THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

JULY 1903.

I. *On Electrostriction.* By LOUIS T. MORE, *Ph.D., Professor of Physics in the University of Cincinnati* *.

[Plate I.]

IN a previous paper †, I published an account of some experiments made on the supposed elongation of a dielectric placed in an electrostatic field. The conclusion reached was that glass and hard rubber do not elongate perpendicularly to the field, and that the effects observed by others were due to different causes. This conclusion was something of a disappointment, as I had undertaken the work for the purpose of solving a problem based on this effect of electricity.

The laws of electrostriction had been so elaborately formulated, and had been substantiated by so many experimenters, that their results were accepted and an apparatus constructed which would record changes of the given magnitude, allowing for a margin of safety. In the early stages of the work, when the tubes of glass operated on were charged, elongations were observed and recorded in my notes which agreed fairly with those of my predecessors. But combined with them were, at times, irregularities and discrepancies. The motion of the recording image was jerky, and the return of the tube after discharge sometimes great and at other times small; occasionally a tube was used which showed practically no deflexion when charged. After considerable difficulty these

† Communicated by the Author.

* *Phil. Mag.* vol. 1. pp. 198-210 (1900).

disturbances were located, and proved to be due principally to heating of the glass and to a slight bending of the tube, produced by a field of force not absolutely uniform in all directions.

As these discrepancies threw doubt on my results, the work of others was compared, and so many inconsistencies became evident that I abandoned my original plan and turned my attention to the subject of electrostriction itself.

From experiments made by Quincke* it could be inferred that rigidity and accuracy of alignment were very essential, and my apparatus had, as I thought, been constructed to obviate errors due to the neglect of these conditions. But repeated trials convinced me that the utmost care was necessary to eliminate their influence. I finally did get such uniform tubes in so accurate and rigid adjustment as to reduce them to a minimum. But with the elimination of these extraneous phenomena the entire elongation disappeared also. So I concluded that either the effect did not exist or was much smaller than was supposed.

Some time afterwards Dr. Sacerdote called my attention to his own memoir on the theory and to the experimental work of Cantone. He argued that the absence of the effect in my case resulted from employing an apparatus not sufficiently sensitive. His criticisms were afterwards published†. In a reply‡ I attempted to show that Dr. Sacerdote had misunderstood certain important statements of mine, and that my apparatus did have a sensibility great enough to have indicated the effect, assuming the values given by Cantone. The results of others, it was stated, were not quantitatively reliable.

After some correspondence, Dr. Sacerdote has asked me to repeat my work, using a more sensitive apparatus. The delay in doing so has been somewhat great; but it has the advantage of allowing time for the discussion of the subject which has occurred.

It is hardly necessary to give an account of the experimental results on this subject, as they have already been described and discussed by myself and others, excepting the work of the more recent investigators.

Prof. Cantone §, alone and in collaboration with Sozzani ||,

* Quincke, *Wied. Ann.* x. pp. 161, 374, 513; xix. pp. 545, 705; xxviii. p. 529; xxxii. p. 530.

† Sacerdote, *Phil. Mag.* vol. i. p. 357 (1901).

‡ *Phil. Mag.* vol. ii. p. 527 (1901).

§ Cantone, *Rend. della R. Acc. dei Lincei*, t. iv. pp. 344, 471 (1888).

|| Cantone and Sozzani, *Rend. della R. Ist. Lombardo*, t. xxxiii. (1900).

has published many experiments on the behaviour of glass condensers when charged. For the purpose he used cylindrical tubes about 70 centimetres long and 7 millimetres in diameter, ranging from 0.045 to 0.2 centimetre in thickness. The potentials to charge them occasionally were great enough to give a 10 millimetre spark. The armatures of the condensers were adherent to the glass, and the elongations were measured by the displacement of interference-bands. The greatest displacement noted is 2.6 bands, but the great majority were less than one band, and they were measured to one-hundredth. The elongations, reduced to unit dimensions, average

$$\frac{\delta l}{l} \times \frac{d^2}{V^2} = 6 \times 10^{-13}.$$

This result is calculated from the deflexions observed when the tubes were charged; the return, when the tubes were discharged, sometimes equalled the above, and was sometimes less than half. The time of charging also affected the amount of the elongation; but no law could be formulated, and no reasons are given for the time of charging determined on to give the correct value.

In spite of the carefulness which Cantone's experiments show, there are evidences that the values given are, at least, partly due to causes other than the one he ascribes them to.

In the first place, the amount of the elongation increases with the time of charging and to an unknown extent.

Secondly, they are partly due to heating of the tube. Cantone himself attributes the erratic return of the tubes to the heat evolved; and this varies between a complete return and a value of only one-half. If so, surely part of the elongation in charging must be due to the same cause.

Thirdly, the armatures touch the glass, and their attraction will elongate the tube, using Poisson's ratio, approximately one-third the observed amount.

Fourthly, tubes of the dimensions given are flexible, rarely straight, and do not have a uniform wall-thickness. All of these defects tend to strain the tube when it is charged, as the field is not uniform. The amount of this strain is large, and even with unusually accurate tubes it is difficult to avoid an apparent elongation greater than the supposedly true effect.

Unless these causes of possible elongation are eliminated, the elongation observed is subject to criticism.

In answer to a statement made in my paper, that the true effect could be obtained only when free charges were not placed on the dielectric; or, in other words, when the armatures of the condenser were separated from the dielectric and

the intervening spaces filled with a non-conducting fluid having a dielectric constant equal to that of the solid dielectric, Prof. Ercolini* made a series of experiments with such armatures.

The arrangement of his apparatus was similar to mine, except that the changes of length were measured by the displacement of interference-bands.

A comparative table of the dimensions of the apparatus will show their relative advantages.

TABLE I.

	Ercolini.	More.	
		Former.	Present.
Outer diameter of glass tube (A) ...	16.33 mm.	57.00 mm.	44.00 mm.
Inner " " " " ...	15.30 "	52.00 "	40.00 "
Thickness " " " " ...	0.52 "	2.50 "	2.00 "
Distance between A and armatures .	2.00 "		
" " armatures	4.52 "	7.50 "	4.00 "
Length of armatures	550.00 "	440.00 "	550.00 "
Value of least reading	0.01 fringe	1.5×10^{-5} ,"	3×10^{-6} ,"

Ercolini states that each of the tubes operated on experienced at the moment of charge an elongation, which disappeared with the discharge. But that these results were so small and disturbed by oscillations occurring when the spark passed that he could not obtain numerical results. Together with this elongation of the tube were *lateral displacements* shown by irregular movements of the bands, which could never be avoided. These were probably due to irregularities in the thickness of the glass and to lack of coincidence in the axes of the three tubes.

Just how the writer could distinguish between the regular and irregular displacement of the bands, it is difficult to see, since they must have occurred simultaneously ; nor does he take any notice of effect of the heating of the tubes.

Dr. Sacerdote † considers this case to require the special equation

$$\frac{\delta l}{l} = k_1 \frac{\overline{KH}^2}{8\pi}.$$

That is, the elongation is dependent only on the difference of potential, the dielectric constant of the glass, the variation of this constant with pressure k_1 , and the distance between

* Ercolini, *Nuovo Cimento*, ser. 5, t. ii. p. 5 (1901).

† Ercolini, *Journ. de Phys.* sér. 4, t. i. p. 40.

the armatures ; but that it is independent of the nature of the non-conducting fluid.

These experiments are adduced by Ercolini and Sacerdote as proving that the coefficient k_1 is positive. That is, the dielectric constant increases with a tractile force acting perpendicularly to the field of force. In a note it is stated that the elongation was too small to be even noted by my apparatus. Possibly that is dismissing the matter too curtly unless the estimate of the sensibility of my apparatus made by Dr. Sacerdote is the correct one. In my own opinion, formed after long experience with it, this sensibility is at least $3\frac{1}{2}$ times as great as he assumes it to be ; and I thought such was evident from my explanation published after his first criticism. Even if this criticism were valid in respect to my former apparatus, it certainly is not to my present one. The only apparent advantage of Ercolini's arrangement is that he used thinner tubes ; but by doing so he increased the effect of the irregular movements he describes. Also it must be remembered that I used potentials great enough to rupture the tube, and a more intense field could hardly have been obtained. Yet, as will be shown later, no elongation was observed with non-adherent armatures when lateral effects were eliminated. The conclusions of Ercolini's work have been questioned by Corbino, and, I think, rightly.

The most recent work in this field is by Dr. Shearer*. The writer experimented on condensers of glass and hard rubber with adherent armatures. The method of measuring the effect was by the displacement of interference-bands. His results on rubber and glass may be summarized as follows,—the supposed elongation being for these substances of the order 10^{-8} and 10^{-13} per unit dimensions respectively.

Rubber plate condenser 40 cms. long and 3 mm. thick. Difference of potential sufficient to puncture glass 3 mm. thick. A displacement of approximately one-fourth of a band was observed, increasing gradually, and returning to the original position after each discharge. This effect disappeared after the first few charges when there was no spark-gap in the circuit. A very slight twist of the hard rubber, by reason of non-uniformity of the field, might easily cause this effect. When a spark-gap was inserted a considerable displacement of the bands, increasing greatly with the time, occurred. On looking between the condenser-plates the whole region appeared filled with a glow-discharge. It may be inferred that this motion was due to a gradual heating of the hard rubber.

* Shearer, *Physical Review*, vol. xiv. p. 89 (1902).

Glass rod with capillary bore, length 45 cms., diameter 5 mm. Potential from 14 to 80 kilovolts. No movement whatever was observed except when a spark passed.

Glass tubes, 75 cms. long, wall-thickness from 0.08 to 0.12 cm. Potential in each case great enough to rupture the tube. No movement so large as one-half a band-width was observed, when the conditions were such as to avoid a direct heating effect. (With tubes of the same length and 0.13 cm. thick, Cantone records a displacement of 1.4 band for a spark-length of 1 centimetre.)

Glass tubes, silvered part of the length 73 cms., wall-thickness 0.24 cm. Potential produced a spark 5 cms. long, at which distance the tubes broke. The writer shows that, compared with Cantone's results, the expansion should have been far within the limits of possible readings with the instrument. No effects were observed other than those plainly due to temperature changes. Not the least motion of the bands could be detected when the potential was instantaneously raised by connexion being made with a set of highly charged jars.

Hard rubber tubes, length 100 cms., wall-thickness 2.5 mm. Potentials increased to 4 cms. spark, when tubes broke. According to Korteweg and Julius, the expansion of rubber is of the order 10^{-8} , instead of 10^{-13} , the coefficient of glass. When charged for one minute, there was a slow motion of 18 bands across the field. About ten minutes were required for the complete return. This motion was shown to be due to heating. A long piece of fine copper wire was wound spirally about a similar tube. The wire was inserted in a Wheatstone-bridge and the resistances balanced. The tube was then charged with a potential to give a 2-cm. spark, the displacement of the fringes noted, and the tube discharged. On again measuring the resistance of the wire it was found to be greater, and the increase was measured. Calculation of the rise in temperature of the wire proved that the heating of the tube was of the same order as the elongation of the tube, if it were due to heating alone.

Dr. Shearer concludes from his experiments that the observed changes in length may be readily accounted for without reference to other causes than heat and slight distortions due to an unsymmetrical field of force; and that the production of changes of form and dimensions in matter by purely electrostatic charge is extremely improbable.

For the theoretical development of this subject the reader may refer to the memoirs of Sacerdote and Shearer, who have given complete bibliographies.

The greatest confusion in the correct interpretation of the formulæ arises, I think, from what may be called a too material view of the æther. It has proved so convenient a medium on which to hang our equations that we are apt to forget that it has, and must necessarily have, so far as we are concerned, a hypothetical existence; and a mechanical interpretation of one of its functions invariably breaks down either *per se*, or when it is compared with other functions. Witness Maxwell's attempt to explain mechanically displacement-currents of electricity, or the abandonment of the elaborately developed elastic-solid theory of light. But if the functions ascribed to the æther itself are contradictory, they become hopelessly involved when the æther is associated with matter. Even if Faraday's hypothesis, that an electric charge produces a strain in the æther, be granted, it does not at all follow that it will be accompanied by unbalanced mechanical forces in matter immersed in the æther. In fact, every experiment has failed which attempted to show a connexion between the two when the effects were static or kinetic, if the velocity was comparatively small. When the velocity of the kinetic energy involved is as great as or greater than that of light, the presence of matter in the æther does seem to affect the action. Certainly Kerr's effect, and others of the same kind, can be as readily explained as purely mechanical or thermal actions.

The most recent and complete of these theories of electrostriction is by Sacerdote, who gives for the relation between the electrical field and the elongation of a cylinder having adherent armatures

$$\frac{\delta l}{l} = (a + k_1) \frac{\bar{K} V^2}{8\pi d^2}.$$

In this equation a is the inverse of Young's modulus and k_1 the coefficient of variation of the dielectric constant $\bar{K}$, produced by a tractile force acting perpendicularly to the line of force. The remaining letters have their usual significance. The value of k_1 has never been determined with any accuracy. Indeed, even its sign is a matter of dispute, some obtaining a positive value, and others a negative one. It is introduced into the equation on the ground that the relation is a reciprocal one. If the dielectric constant of a substance increases with a tractile force applied perpendicularly to the lines of force, it is assumed, when a dielectric is charged, that the field of force should produce a tractile force, accompanied by a corresponding elongation. But it should be remembered that this coefficient is not yet determined, and even if it were, the reciprocal relation may not exist. In a general way, the

dielectric constant is greater, the denser the substance ; it may well be that if a gas, or a solid to a less degree, were compressed, and the molecules thus become closer together, the dielectric constant might increase. It does not at all follow that the increase is accompanied by a repulsion of the molecules at right angles to the field, unless we make the assumption that an electrical charge produces in some manner a mechanical strain. If the dielectric constant is shown by experiment to increase with a tractile force, not because the molecules of matter are incidentally brought closer together, but because of the stress itself, then the reciprocal relation that a charge causes a strain perpendicularly to the field will hold, or *vice versa*. But until one or the other is indisputably proven, to introduce either one into the equation to prove the other is tacitly assuming the whole question of the mechanical relations between æther and matter.

In a recent paper Dr. Sacerdote* discusses theoretically the special case when the armatures are separated from the solid dielectric by spaces filled with a non-conducting fluid. He arrives at the conclusion that the modulus of elasticity does not enter, the equation being

$$-\frac{\delta l}{l} = k_1 \frac{KV^2}{8\pi d^2}.$$

The letters have here the same significance as before, and the dielectric constant of the fluid is immaterial.

This is, however, only apparently so, for this constant would be involved in the calculation of the field-strength in the solid dielectric. For if the dielectric constants are different, the fall of potential between the armatures is no longer uniform. The resultant field in the solid would be equivalent to a field produced by the potentials of the armatures acting through a homogeneous dielectric *plus* a field produced by free charges assumed to be *on* the faces of the solid dielectric. These latter evidently require the treatment for a condenser with *adherent* armatures, and the introduction of the coefficient of elasticity.

In this paper he again asserts that my experiments gave nugatory results because of lack of sensibility, and Ercolini's † show k_1 to be positive. Granting this criticism to be true, it should not be overlooked that I also experimented with hard-rubber tubes and obtained no elongation, although the effect is supposedly much greater than for glass. The results, to be given later, using thinner glass, a less distance between

* Sacerdote, *Journal de Phys.* (3) t. x. p. 196

† Ercolini, *loc. cit.*

the armatures, and a more sensitive apparatus than Ercolini's, confirm my former ones.

As my previous experiments did not convince my critics, there was no alternative but to repeat them, and as conclusively as possible. In doing so the following points were kept in view: the apparatus should be more sensitive and more carefully adjusted; extraneous effects guarded against or eliminated; effect of heating measured; and the mechanical attraction of the armatures, when adherent, calculated. The task has been an extremely difficult one; but perhaps in all four particulars something has been done.

In a general way, the apparatus is similar to the one previously employed; but it was certainly more efficient because of better construction. Whether the interferometer method is better or not, I do not know; but the errors do not result from any lack of sensitiveness or accuracy of the magnifying machine used.

A brass tube A (Pl. I. fig. 1) was screwed in a heavy iron flanged collar, and this to a wooden base. The experimental glass tubes were placed coaxially over this brass tube and cemented to a brass collar C at the base. The brass tube thus formed one armature and the glass tube the dielectric of a condenser. For the other armature several devices served, either a sheet of tinfoil was pasted on the outer surface of the experimental tube; or a glass tube, silvered on its inner surface, cemented in a position coaxially to the other two; or a brass tube F capped with short glass tubes at the top and bottom, as shown in the figure. The brass tubes were seamless, quite straight, and of uniform thickness. The glass tubes were obtained from Messrs. Eimer and Amend, of New York, who kindly examined their entire stock in order that they might supply me with perfect pieces.

The dimensions of the tubes are given in the table.

TABLE II.—Experimental Glass Tubes.

No.	Length.	Outer Diam.	Inner Diam.	Thickness.
1	65 cms.	4.40 cms.	4.05 cms.	1.8 mm.
2	65 "	4.48 "	4.05 "	2.2 "
3	65 "	4.43 "	4.03 "	2.0 "
4	65 "	4.55 "	4.13 "	2.1 "
5	65 "	4.53 "	4.15 "	1.9 "
6	65 "	4.67 "	4.23 "	2.2 "

Inner brass tube A, Length 64 cms., outer diam. 3.9 cms.

Outer glass " F, " 62 " inner " 4.9 " base to silver 6 cms.

Outer brass " F, " 55 " " " 4.7 " " tube 5 cms.

that is, one-fifth that of the present form. And this minimum deflexion could be observed readily and measured accurately.

When electrically charged the inner armature and the tripod table were always grounded, and the outer one charged by a powerful static generator, capable of giving a spark eight inches long and of great volume. From one to four large leyden-jars were placed in the circuit to prevent irregularities in the charging. The potential-differences were measured by a spark-micrometer with balls two centimetres in diameter. With all the apparatus in the circuit the highest potentials were reached in from ten to fifteen seconds. To prevent any charge from reaching the tripod table, it was inclosed in a cage of fine copper gauze wire, which fitted over the top of the experimental tube and was grounded. When the tube was disconnected from the circuit, no disturbance reached the mirror, even though long sparks passed across the terminals of the spark micrometer.

The entire apparatus was fastened to heavy beams, bolted to masonry piers in a basement room. No experiments were made when the image showed any motion due to temperature changes in the room during a period of time five or six times that of charging.

A convenient method of testing the apparatus is to make a determination of the modulus of elasticity. The weights for compressing the tubes were placed on a table with three legs which rested on the glass tube only. Deflexions are expressed in divisions of the micrometer.

TABLE IV.—Modulus of Elasticity.

	Tube 3.	Tube 4.
Weight, 1000 gr.	60.5 div.	45.0 div.
	61.0	55.0
	58.0	55.0
	56.0	—
	60.0	51.7
	—	—
	59.0	—
	—	—
Weight, 500 gr.	31.0	25.0
	30.5	25.0
	30.5	—
	—	25.0
	30.7	—
	—	—

Employing the ordinary formula for Young's modulus, we have from the data,

	3.	4.
Length of tube	65 cms.	65 cms.
Area of cross-section	2.6 cms.	2.9 cms.
Change of length per kg. . . .	60.0 div.	50.0 div.
Value of a division	6×10^{-6} mm.	6×10^{-6} mm.
Young's modulus (μ)	6.8×10^{11}	7.3×10^{11}

These results agree so closely with each other and with the accepted value for this modulus for glass that there should be no hesitation in accepting the readings of the apparatus.

Passing now to the experiments made when the condenser was electrically charged, it is more convenient to give the results in a series of tables without comment, and to discuss them later. They may be divided into two classes, those with adherent armatures and those when the armatures are separated from the dielectric by a non-conducting fluid.

Adherent Armatures.

Tube 5 was cemented in place and covered on the outer surface with tinfoil for a distance of 50 centimetres. The space between this tube and the grounded brass tube was filled with a concentrated solution of sodium carbonate in water, which served to ground the inner surface of the glass tube. A second glass tube, larger in diameter and without a metal coating, was cemented coaxially with the others, and the space between the glass tubes, about a millimetre, was filled with pure lard oil. The oil and outer tube merely prevented the charge from leaking off the tinfoil. A fine wire, connected with the generator and spark-micrometer, was attached to the foil; first passing through a hole bored in the outer tube, half-way up. The hole was then closed with a rubber stopper.

Observations are recorded only for days when the room temperature was so constant that the image remained steady. The drum of the micrometer contained 250 divisions, and a change from a greater to a less number denotes an increase in the length of the tube. Both tilting-mirrors were used, but as the readings agreed they are all reduced to the dimensions of the one with feet 6 millimetres apart. Observations are recorded in divisions of the micrometer, one division being equivalent to 6×10^{-6} millimetres.

From the final reading 243 with a 15 mm. spark, the readings were made as rapidly as possible, shown by the final reading of one being the same as the initial of the next.

When the spark-length was altered, a considerable time necessarily elapsed between readings, as the difference between consecutive values indicate. Before this series of experiments, the image was perfectly stationary, and it was not until the tube had been charged that a continuous increase in length occurred.

The up and down deflexions were always accompanied by a slight horizontal motion. The deflexion begins slowly, and increases rapidly during the latter part of the charging, being especially rapid just before and at the time the spark passes. After the spark, the mirror oscillates rapidly and then comes to rest, nearly always at the final reading; the return is slow and never to the full amount of the original deflexion.

TABLE V.—Tube 5. Length of Armature, 50 cms.

Spark-length.	Initial Reading.	Final Reading.	Deflexion.	Remarks.
12 mm. ...	190 div. 145 135	175 div. 125 120	15 div. 20 15 (?)	1 jar in circuit. Time to charge $\frac{1}{2}$ minute.
Discharged and left standing 7 minutes.				
12 mm. ...	62 18	25 233	37 35	Time to charge 1 minute.
Discharged and left standing 1 minute.				
12 mm. ...	213 173 163	193 163 130	20 10 (?) 33	Time to charge 1 minute.
Discharged but time not measured.				
12 mm. ...	43 25	38 18	5 7	3 jars in circuit. Time to charge less than 15 seconds.

TABLE VI.—Tube 5. Length of Armature, 50 cms.

Spark-length.	Initial Reading.	Final Reading.	Deflexion.	Remarks.
16 mm.	50	35	15	Discharged 5 min. Discharged 1 min. Discharged 10 min.
	193	167	26	
	123	82	41	
	75	57	18	

The tube was ruptured when the spark was increased to 2 cms.

TABLE VII.—Tube 2. Length of Armature, 52 cms.

Spark-length.	Initial Reading.	Final Reading.	Deflexion.	Remarks.
10 mm. ...	88	75	13	1 jar in circuit.
	63	38	25 (?)	
	18	3	15	
15 mm. ...	218	160	58	For the remainder of experiment no time elapsed between successive charges.
	30	243	37	
	243	214	29	
	214	185	29	
20 mm. ...	148	100	48	Zero changed.
	10	230	30	
	230	199	31	
15 mm. ...	185	168	17	
	168	158	10	
	143	133	10	
10 mm. ...	102	97	5	
	90	84	6	

The first series of observations, Table V., shows the influence of the duration of the charge, of discharge, and of the repetition of the charge when the potential-difference is maintained the same. When a quick charge is first given to the tube, it apparently increases in length about 15 divisions for a potential-difference of 12 mm. spark, or 110 c.g.s. If the tube is charged more slowly, about a minute, the increase in length is more than double, and, on the other hand, a less duration of charge diminishes the effect to a very small displacement. Table VI. shows the first effect of a charge at a greater potential to be the same as for a less.

In Table VII. the effect of varying the potential is given. With an increase of potential the elongation increases, but not uniformly, and a return to the lower potentials shows a much diminished value.

Before a true interpretation of this elongation normal to the field can be made, at least two corrections must be allowed for. In the first place, since the armatures are in contact with the glass, their mechanical attraction will compress the glass in the direction of the field, and this compression will be attended by an expansion in the direction of the axis of the cylinder, according to Poisson's theory. Finally, charging the armatures will certainly heat the tube to some extent.

The formula for the first correction in my former paper

is incorrect, as it should be

$$\delta l = \frac{1}{\mu} \frac{F}{S} \sigma l = \frac{1}{\mu} \frac{KV^2}{8\pi d^2} \sigma l,$$

where μ = modulus of elasticity = 7×10^{11}
 K = dielectric constant = 7
 V = potential-difference = 110 c.g.s.
 σ = Poisson's ratio = 0.25
 l = length = 50 cms.
 d = thickness = 0.2 cm.

or $\delta l = 1.5 \times 10^{-6}$ cm. = 3 divisions of micrometer.

This effect is then, at least, one-fifth the elongation when a tube is charged for the first time, and about one-half the elongation if it has been previously charged to a higher potential.

There can be no doubt that the charges on the armatures are sufficient to heat the glass an appreciable amount, for as soon as the tube has been discharged, the image begins to move continuously in the direction to show an elongation. The values in Table VI. give the magnitude of this effect. With a spark-gap of 16 millimetres, an initial deflexion of 15 divisions was obtained, the same as for a 12 millimetre spark. The tube was then discharged and allowed to stand five minutes, during which time the image moved 92 divisions (from 35 to 193). A second charge gave a deflexion of 26 divisions and a discharge for one minute 44 divisions. The change of seven divisions during the ten minutes following has no significance, as it was necessary to readjust the apparatus. But the effect of heat in expanding the tube is even more apparent from the values obtained when the tube was charged. The first reading, with an unheated tube, is 15 divisions. If it is then allowed to cool for ten minutes before charging again, the elongation is 18, and for five minutes 26; while if the previous cooling is only for one minute the expansion is 41. It seems evident that the increase of length, due to heat evolved during the charging, is of an order equal to the observed effect.

It is easy to calculate the rise in temperature which would expand the tube the entire amount, assuming 15 divisions or 9×10^{-6} centimetre for a 12 millimetre spark. The formula is

$$\delta t = \frac{\delta L}{\lambda L},$$

where δL = observed expansion = 9×10^{-6} cms.
 λ = coeff. of expansion = 9×10^{-6}
 L = length of tube = 60 cms.,
 then $\delta t = 0.017$ C.

The rise in temperature of the glass is from two causes ; the ohmic current through the glass from armature to armature, and the discharge through the oil and along the surface of the glass to the top and bottom of the apparatus. Of the two the latter is undoubtedly the greater, as the oil is a better conductor. The liquid is always put in violent commotion by the charge and, at times, the discharge vertically is so great as to cause a glow-discharge along the glass from the surface of the oil to the wire-gauze cage on the top of the tube. The deflexions also indicate the same thing as they lag behind the potential, showing that the passage of heat from the oil to the glass requires a small interval of time.

Although the rise in temperature of the glass itself cannot be measured, it can for the oil. For this purpose one junction of a platinum-iron thermoelectric element was inserted in the oil and the other in ice. The galvanometer was first calibrated, and gave a permanent deflexion of 2 centimetres for an increase of 6° . The tube was then charged ten times to a potential giving a 12 millimetre spark, when the deflexion was 0.3 centimetre, an increase of $0^{\circ}.9$. On another day, the same number of similar charges indicated a rise of $0^{\circ}.4$. This is much greater than is required to expand the glass by the observed amount, and will account for the continued increase in the length of the tube after it has been charged and discharged a number of times.

Non-adherent Armatures.

To experiment with non-adherent armatures, a new glass tube, number 3, was cemented in place and coaxially with it a larger glass tube, 4.9 centimetres inner diameter, which had been silvered on its inner surface a length of 50 centimetres. The sodium carbonate solution was not used, but, instead, both spaces between the three tubes were filled with lard oil. Electrical connexion was made with the silvered surface by a strip of tinfoil, carried through a hole in the outer tube, which was then closed by a rubber stopper. The other arrangements were as before. Both mirrors were used, but deflexions have been reduced so that one division equals 6×10^{-6} mm. as before.

The tube was now charged to a potential producing a 12 millimetre spark, with one jar in the circuit. The image moved to the left, entirely out of the field of the microscope, and this motion was accompanied by a large apparent elongation of the tube, too irregular and jerky to measure.

No care that I could employ to make the three tubes rigid and parallel would overcome this difficulty until the

tops of the tubes were kept in their proper positions by wedges. This was accomplished by inserting three little wedges of wood, or, preferably, of glass, between the experimental tube and each of the armatures at the top. Lest the friction might lessen the full effect of the expansion, new determinations for the modulus of elasticity were made and gave the same values recorded in the previous table for that experiment.

With the wedges in place, a potential of 12 millimetre spark produced some lateral bending, but not nearly so much and the motion was uniform. It was accompanied by an apparent elongation of 60 divisions. That this elongation was due to the side-wise motion was proved by having an assistant press gently against the top of the tube in a horizontal direction until the lateral motion was the same as before—the mirror again indicated an elongation of 60 divisions.

It was noted, during the experiments with adherent armatures, that after a tube had been charged a few times the image moved steadily to indicate a gradual elongation of the tube. This motion was ascribed to the effect of heat. Experiment shows this effect to be more pronounced with the present arrangement.

The room temperature being constant, a series of charges were given the condenser by exciting the generator continuously until a number of sparks passed, after which it was short-circuited and the image observed; then the deflexion was recorded at successive intervals of time.

Spark-length, 12 mm.—Number of Sparks, 10.

Deflexion, 1.5 minutes after tube was discharged ...	50	div.
„ 4.0 „ later	100	„
„ 5.5 „	150	„

Spark-length, 12 mm.—Number of additional Sparks, 10.

Deflexion, 1.5 minutes after discharge	130	div.
„ 4.0 „ later.....	225	„
„ 5.0 „ „	245	„
„ 5.0 „ „	285	„
„ 1.0 „ „	80	„
„ 16.5 „	965	„

The outer tube broke with further charging. It was not altogether satisfactory, as it was not accurately cylindrical and straight and was unnecessarily large. It was replaced by a

brass tube 55 cms. long, 4.7 cms. inner diameter, and 2 mm. thick. To insulate it, two glass caps, slightly larger in diameter, were added to the ends, as shown in fig. 1. When the glass wedges were in place, I succeeded in overcoming the lateral motion entirely.

With a spark-gap of 12 mm., the image remained quite at rest until the spark passed, when there was a sudden motion of 25 divisions. Keeping the potential constant, a number of charges were given, the jerk as the spark passed grew successively smaller, and finally ceased altogether. The spark-gap was increased to 2 cms., but produced no deflexion. Higher potentials could not be used as the upper cap broke.

When the silvered glass armature was used, the electricity leaked rapidly from the sharp edges of the silvering, causing a violent motion of the oil and a glow-discharge between the surface of the fluid and the wire cage covering the mirror. The edges of the brass-tube armature were well-rounded and the discharge materially lessened; so much so that the generator reached its highest potential in far less time, and the tube retained a high charge for many minutes. The heating was correspondingly less and slower.

Heating Due to 10 Sparks of 12 Millimetres.

Deflexion	2 minutes after discharge		22 div.
"	2 "	later.....	20 "
"	6 "	"	34 "
<hr/>			
"	10 "	"	76 "

The following experiments were made when the room temperature was constant, and sufficient time was allowed between two chargings for the tube to cool off and the image to come to rest again.

With the spark-gap set for 12 mm. a spark passed through the oil from the upper edge of the armature to the wire cage. Just before and at the time of the discharge, the tube expanded 27 divisions. The generator was short-circuited, and the heat effect for one minute was 11 divisions.

The discharge was a second time through the oil, the accompanying expansion, 13 divisions, and the motion during one minute after discharge, 11 divisions.

Spark-gap 10 mm., the discharge was across the spark-micrometer. There was no motion whatever of the image. Heating effect for one minute after discharge, 9 divisions.

Spark-gap 11 mm. No effect except a slight and rapid

oscillation as the spark passed. Heating for one minute, 15 divisions.

Spark-gap 12 mm. Discharge was not through the oil, and there was no deflexion except the oscillation described before.

Spark-gap 15 mm. Discharge was through the oil, but there was no deflexion except a permanent displacement when the spark passed, caused by the jar of the explosion.

The experiments described confirm the results of my former paper, that the elongation which may occur when a dielectric is charged can be fully accounted for by causes other than electrical stresses. Of these, heating is undoubtedly the most important, although the mechanical attraction of adherent armatures and the slightest non-uniformity in the field also produce appreciable errors.

The heat developed is probably, at least in my experiments, due more to currents along the surface of the dielectric than those through it, if the substance is a good insulator. As shown by my observations, the elongations, if they occur, lag behind the potential, and this effect is much diminished when a brass tube with rounded edges is used for the charged armature instead of tinfoil or silvered glass. When such an armature, separated from the dielectric, is charged, the heating effect is apparently so much retarded as not to interfere with rapid readings, and as a consequence no elongation occurs.

Until these extraneous causes can be eliminated or balanced, and an elongation observed which is indisputably due to the electrical field, I am compelled to believe either that the effect of electrostriction on matter is zero or that it is very much smaller than has been supposed.

University of Cincinnati,
February 1903.

II. *On the Potential and Lines of Force of a Circular Current.* By H. NAGAOKA, Professor of Physics, Imperial University, Tokyo*.

1. **T**HE potential of a circular electric current or of a vortex ring of infinitely small section was investigated by Lord Kelvin†, Maxwell‡, Hicks§, and Minchin||. They express the solid angle subtended by the circle by means

* Communicated by the Author.

† Lord Kelvin, Trans. R. S. E. (1869).

‡ Maxwell, 'Treatise on Electricity and Magnetism,' vol. ii. chap. 14.

§ Hicks, Phil. Trans. 1881, p. 628.

|| Minchin, Phil. Mag. xxxv. (1893).