reverts to the second part of the proposition.

[To be continued.]

XXVIII. Re-