Philosophical Magazine Series 5

ISSN: 1941-5982 (Print) 1941-5990 (Online) Journal homepage: <http://www.tandfonline.com/loi/tphm16>

I. On torsion

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To cite this article: Gustav Wiedemann (1880) I. On torsion , Philosophical Magazine Series 5, 9:53, 1-15, DOI: [10.1080/14786448008626791](https://doi.org/10.1080/14786448008626791)

To link to this article: <http://dx.doi.org/10.1080/14786448008626791>

Published online: 28 Apr 2009.

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THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

—◆—
[FIFTH SERIES.]

—
JANUARY 1880.
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I. *On Torsion.* By GUSTAV WIEDEMANN*.

[Plate I.]

1. **S**INCE M. Weber, in 1835, studied the laws of the temporal course of the permanent elongation remaining after the cessation of a temporary stretching of a thread (the elastic effect)†, and in 1841 advanced as the cause of the elongation “the difference of the position of the axis of elasticity of the smallest particles at any instant from that which corresponds to, and is determined for each tension at, perfect equilibrium,” an opportune observation by R. Kohlrausch (in 1847)‡ has been followed by a more extensive series of investigations upon the relations of imperfect elasticity. These have been carried out essentially in two different directions. In the one set the temporary and the permanent alteration of form are in part observed directly, as well as in their temporal course; in the other the diminution of the length of swing of oscillating bodies.

In the experiments of the latter sort (for example, on torsional oscillations) the molecules are immediately transferred from the positions assigned to them by the forces acting at any instant to other positions, before the elastic afteraction has ended; and consequently the phenomena are highly complicated. It therefore appeared to me advisable, before studying

* Translated from Wiedemann's *Annalen*, 1879, No. 4, vol. vi. pp. 485–520.

† Pogg. *Ann.* xxxiv. p. 247 (1835), liv. p. 1 (1841).

‡ Pogg. *Ann.* lxxii. p. 393 (1847).

relations of this sort, first to investigate the positions of equilibrium of bodies deformed by forces, or the torsions of their molecules which then takes place.

With this intention I have already, in the years 1858–1862*, published a series of treatises both on torsion and bending considered apart, and also on the analogies between mechanical and magnetic phenomena, as well as the reciprocal actions between the two.

I had endeavoured *inter alia* to determine in general outlines the ratio between the permanent deformations of a body and the temporary deformations occurring during the action of the deforming force, and had shown that, as Wertheim† had already observed, the former occur even after the slightest temporary deformations, so that in a body not previously deformed a limit of elasticity does not properly exist, but bodies which have sufficiently often been deformed within certain limits by determinate forces, on the repeated action of the same or less forces in the same direction as that last applied, constantly again receive the same permanent deformation. With this agrees the result subsequently (1865) communicated by Thalén‡, that by repeated stretching of rods a raising of the limit of their elasticity, increasing with it, takes place§. Moreover, with the first deformation the permanent alterations of form increased much more rapidly than the temporary, which is not the case in an equal degree with repeated deformations. I had in like manner shown that, when left longer in a state of rest after repeated deformations, bodies partially return to their previous condition||, and behave similarly under deformations which exceed the limits of the first. I have also studied the behaviour of bodies under consecutive oppositely directed deformations¶, &c.

In addition to these experiments I take leave to communicate some older and newer observations by which their results should be extended.

2. *Apparatus*.—The influence upon a wire of repeated tor-

* Pogg. *Ann.* ciii. p. 563 (1858), cvi. p. 161; Basle *Verhandlungen*, ii. Heft 2, p. 169; Pogg. *Ann.* cvii. p. 139 (1859).

† *Ann. de Chim. et Phys.* xii. p. 385 (1842); Pogg. *Ann.* Ergbd. ii. p. 70 (1848).

‡ Pogg. *Ann.* cxxiv. p. 600 (1865).

§ Similar results also by Tresca, *Compt. Rend.* lxxiii. p. 1163 (1874); Uchatius, *Beibl.* i. p. 273; Bauschinger, *ibid.* p. 380 (1877); and others.

|| Similarly Bauschinger, *l. c.* (1877).

¶ A series of the results of torsion and bending obtained by me have, certainly without mention of them, been confirmed by W. Voigt (*Dissertation d. Univ. Königsberg*, 1874?) in the bending of sticks of rock-salt.

sions in the same direction was first more fully investigated. The apparatus employed was the same that had served for the previous experiments. Pl. I. fig. 1 represents it with some more recent modifications. The point which, in the former experiments, held fast in the axis during the torsion the lower extremity of the wire and the circle, bow, and metal rod suspended from it, instead of working as before in an agate bearing, passes now through a small well-polished steel ring fixed on the axially pierced screw h_1 . Further, to the frame with roller r and weighing-basin s (by the load of which the wire is twisted) first a shorter brass wire vv is soldered, on which, by means of two silk threads, a longer, very thin and light, smooth horn rod xx is suspended beneath the basin, and, on the thread twisting, strikes against the vertical wires ww as soon as, the string AA being let run down, the roller is no longer supported by the latter, but by the thread o , wound round the circle on the wire. This arrangement is more suitable than the former one, in which the wire vv struck direct against the wires ww , since thereby the shakings that might take place during the slow lowering of the roller, in consequence of the elastic afteraction in the temporarily twisted wire, and in the thereby occasioned sliding of the wire vv on ww , are avoided. Finally, the string AA is fastened at t , and carried over a pulley at u to the observer, where its end is put round an axle Z of 5 millims. thickness, which can be turned by means of a winch. Thus we can still better, when the scale-pan is loaded with the weights, let down and raise the roller very slowly, and twist and untwist the wire without a shock. The wheel p turned so readily on its axis that when a string loaded at each end with 200 grams was suspended on it, an excess of less than 1 gram caused it to rotate.

The mirror-reading was now constantly employed for reading off the temporary and the permanent torsion within wider limits. A strip of tin of 1 decim. height, bent to a semicircle of 1 metre radius, and kept in this position by cross bands, was placed concentrically upon the wire and lined inside with a millimetre-scale, which, as usual, was observed in the mirror d attached to the clamp c , by means of a telescope. A displacement of the image of the scale 34.9 divisions corresponds to 1° of rotation of the mirror.

To the rod g a small bow with its edge horizontal was fastened beneath, into which a broad hook was inserted carrying on an iron rod 1 centim. thick, and 30 centims. long, a horizontal circular iron plate of 20 centims. diameter. Upon the latter were laid, very cautiously, in order not to cause any torsion, flat circular iron weights slit radially on one side,

Fig. 1.

Fig. 2.

which stretched the wire. Before placing them the wire was annealed under feeble tension.

3. *Repeated Torsion in the same Direction*.—First a brass wire of 480 millims. length and 2 millims. thickness, which was loaded beneath with altogether 10460 grams, was repeatedly twisted by increasing weights G , and the first temporary torsion T which took place immediately after the initial complete action of the weights was, so far as possible, constantly observed (the commencement of which was known by the little rod x striking against the wires $w w$), as well as the final temporary torsion T_1 , which was definitively produced after a long time. Likewise on lifting the torsion-weights the torsion P which at first set in, and the finally constant permanent torsion P_1 , were measured. The values of T and P cannot be exactly determined by any method, since the twistings of the wire that take place on lowering and winding up the weights immediately pass over, at the full action and the complete lifting of the weights, into the deformations produced by the aftereffect.

The results of the observations are contained in the following Table, in which Δ denotes the differences between the consecutive values of $T_1 - P_1$:—

I.													
W.	0	30	40	50	60	70	80	90	100	110	120	130	140
T	0	312	416	523	628	733	844	956.5	1070	1191	1338	1510	1725
T_1	0	312	416	523	628	734	845	959	1074	1204	1351	1543	1820
P	0	0.6	1.3	2.3	3.8	5.8	9.8	16	26.3	44	73.3	132.8	253
P_1	0	0.6	1.3	2.3	3.8	5.8	9.8	16	26.3	43.3	71.8	130	245
$T_1 - P_1$		311.4	414.7	520.7	624.2	728.2	835	943	1048.7	1160.7	1279.2	1413	1575
Δ		3.1038	103.3	106	103.5	104	104.8	108	105.7	112	118.5	133.8	162

Thereupon the wire was again twisted sixteen times by the weight 140. Then was

XVII.

T .	T_1 .	P .	P_1 .	$T_1 - P_1$.
1913	1919	373.8	372	1547

If now the same wire was again twisted by increasing weights, its permanent torsion remaining constantly $P_1 = 372$, then was

XVIII.

W.	0	30	40	50	60	70	80	90	100	110	120	130	140
T	372	682	787	892	999.5	1107	1217	1329.5	1442	1555	1673	1793	1909
T_1	372	682	787	892	1001.5	1109	1219	1332.5	1445	1559	1677	1797	1914
$T_1 - P_1$		310	415	520	629.5	737.5	847	960	1073	1187	1305	1425	1542
Δ		3.103	105	105	109.5	107.5	110	113	113	114	118	120	119

Thus the temporary torsions increase, as was previously found, at the first operation of the increasing weights, at first more slowly, and then always more rapidly, up to a determined value corresponding to the maximum loading. This latter value rises on the repeated employment of the maximum loading, at first more rapidly, then more slowly, up to a certain maximum*.

The permanent torsions exhibit the same behaviour; only they rise, on the operation of increasing weights, as already mentioned, more rapidly than the temporary torsions; they are also more increased, on repeated torsion by the same maximum weight, than the latter.

The result previously observed with alternately directed torsions is here confirmed—that the limits up to which oppositely acting forces temporarily twist wires always become narrower with repeated torsions, and gradually approach a constant minimal distance. In the present case those forces are the temporarily twisting maximal weight and the force *nil*, and the distance between the limits is the corresponding value $T_1 - P_1$.

At the same time, with repeated torsions the differences $T_1 - P_1$ that obtain for the various twisting weights become constantly more proportionate to the twisting weights; they rise a little with increasing weights, as the differences Δ of the consecutive values $T_1 - P_1$ show.

4. *Repeated Torsion in opposite Directions*.—In order to pursue this latter relation within wider limits, especially when the wires are not only unilaterally twisted repeatedly in one determined direction, but their molecules are turned aside by alternately acting forces in opposite directions, a wire of the same dimensions as the former ones was loaded with 8175 grams, repeatedly put into oscillation, and thereupon twisted ten times hither and thither by the weights ± 114 , when its temporary and permanent torsions no longer perceptibly altered; their amounts, in scale-divisions, were:—

W = +80	T 3753	P 1486.2	T - P = 2267
W = -80	T 1122	P 1387.5	T - P = -1265.5

Upon this the wire was twisted by increasing weights (-)W; and then the numbers were:—

W =	44	64	84	104	124
T	943	740	533	331.6	124.5
P	1387.5	1387.5	1387.5	1387.5	1387.5
T - P	444.5	674.5	854.5	1056	1263
Δ		203	207	201.5	207

Finally the wire was twisted in the opposite direction by

* See an analogous observation on stretching: Thalén, *l. c.* p. 682.

increasing weights + W, when the following numbers were obtained :—

W=	44	64	84	104	124
T	1857	2077	2295·8	2524·8	2753
P	1408·5	1425·5	1443	1464	1486·2
T-P	448·5	651·5	852·8	1060·8	1267
Δ		203	201·3	203	206·2

Here, as in the previous experiments, after repeated twistings and untwistings the temporary and the permanent final states of the wire were much more quickly reached than at the first action of the twisting forces.

5. *Torsion of a partly untwisted Wire*.—Another, similar wire was again and again twisted to and fro by the weights ± 120 , till its torsions were no longer altered on the renewed application of the same forces. Again the limits within which the temporary and the permanent torsions were confined continually approached one another. Thus, at the first and the sixth torsion they amounted to :—

	I.		VI.	
W	- 120	+ 120	- 120	+ 120
T	-1343	+1415	-1328	+1401
T ₁	-1366	+1431	-1334	+1406
P	- 75	+ 145	- 52	+ 119
P ₁	- 72	+ 143	- 50	+ 118
T ₊₁₂₀ -T ₋₁₂₀	2758	2797	2729	2740
P ₊₁₂₀ -P ₋₁₂₀	220	215	171·5	168

The wire was next twisted once more by increasing weights, when the permanent torsion remained +118; the temporary torsions amounted to :—

W=	+30	+40	+50	+60	+70	+80	+90	+100	+110	+120
T=	430	535	642	749	857	964	1072	1180	1288	1396
T-P	312	417	524	631	739	846	954	1062	1170	1278
Δ	105	107	107	108	107	108	108	108	108	108

After this the permanent torsion (118) of the wire was reduced to lower values by the weights -20 to -120, and the wire each time again twisted by increasing weights in the positive direction. The operation was afterwards reversed. The results are contained in the following Table, in which T₀ and P₀ denote the temporary and permanent torsions obtained at the reductions.

The results of the first observations (reduction by negatively acting weights) are more exact, because the finally valid action of the weights was longer waited for. They are on that account delineated in the curves in Plate I. fig. 2, of which the ordinates give the permanent torsions P-P₀, and the abscissæ the twisting weights. Moreover it follows immediately from the numbers of both Tables that, reckoning from the torsion P which remained at the reduction, the tem-

porary as well as the permanent torsions are substantially the same in both cases.

Reduced by -40 . $T_0 = -350$, $P_0 = +80.5$.					Reduced by $+40$. $T_0 = -409$, $P_0 = -8$.				
	30	60	90	120	- 30	- 60	- 90	- 120	
T	398	724	1056	1395	-729	-657	-984	-1338	
P	84.8	93	105	118	- 12	- 20	- 33	- 54	
$P - P_0$	3.5	12.5	24.5	37.5	- 4	- 12	- 25	- 46	
$T - P$	314	631.5	951	1277	-317	-637	-951	-1274	
Δ	317	320	326		320	314	323		

Reduced by -60 . $T_0 = -587$, $P_0 = +54$.					Reduced by $+60$. $T_0 = -643$, $P_0 = 14$.				
T	375	705	1043	1394	-308	-642	-975	-1334	
P	61	74	93	118	7	- 5	- 24	- 55.5	
$P - P_0$	7	20	39	64	- 14	- 19	- 38	- 69.5	
$T - P$	314	631	950	1276	-315	-637	-951	-1278.5	
Δ	317	319	326		322	314	327.5		

Reduced by -80 . $T_0 = -820$, $P_0 = 27.5$.					Reduced by $+80$. $T_0 = 884$, $P_0 = 41.6$.				
T	351	687	1034	1394	-285	-623	-963	-1333	
P	38	55	82.5	118	- 32	- 14	- 13	- 54	
$P - P_0$	10.5	27.5	55	90.5	- 9.6	- 27.6	- 54.6	- 95.6	
$T - P$	313	632	951.5	1276	-317	-637	-950	-1279	
Δ	319	319.5	325.5		320	313	329		

Reduced by -100 . $T_0 = -1068$, $P = -6.5$.					Reduced by $+100$. $T_0 = 1130$, $P_0 = 76$.				
T	326	671	1027	1397	-255	-602	-948	-1333	
P	12	36	72	118	59	32	- 2	- 57	
$P - P_0$	18.5	42	78.5	124.5	- 79	- 44	- 78	- 127	
$T - P$	314	635	955	1279	-314	-634	-946	-1272	
Δ	321	320	324		320	312	336		

Reduced by +120. T = -1327, P = -51.					Reduced by +120. T ₀ = 1393, P ₀ = 116.				
T	290.6	648	1013	1396	-223	-585	-940	-1332	
P	-21	17	62	115	90	50	8	- 47	
P - P ₀	30	68	133	166	- 26	- 66	-124	- 161	
T - P	312	631	950	1281	-313	-636	-948	-1284	
Δ	319	320	330		323	312	336		

6. *Results*.—If, therefore, by determined forces $\pm W_m$ acting in opposite directions, a wire is repeatedly twisted in alternate senses, till the temporary and permanent torsions $T_{\pm m}$ and $P_{\pm m}$ attained by those forces are constant, and if the wire be again twisted by increasing weights in the sense of the last torsion, wherein its residual permanent torsion P_p remains constant, its *transient* torsions $T_p - P_p$, reckoned from the latter, increase nearly in proportion to the acting weights. A very slight rise in the values of Δ is perceptible with the greater torsions.

If the permanent torsion P_{+m} produced in a wire by the weight W_{+m} be reduced by a counteracting weight to a lower value P_n within the limits $P_{\pm m}$, and then the wire be again twisted by increasing weights $W_{(+)}$, the permanent torsion will gradually increase from P_n to P_{+m} . If this be subtracted

from the temporary torsions produced by the above weights W , the differences $T-P$ will again be the same to whatever value P_n within the first limits P_{+m} and P_{-m} the torsion P_{+m} has been reduced. They rise hardly quicker than corresponds with the proportionality to the acting weights, and are equal to the temporary torsions $T_m - P_p$ produced by the same weights in the ascending direction. Within the limits $P_{\pm m}$ and $T_{\pm m}$, therefore, the wire is almost perfectly elastic, if the temporary deformations produced by the forces be reckoned from the actual zero-position, and so the *transient* torsions alone be taken into account. It is consequently only the displacement of the position of permanent torsion toward one side that conditions the deviation of the total (reckoned from the original position of equilibrium) temporary torsions from proportionality with the acting forces. Therewith the permanent torsion P_n increases up to P_m , at first more quickly, and then more slowly, with the rising forces.

From the above-cited experiments it immediately follows, further, that the wire which, after repeated torsions, has acquired a permanent torsion P_{+m} by forces $W_{(-)}$ acting in the opposite direction to that of the last applied twisting force $W_{(+)}$, receives stronger temporary torsions $T_{(-)}$ than when the forces $W_{(+)}$ act repeatedly in the same direction as the force $W_{(+)}$. The difference corresponds to the shifting of the position of permanent torsion in the negative direction.

7. *The Permanent Torsion of a Wire that is frequently twisted as far as a determined Limit.*—In other experiments a metal plate directed vertically downwards was soldered in a radial position to the horizontal disk borne by the wire; and horizontal screws were arranged so that at the torsion of the wire the plate struck against them. When in this way a brass wire of 2 millims. thickness, 450 millims. long, and loaded beneath with 9200 grams, was repeatedly twisted by the torsion-weights, the deflection amounting to T scale-divisions, its permanent torsions P_1 were found to be:—

	I.				II.		
	1.	2.	3.		1.	2.	3.
T	1387	1387	1387		2700	2661	2607
P_1	99.2	106.9	110		1056	1120	1136

If the wire in experiment I. was again temporarily twisted to 1387, struck gently with a flat wooden bar, and untwisted, P rose to 115.2, P_1 to 115.

Thus also with repeated *equal* (or, indeed, somewhat diminishing) temporary torsions of a wire the succeeding permanent torsions increase. The latter also increase after agitation of the temporarily twisted wire.

8. *Influence of the Load*.—I have already, in the year 1858*, shown that, just as the breaking-off of a permanently twisted iron wire by torsion through magnetizing is independent of the weights with which the wire is loaded, so also, within certain limits of loading (in oscillation-experiments), the toughness of iron wires has the same independence of their tension†, and mentioned that this is not the case with silver wires, especially at a higher temperature. A brief statement of similar results respecting the increase of the toughness with the load was published in 1865 by Sir W. Thomson‡. Braun also has recently§ made experiments in the same direction, concerning the action upon bodies of two deforming forces in directions independent of one another||.

In connexion with those observations, to be communicated later on, I first made some further experiments, on the static conditions of wires which had been subjected to various forces both of tension and torsion. For these experiments the weighted iron rod suspended beneath the rod *g* of the torsion-apparatus was replaced by a zinc beaker 21 centims. deep and 23 in diameter, hanging upon a horizontal bar. The beaker was parted in the middle by a perforated zinc plate, in order to avoid rotatory oscillations of the water with which it was filled. Near its axis a glass tube of 5 millims. diameter, nowhere in contact with the beaker, descended into it to about 2 centims. from the bottom; the glass tube was connected with a vessel full of water by a caoutchouc tube; and by raising and lowering this vessel the level of the water in the beaker could be varied. To the bottom of the tube was cemented a setting which supported, concentric with the end of the tube, a hemisphere of sheet brass about 2 centims. in diameter. This, as well as the lower part of the tube, was bound round with thick flannel; so that the water flowed through the tube into the beaker with very little velocity, and the weight acted upon the wire, as far as possible, without shock—which in experiments of this kind must be particularly attended to, and cannot be secured by screw arrangements, appending the weights by hand, &c. The mirror attached to the torsion-apparatus remained in fact perfectly stationary while, the wire being not twisted and the point *h* hanging freely, water was introduced through the tube into or let out

* Pogg. *Ann.* ciii. p. 575 (1858). † *Ibid.* cliii. p. 391 (1874).

‡ Phil. Mag. [4] xxx. p. 63 (1865).

§ Pogg. *Ann.* clix. p. 337 (1878).

|| Here belong indirectly the observations upon the stronger deadening of the vibrations of strings and rods with greater tension and height of tone. Compare A. Helmholtz, *Tonempfindungen*, p. 122; Warburg, Pogg. *Ann.* cxxxix. p. 89 (1870); Braun, *ibid.* cxlvii. p. 64 (1872), "Dissertation," and cli. p. 260 (1874),

of the beaker. The total weight of the metal ring hanging to the wire, with its holdfast &c., as well as that of the beaker filled with water up to the end of the glass tube, amounted to 3220 grams.

9. *Loading during Permanent Torsion*.—First, an annealed brass wire, of 2 millims. thickness and 480 millims. length, was twisted ten times, five minutes each time, up to the stroke of 2430 scale-divisions, while it was loaded beneath with 3220 grams. The permanent torsions, five minutes after each detorsion, were:—

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
721	733	737	740·8	743·2	745·2	747	748	749	750

6000 grams of water being then admitted into the beaker suspended beneath the wire while the latter *was in the position of permanent torsion*, the permanent torsion was reduced by the shaking to 748·7. If the wire was then again brought to the previous temporary torsion and untwisted, the permanent torsion rose to 764—and on a second torsion and detorsion, to 770. When the water was let out, the permanent torsion was lowered to 762·6 by the shaking, but rose again up to 765 at a single torsion and detorsion.

10. *Loading during Constant Temporary Torsion*.—When in like manner a new wire was temporarily twisted and untwisted to the stroke of 1952 scale-divisions ten times by the lesser loading, the permanent torsions amounted to:—

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
351	387·5	401	409	434	447·5	420·2	422·3	424	425·2

The wire was then again twisted up to striking; and *while it was in the position of the temporary torsion*, 6 kilogs. of water was admitted into the beaker with which it was loaded. After the detorsion the permanent torsion had risen to 452; after a repeated torsion and detorsion, to 461.

The same experiments were performed inversely; that is to say, the wire was first twisted and untwisted ten times while loaded, and was then unloaded in the positions both of the permanent and of the temporary torsion. The results were:—

I. Loaded wire temporarily twisted up to 2283 scale-divisions ten times.

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
P...	600	644	651	655	658	660·6	662·2	663·5	664·8	665·8

While the wire was in the permanent position the water was let out, when P fell to 662. The wire being again twisted up to 2283 scale-divisions, and untwisted, without loading, P rose to 664—and on a second torsion and detorsion, to 665·6.

II. Loaded wire ten times temporarily twisted to 1839 scale-divisions.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	
P ...	350	385	397	304	309	312·5	315	316·8	318	319·5

When the wire in the temporary position was unloaded and untwisted, the permanent torsion was 319·5, and rose to 320 on a second torsion and detorsion of the unloaded wire.

Therefore, if a load acts upon a wire while it is temporarily twisted only up to a determined limit, an increase of the permanent torsion appears, similarly as above, at the shaking of the wire. This takes place both when, after repeated temporary torsions, the weight is hung on the wire in the position of the permanent and when in the position of the temporary torsion. The unloading acts (though scarcely) in the same direction.

11. *Loading during Variable Temporary Torsion*.—Analogous experiments were made on a brass wire of the same dimensions; but the wire was temporarily twisted by a weight (110 grams) freely, *without striking*, and the load was altered *in the position of the permanent torsion*. In order to render the differences of behaviour with different loads more conspicuous, the wire was heated to 100° C. For this purpose it was suspended, with its holdfasts, in a vertical brass tube of 15 millims. diameter, which was surrounded by a somewhat shorter brass tube 6 centims. wide. The ends of the latter were connected with those of the former by conical additions. This entire heating-apparatus was divided lengthwise into two halves, which were soldered together along the surface of the longitudinal section, and were joined to one another by hinges, in order that the wire might be more easily suspended in it. Steam was introduced simultaneously into both halves, through side tubes, from above, and carried off from beneath. The following were the temporary and permanent torsions (P_0 noting the permanent torsion after alteration of the loading weight):—

W.	P_0 .	T.	T_1 .	P_1 .	$T_1 - P_1$.
3220		1475	1502	96	1406
"		1501	1515	120	1395
"		1511	1518	130	1388
"		1516	1521	137	1384
"		1519	1522	141	1381
"		1522	1524	144	1380
"		1524	1525	146	1379
"		1524·2	1525·2	148	1777
"		1524	1525·2	149	1376
9220	140	1570	1627	241	1386
3220	235	1620	1626·5	242	1384
9220	237	1636	1671	282	1389
3220	265	1659	1667	279	1388
9220	272	1673	1685	298	1387
3220	285	1677	1681	293	1388

Consequently, when the state of the wire, after repeated torsions and detorsions, has become constant, an increase of the loading, imparted to it during its detorsion, effects an augmentation of the following temporary and permanent torsions; on the subsequent diminution of the load the final temporary and permanent torsions remain almost unaltered (*conf.* § 14). If, however, we take the differences between the temporary and the permanent torsions after the wire had been once loaded, they remain almost perfectly equal. By the loading, therefore, only the permanent position of equilibrium is displaced (as above, § 5), reckoned from which the transient torsions produced by the torsion-weight are constant. In this sense the wire which has arrived at a constant state possesses equal elasticity with different loads.

12. *Torsion of a Wire alternately loaded with different Weights.*—A similar action of different loading of the wire is shown when it undergoes torsion by increasing weights w at the same time that it is loaded with alternately lighter and heavier weights W . The experiments were made on a brass wire of the same dimensions as the former ones, and at the temperature of 100° C. The results were as follows:—

w .	W .	P_0 .	T .	T_1 .	P_1 .	$T_1 - P_1$.	ΔT_1 .	ΔP_1 .	$\Delta(T_1 - P_1)$.
44	3220	...	470	471	3.2	467.8	113	5.3	107.7
54	9220	3	583	584	8.5	575.5	105	1.8	103.2
64	3220	6.5	689	689	10.3	678.7	117	6.7	110.3
74	9220	9.5	804	806	17	789	111.5	3.5	107
84	3220	16	916.5	917.5	21.5	896	128.5	18.3	110
94	9220	21.3	1041	1046	39.8	1006.2	115	9.7	111.3
104	3220	38.5	1161	1167	49.5	1117.5	150	38.2	115.8
114	9220	48.5	1311	1323	87.7	1235.3	148	35.3	120.0
124	3220	84	1459	1468	113	1355	220	107	140
134	9220	111	1679	1715	220.1	1495			

The differences Δ show distinctly that the values T_1 and P_1 constantly increase relatively more rapidly at the transition from the unloaded to the loaded wire than in the opposite case, while, on the contrary, the values $T_1 - P_1$ increase with almost perfect regularity.

13. Lastly, we notice the influence of different stretchings with torsions in alternately different directions. A brass wire of the former dimensions was, under alternately lighter and heavier loading, turned both ways up to a determined temporary torsion $\pm T$, and then the permanent torsion $\pm P$ was observed. Thus:—

	T = -1479.		T = +1479.			T = -1479.		T = +1479.	
	W.	P.	W.	P.		W.	P.	W.	P.
1....	3220	-152	3220	+160	8...	3220	-163	3220	+175
2....	"	-154	"	+166	9...	"	-152	"	+170
3....	"	-150	"	+168	10...	"	-150	"	+171
4....	"	-151	"	+166	11...	9220	-171	9220	+187
5....	9220	-188	9220	+207	12...	"	-163	"	+182
6....	"	-183	"	+200	13...	3220	-157	3220	+171
7....	"	-174	"	+191					

Consequently, if the wire in the alternately directed torsions with a lighter load is brought into such a state that it constantly takes up again the same positive and negative permanent torsions, these will be increased with a heavier loading, but will gradually be diminished with repeated temporary torsions, and approach toward those obtained with the lighter loadings.

14. *Effect of very frequently repeated Loadings.*—The hitherto mentioned effects of loading the wire with different weights always manifested themselves before the wire had arrived at the constant state in which it always behaves in the same manner under like conditions. There still remained for investigation the question whether, after the expiration of this variable state of the wire, alterations of the load still exercise any influence upon its torsion.

For this purpose a well-annealed brass wire of 2 millims. diameter and 480 millims. length was put into the apparatus, loaded beneath with the empty beaker, again annealed and then repeatedly temporarily twisted and untwisted by 100 grams, and, as before, the values T and T_1 as well as P and P_1 read off, as follows:—

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.
T	1952	2078	2113	2133	2138	2144	2146	2148
T ₁	2106	2123	2141·5	2146	2149	2153	2154·5	2160*
P	409	445	479	489	498	503	507	517
P ₁	386	430	472	484	492	498	503	509
T ₁ - P ₁	1720	1693	1669·5	1662	1657	1658	1651·5	1651

	IX.	X.	XI.	XII.	XIII.	XIV.	XV.
T	2154	2155	2158	2157	2158	2165	2161
T ₁	2160	2161	2162	2163	2164	2169	2168
P	516·5	517	521	523	523	526·5	
P ₁	511	513	516	518	520	522·5	
T ₁ - P ₁	1649	1648	1646	1645	1644	1646·5	

After the wire ceased to be in this way essentially altered, 7 kilograms of water were alternately let into and out of the

* A slight shaking.

beaker during the temporary torsion of the wire. The temporary torsions Tz and Ta were then :—

	I.	II.	III.	IV.	V.	VI.
Tz	2167	2179 (2188)	2191	2194	2194	2194·5
Ta	2181	2194	2200	2198	2200	2203

Hereupon the wire was allowed to untwist by lifting the twisting-weights; its permanent torsion was immediately 559, and after a longer time 552. If water was again let into and out of the beaker, the permanent torsions Pz and Pa were :—

	I.	II.	III.	IV.
Pz	542	542	541	541
Pa	549	549	549	549

In the temporary torsion, therefore, the first loadings and also the unloadings produce first of all an increase; in the permanent torsion the first loading effects a diminution. If this effect has run out, in the present experiments a decrease *apparently* takes place in both cases at the loading, and an increase at the unloading, to the slight amount of 6–9 scale-divisions. In order to ascertain if this trifling effect might not result from the construction of the apparatus notwithstanding the utmost care in its arrangement, the wire was again unloaded. The positions of the mirror before and after the loading corresponded to 584·7 and 579·2 scale-divisions respectively. The difference amounts to +5·5 in favour of the unloading. If, on the other hand, the steel ring was screwed beneath, in which the point that was fastened under the wire and sustained centrally by it was suspended, the position remained perfectly unaltered at 587. If the wire was then temporarily twisted up to 2212 scale-divisions, and then slowly carried over to the permanent torsion-position 494, and after a longer time to 488, and then alternately loaded and unloaded beneath, I obtained the following :—

	I.	II.	III.	IV.	V.	VI.
Pz	482·5	480·8	480·1	479	478·8	478·8
Pa	481·8	480·2	480	478·8	478·8	478·8

Here, therefore, after the first effects have run out, which at first produce at each change of tension a diminution of the permanent torsion, the latter remains perfectly invariable. The above-observed small final variations of Pz and Pa , as well as of Tz and Ta , are consequently to be attributed to slight rollings of the steel point in the ring, which must always occur if at the action of the twisting-weights it is pressed to one side of the steel ring, rotated on it, and then at the change

of load caused to approach the centre or another side of the ring.

Accordingly, if the wire has attained its constant state, variations of the loading within the limits of the experiments exercise no sensible influence upon either its temporary or its permanent torsion.

[To be continued.]

II. *On certain Vibrations of Solids.*

By FREDERICK GUTHRIE*.

[Plates II. & III.]

§ 1. **T**HE transverse vibrations of rods, prisms, or laths have been examined by Euler, Poisson, Cauchy, Seebeck, and Chladni. These calculators and experimenters have considered the notes and nodes in such bars when both ends are free. The following numerical experimental results may be of use to those studying the vibration of such solids. They to a considerable extent are in accord with the results obtained by previous experimenters.

I propose to consider here :—(I.) The actual position of the transverse nodes in a lath rigidly clamped along one narrow edge and vibrating in segments ; and (II.) the torsional and torsio-transverse vibrations of a lath similarly clamped.

I. The material used was hammered sheet-brass cut into strips or laths, clamped firmly in a heavy horizontal vice. The brass was 2·5 millims. thick. The free ends were always, unless otherwise stated, 280 millims. long. Three such laths were used, which were of various widths. The laths will in the sequel be called A, B, and C.

A was $280 \times 2\cdot5 \times 9\cdot4$ millims.

B „ $280 \times 2\cdot5 \times 22\cdot0$ „

C „ $280 \times 2\cdot5 \times 32\cdot0$ „

The total length of each lath was 305 millims. (say 1 foot). That the effect of the tail of the lath behind the vice's jaws is negligible will appear in § 4. The nodes were determined in the usual way by fine dry sand, the lath being vibrated by a bow. Let us call the line of the vice's jaws the artificial node, and the other nodes the natural ones ; and let us call the piece of lath from the free end to the first node the first segment, and so on, the last segment being the piece bounded at one end by the vice's jaws. We may represent in the cases of segmental vibration the fixed point by a double transverse

* Communicated by the Author.