THE USE OF ALTERNATING CURRENTS FOR SUBMARINE CABLE TRANSMISSION.*

BY

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ABSTRACT.

THIS paper contains the results of experiments performed by members of the Signal Corps Research Laboratory, Bureau of Standards, in an attempt to develop a method or methods of signal transmission over submarine cables which would permit of a more effective use of existing cable systems.

In the method proposed and developed, signals are transmitted by means of alternating currents simultaneously with the normal battery or direct-current operation of the cable. Currents of several different frequencies may be used, reserving a given frequency for each message, and by tuning or equivalent methods at the receiving end these messages may be separated, so that the simultaneous transmission of a number of messages is rendered possible.

Accurate determination of the fundamental physical properties of a particular cable for different frequencies has been accomplished, and the results are given herewith. From these physical properties accurate calculations for any length of cable can be made, so that the design of proper apparatus becomes purely a laboratory problem.

Actual trial of the proposed alternating-current method was made covering distances up to approximately 700 kilometres, with results which were in all cases completely satisfactory, indicating that the method is an entirely feasible one, and one which will render cables of this length, and undoubtedly of several times this length, much more useful.

I. THE GENERAL PROBLEM.

In March, 1919, in response to a request by the Chief Signal Officer for a consideration of the possibility of improving cable communication methods, the Signal Corps Research Laboratory

* Communicated by Maj. Gen. George Owen Squier, Chief Signal Officer, U. S. A.

commenced work with a view to ascertaining whether any practicable scheme could be devised which lay within its power to develop. After some deliberation it was decided that a possibility existed of using alternating currents as a means for transmitting signals over a submarine cable. The plan was not to use very low frequencies (5 to 10 cycles per second) and the ordinary type of receiver galvanometer, or equivalent apparatus, such as constitute the Squier Alternating Current Method ("On an Unbroken Alternating Current for Cable Telegraphy," George O. Squier, *Proceedings of the Physical Society of London*, August 15, 1915), but to use a frequency sufficiently high to produce an audible tone which could be received by means of an ordinary telephone receiver and further, so that several such alternating currents of different frequencies could be sent over the cable at the same time and separated at the receiving end by any one of the several well-known methods of tuning. Success in such a proposed scheme would make possible the multiplexing of submarine cables and, of course, provide for transmitting many more messages in a given time than can be done with present methods. Whatever success might be attained would be a net gain to the art, because the regular operation of the cable with direct current would proceed uninterruptedly.

As has long been known, the attenuation of alternating currents in or along submarine cables is very great on account of the resistance and capacity. This fact was considered at the time of formulating ideas in regard to possible methods, and calculations were made to determine the approximate magnitude of the attenuation for typical lengths and constructions of cables. The results of all calculations indicated that it would only be through the medium of powerful amplification at the receiving end that any success could be hoped for in the use of alternating currents of a frequency sufficiently high to permit of tuning, and over cables of such length as to make the method of military and commercial value. Had accurate information been available as to the values of the fundamental electrical properties of a cable when acted upon by an alternating electromotive force, rigorous calculation of the total attenuation could have been made, and upon this as a basis the amplifier design worked out purely as a laboratory problem.

As a matter of fact, little information was available concern-

ing the fundamental properties of a cable subjected to alternating currents of different frequencies. In common with all electrical systems the resistance of a cable should increase with increase in frequency, but the amount of this increase could be determined only by experiment. The magnetic field produced by the current within the cable sets up currents in the surrounding water and steel armor so that, in a general way, the increase in resistance with increase in frequency might well be expected to be similar in type to that observed in the case of a simple coil affected by the immediate proximity of another short-circuited coil. The direct-current, or zero-frequency, value of resistance is accurately known for any cable.

In regard to the self-inductance (coefficient of self-induction) of a cable no information could be found. The immediate presence of the steel armor and the uncertainty as to the geometrical distribution of the return current are factors which would make its calculation quite unreliable, if not worthless. Whatever its value might be, however, its variation with frequency should be of the same type as the variation of the apparent self-inductance of the simple coil system referred to in the preceding paragraph.

Because of the fact that the dielectric portion of the cable is well localized geometrically and of quite definite physical properties (being composed of either rubber or gutta percha) it would normally be expected that the apparent capacity and the dielectric loss coefficient would be dependent upon the frequency in the same way as in the case of a simple condenser, *i.e.*, a slight decrease in capacity and a practically constant power-factor angle.

The above general analysis is amply substantiated by the experimental results obtained from the measurements made on an actual cable. As the Signal Corps has available and operates the cable system between Seattle, Washington, and Sitka, Alaska, as well as a number of short cables in southeastern Alaska, it appeared that the most desirable procedure would be to utilize these various lengths of cable, as already laid, for the investigation of the problem in hand. The most important part of the experimental work lay in the determination of the four fundamental electrical properties previously mentioned, for such results are applicable generally. With such apparatus as could be developed in the short time available, however, excellent results

were obtained in the practical application of this method of signalling.

The description of apparatus used and the results obtained follows:

II. POWER SOURCES.

Whatever form the electrical signaling methods may take, safety to the cable insulation demands that the voltage impressed between cable conductor and sheath be limited to 70 volts. With an alternating emf. of sinusoidal wave form this means a limitation to 50 volts effective value. Throughout the range of

PLATE I.

frequencies from 0 to 1400 cycles per second preliminary approximate calculations indicated that a power of not over 40 watts would be required to impress this voltage on the cable. Preliminary calculations also showed that it would not be reasonable to expect much success from the use of frequencies higher than 1400 cycles per second.

For frequencies of from 7 to 40 cycles per second an inverted rotary converter was used as the power source. This machine was made by mounting a pair of slip rings on the shaft of a small direct-current motor and connecting them to electrically opposite points in the armature winding. The converter was driven and the voltage controlled by using the proper number of dry cells. Plate I shows the converter with its slip rings, and Plate II shows

the wave forms of voltage and current obtained when it is connected to a capacity load. The higher harmonics in the current wave *I* are considerably magnified when the machine feeds thus into a condenser, but since along a cable the higher frequencies are damped or attenuated much faster than the lower, these higher frequencies in the current are of little consequence. As matters turned out, this low-frequency generator was used very little.

For higher frequencies, an oscillating vacuum-tube generator capable of delivering 50 watts at any frequency above 20 cycles per second was developed. In this set standard Signal Corps

PLATE II,

50-watt vacuum tubes were used which, in order to gain flexibility of control, were arranged with one tube in an oscillating circuit and one tube in an amplifier circuit which was actuated by the first tube. From the last, or amplifier, tube the power was withdrawn by means of an iron-cored transformer.

In order to secure constancy in the frequency calibration an air-core inductance coil was used in the oscillating circuit. This coil, wound with No. 18 B. & S. double cotton-covered wire, contained approximately 3500 turns. Its outside diameter was 28 cm. and its axial length 7.6 cm. Taps were brought out at intervals to provide for flexibility in the adjustment of the circuit conditions. When oscillating vigorously, the high voltage developed across the coil made it necessary to use a mica condenser capable of withstanding a working stress of 1000 volts, effective

alternating. A ten-microfarad condenser, adjustable in steps of 0.01 microfarad, gave excellent control of the frequency. Fig. 1 shows the essential features of the circuit arrangement.

The power output of the set was adjusted by means of the rheostats in the filament battery circuits. Separate storage bat-

FIG. 1.

PLATE III.

teries were used for lighting the filaments, while a single battery of approximately 600 dry cells was used in the plate circuits of both tubes. For all ordinary purposes it was found possible to calculate the frequency from the known values of capacity and inductance in the oscillating circuit, using the usual formula. This is true for the kind of circuit used when the condenser capacity is large in comparison with the capacity of the coil. The

following tables give the principal electrical properties of the oscillating circuit, and show how a complete range of frequencies can be covered by the use of only two of the taps in the coil:

TABLE I.
Inductance of Oscillator Coil.
Taps Counted from Inside of Coil Outwards.

Taps	Turns	Inductance henrys	
0-1	324	0.0142	Outside diameter, 28 cm. Axial length, 7.6 cm.
0-2	648	0.0587	
0-3	972	0.138	
0-4	1296	0.254	
0-5	1620	0.408	Winding of No. 18 B. & S. gage, double cotton insulated wire.
0-6	1944	0.605	
0-7	2268	0.846	
0-8	2592	1.125	
0-9	2916	1.448	
0-10	3240	1.818	
0-11	3456	2.094	

TABLE II.
Frequency Calibration of Oscillator.

Frequency in Cycles Per Second when Condenser is Connected Across Taps 0-11 and Across Taps 0-3.

Capacity mf.	Taps 0-11 frequency	Taps 0-3 frequency
30.2	20.00	
11.1	33.01	
10.0	34.78	135.5
9.0	36.66	142.8
8.0	38.88	151.5
7.0	41.57	161.9
6.0	44.90	174.9
5.0	49.19	191.6
4.0	54.99	214.2
3.0	63.50	247.4
2.0	77.77	303.0
1.5	89.80	349.8
1.0	109.9	428.4
0.9	115.9	451.6
0.8	122.0	479.0
0.7	131.5	512.1
0.6	142.0	553.1
0.5	155.5	605.9
0.4	174.0	677.4
0.3	200.8	782.2
0.2	245.9	958.0
0.1	347.8	1355.0

Plate III shows the wave forms of current and voltage produced by the oscillator when operating at a frequency of 200 cycles per second and when feeding into a capacity load.

Plate IV shows the arrangement of the panel containing the various elements of the oscillator. The coil and condenser are included in the photograph. The oscillator here described was used for the major portion of all experimental work. In certain

PLATE IV.

cases, where only a small amount of power was required, as in the calibration of apparatus, a similar but lower-powered oscillator was used.

MEASUREMENT OF THE ELECTRICAL PROPERTIES OF THE SEATTLE-SITKA CABLE.

Upon arrival at Seattle, Washington, in June, 1919, steps were taken to proceed immediately with determination of the fundamental electrical properties of the cable and of their dependence upon the frequency. Several methods are available for making such determinations, but the one selected as being most suitable for the case in hand was that in which the apparent impedances of a moderate length of cable are measured by means of an impedance bridge, both when the remote end is short circuited and when it is open circuited.

A short length (1.4 kilometres) of cable of the same type as the main Seattle-Sitka cable was available continuously for this experimental work. It extended from the foot of University Street, Seattle, to the cable hut at Fort Lawton, and at both ends the cable passed directly into salt water, making conditions practically uniform and ideal throughout its length. Excellent ground connections at both ends were provided by soldering leads to the several strands of the cable sheath. The conclusion that the physical arrangements were such as to eliminate errors arising from end effects was amply borne out by the results of certain check measurements to be described later.

The technical specifications for the cable upon which the measurements were made are given in Table III. The cable in question is made up of a stranded copper core, rubber insulation bound with tape, a layer of jute, and then the stranded steel armor with its serving of impregnated jute.

TABLE III.
Alaskan Cable Specifications.

Data taken from Signal Corps Manual No. 3, Chap. II, p. 73.

Type, 1903, Deep sea

Outside diameter of cable	2.06 cm.
Weight of whole cable in air	881 kg. per km.
Weight of whole cable in water	569 kg. per km.
Weight of armor wire	577 kg. per km.
Weight of jute and compound	217 kg. per km.
Weight of tape	16.1 kg. per km.
Weight of rubber insulation	43.1 kg. per km.
Weight of copper	32.4 kg. per km.
Breaking stress	6610 kg.
Elastic limit	4990 kg.
Copper resistance at 15.5° C.	4.59 ohms per km.
Dielectric resistance at 15.5° C.	2280 megohms for 1 km.
Electrostatic capacity	0.266 mf. per km.

Before doing any experimental work all apparatus was carefully calibrated. Only two separate standards, namely, a capacity standard and a resistance standard, were required to establish the accurate calibration of all condensers, resistances, and inductances used. For the former, the Bureau of Standards certification of section 0.1 A (0.1000 mf.) of Leeds and Northrup mica condenser No. 47839 was accepted. In order to avoid cumulative errors in the fourth significant figure another cipher was added to the above

value in making up the calibration data. For a resistance standard the Leeds and Northrup precision Wheatstone Bridge No. 3445, installed in the Seattle Cable Office, was used. Before accepting this bridge as standard the sections therein were carefully compared and also checked against other high-grade resistance boxes. In the few cases where the indications of electrical meters are quoted, the meters had been calibrated against auxiliary standards calibrated by the Department of Physics of the University of Washington.

FIG. 2.

Frequencies were determined in terms of capacity and inductance by means of the resonance frequency bridge shown in Fig. 2. In this bridge, the conditions for balance are

$$\left. \begin{aligned} R_1 R &= R_2 r \\ \omega^2 &= \frac{r}{L_f C_f} \end{aligned} \right\} \quad (1)$$

in which $\omega = 2\pi f$ and f = frequency in cycles per second. The L_f or C_f adjustment for balance can be made with nicety, so that the frequency determinations are of the same degree of accuracy

as the values taken for the two standards of capacity and resistance. In some of the field work calibrated tuning forks were used to give fixed frequencies to which the oscillator frequencies were adjusted. Elimination of beats between the sound of the fork and the sound produced by the alternating current in a telephone receiver served, in this method, to determine the frequencies to within one cycle per second, or closer.

FIG. 3.

The type of alternating-current impedance bridge used in making the measurements is shown in Fig. 3, and is more fully described in Sections 103 and 104 of Pernot, "Electrical Phenomena in Parallel Conductors," vol. i. For an inductive impedance with the "IND." arrangement of the bridge, the apparent resistance r , and the apparent self-inductance L , are obtained from the formulæ given in the figure. With the "CAP." bridge

arrangement, the apparent conductance g , and the apparent capacity C , are obtained from the appropriate formulæ. Therefore, impedances Z , or admittances Y are given by

$$\left. \begin{aligned} \text{"IND." bridge, } r &= \frac{R_1 R_2}{R}, \quad L = R_1 R_2 C_0, \quad Z = r + j \omega L, \quad Y = \frac{I}{Z} \\ \text{"CAP." bridge, } g &= \frac{R_1}{R R_2}, \quad C = \frac{R_1}{R_2} C_0, \quad Y = g + j \omega C, \quad Z = \frac{I}{Y} \end{aligned} \right\} \quad (2)$$

For frequencies above 100 cycles per second a telephone receiver was used as a current detector, while for lower frequencies a vibration galvanometer was employed. Insulating transformers were used in both the power input and galvanometer circuits. All of the elements entering into the bridge were carefully calibrated against the previously mentioned standards, and the results in all cases are given in terms of the corrected values.

The essential features of the mathematical analysis leading to a determination of the fundamental line properties from the two impedance measurements, previously referred to, are given here for convenience in reference. The fundamental properties, whose values at the different frequencies are desired, are:

$$\left. \begin{aligned} r &= \text{resistance per unit length} \\ L &= \text{self inductance per unit length} \\ g &= \text{dielectric-loss coefficient (leakance) per unit length} \\ C &= \text{electrostatic capacity per unit length} \end{aligned} \right\} \quad (3)$$

or, as more compactly expressed,

$$\left. \begin{aligned} Z &= \text{impedance per unit length} = r + j \omega L \\ Y &= \text{admittance per unit length} = g + j \omega C \end{aligned} \right\} \quad (4)$$

With the remote end of the line short-circuited, and then with the remote end open-circuited, the respective impedances are (see Pernot, "Electrical Phenomena in Parallel Conductors," vol. i, sections 95 and 100):

$$\left. \begin{aligned} Z' &= U \tanh VI = \text{apparent short-circuited impedance} \\ Z'' &= U \coth VI = \text{apparent open-circuited impedance} \end{aligned} \right\} \quad (5)$$

in which

$$\left. \begin{aligned} U &= \sqrt{\frac{Z}{Y}} = \text{infinite-line impedance} \\ V &= \sqrt{Z Y} = \text{propagation constant} \\ &= \text{length of line or cable.} \end{aligned} \right\} \quad (6)$$

The problem is to evaluate Z and Y in terms of Z'' and Z' , since the two last impedances are those which are determined by measurement. The auxiliary quantities U and V are also desired.

By multiplication of the two equations in (5)

$$U = \sqrt{Z' Z''} \quad (7)$$

Then, from equations (5),

$$V = \frac{1}{l} \tanh^{-1} \frac{Z'}{U} = \frac{1}{l} \coth^{-1} \frac{Z''}{U} \quad (8)$$

The two quantities U and V , which directly determine the behavior of the system in the transmission of alternating currents of the particular frequency in question, are thus immediately determined. The fundamental electrical properties are derivable from them by the aid of equations (6);

$$\left. \begin{aligned} Z &= r + j \omega L = UV \\ Y &= g + j \omega C = \frac{V}{U} \end{aligned} \right\} \quad (9)$$

This completes the solution, for the imaginary portions in equations (9) when divided by ω give respectively the self-inductance L and the capacity C .

In the process of computing, the inverse hyperbolic tangent of the complex quantity $\frac{Z'}{U}$, as required by (8), is most easily obtained by logarithmation after expressing the tangent in exponential form. The result is

$$V = \frac{1}{2l} \ln \frac{U - Z'}{U + Z'} \quad (10)$$

In order to obtain a check on the experimental work, similar measurements were made from both ends of the 14-kilometre length available. As a further check, measurements at different frequencies were made of the apparent impedance at one end when the remote end was closed through a known impedance. Having found the fundamental line properties at these same frequencies, the apparent impedance can be calculated in terms of them and the known impedance at the remote end. The formula for this apparent impedance is

$$Z_a = \frac{Z_o \cosh V l + U \sinh V l}{\cosh V l + \frac{Z_o}{U} \sinh V l} \quad (11)$$

where Z_o is the known impedance at the remote end and Z_a is the apparent impedance.

A comparison of the calculated and observed values of Z_a furnishes a basis for estimating the accuracy of the fundamental determinations.

Tables IV, V, and VI give the corrected values of the quantities observed in the bridge measurements. They lead to values of r and L or of g and C through the formulæ pertaining to the type of bridge used, and the results must, in some cases, be corrected by the conductance of a known resistance shunting the line terminals connected to the bridge. In Table IV the frequencies f were determined directly by adjusting the oscillator frequencies to those of calibrated tuning forks. The other values tabulated are those of the quantities shown in the bridge diagram, Fig. 3. Observations 1, 2, and 3 are not given because they were preliminary, trial settings, not made with much care. In Tables V and VI, L_f and C_f are the corrected values of inductance and

TABLE IV.
Corrected Data from Impedance Measurements Made at Seattle End.

Length of cable, 14,294 km.

No.	Bridge type	Shunt resistance	f cyc./sec.	R_1 ohms.	R_2 ohms.	R ohms.	C_o farads
Cable short circuited at remote end.							
4	IND	Inf.	400	99.96	99.92	106.54	0.53558×10^{-6}
5	IND	Inf.	384	99.96	99.92	107.85	0.56993×10^{-6}
6	IND	Inf.	320	99.96	99.92	113.81	0.68624×10^{-6}
7	IND	Inf.	200	99.96	99.92	123.87	0.93999×10^{-6}
8	IND	Inf.	512	99.96	99.92	98.28	0.32313×10^{-6}
9	CAP	Inf.	824	1000.4	999.6	117.33	0.21960×10^{-6}
10	CAP	Inf.	652?	1000.4	99.92	1111.6	0.00000×10^{-6}
Cable open circuited at remote end.							
11	CAP	200.14	400	1000.4	99.92	1187.0	0.37592×10^{-6}
12	CAP	200.14	200	1000.4	99.92	1700.0	0.38829×10^{-6}
13	CAP	200.14	320	1000.4	99.92	1391.8	0.38143×10^{-6}
14	CAP	200.14	512	1000.4	99.92	923.99	0.36076×10^{-6}
15	CAP	200.14	824	1000.4	99.92	493.44	0.24626×10^{-6}
16	CAP	200.14	640?	1000.4	99.92	664.00	0.32112×10^{-6}
17	CAP	200.14	634?	1000.4	99.92	703.87	0.33055×10^{-6}
18	CAP	200.14	824	1000.4	99.92	492.74	0.24580×10^{-6}
19	CAP	200.14	435	1000.4	99.92	1100.1	0.37224×10^{-6}
20	CAP	200.14	256	1000.4	99.92	1558.9	0.38505×10^{-6}

In observations 10, 16, and 17 the frequencies were determined from the capacity and inductance settings of the oscillator, so that they may be in error by a number of cycles. These observations were not used in the determination of the line properties. Observation 10 was taken particularly in order to observe the change in impedance of the short-circuited cable from an inductive to a condensive nature. At this observation the reactance was zero.

capacity involved in determining the frequency by means of the resonance bridge shown in Fig. 2. In a few cases the frequencies, as determined by direct comparison with tuning forks, are given in place of L_f and C_f .

TABLE V.
Corrected Data from Impedance Measurements Made at Fort Lawton End.

Length of cable, 14.294 km.

No.	Bridge type	Shunt resist.	L_f henrys	C_f farads	R_1 ohms	R_2 ohms	R ohms	C_0 farads
Cable short circuited at remote end.								
21	IND	Inf.	0.31320	0.50387×10^{-6}	99.96	99.92	104.24	0.49839×10^{-6}
22	IND	Inf.	0.14140	0.50387×10^{-6}	99.96	99.92	91.87	0.09316×10^{-6}
26	IND	Inf.	$f = 199.3$		99.96	99.92	122.98	0.92814×10^{-6}
28	IND	Inf.	0.39895	0.95447×10^{-6}	99.96	99.92	118.21	0.78543×10^{-6}
23	CAP	Inf.	0.42433	0.10000×10^{-6}	99.96	99.92	115.11	0.21731×10^{-6}
24	CAP	Inf.	0.24900	0.10000×10^{-6}	499.90	99.92	588.54	0.10050×10^{-6}
25	CAP	Inf.	0.30151	0.050051×10^{-6}	99.96	99.92	107.04	0.72090×10^{-6}
Cable open circuited at remote end.								
29	CAP	Inf.	0.39895	0.95447×10^{-6}	1000.4	99.92	7148.2	0.38458×10^{-6}
30a	CAP	500.5	$f = 199.3$		1000.4	99.92	3459.2	0.38733×10^{-6}
30b	CAP	500.5	$f = 199.3$		1000.4	99.92	3474.3	0.38634×10^{-6}
31	CAP	Inf.	0.31320	0.50387×10^{-6}	1000.4	99.92	2975.0	0.37648×10^{-6}
32	CAP	Inf.	0.14140	0.50387×10^{-6}	1000.4	99.92	1270.7	0.34647×10^{-6}
33	CAP	Inf.	0.42433	0.10000×10^{-6}	1000.4	99.92	746.88	0.28499×10^{-6}
34	CAP	Inf.	0.24900	0.10000×10^{-6}	1000.4	99.92	508.00	0.16155×10^{-6}
35a	CAP	Inf.	0.30151	0.050051×10^{-6}	1000.4	99.92	478.14	0.04416×10^{-6}
35b	CAP	Inf.	0.30151	0.050051×10^{-6}	99.96	99.92	47.86	0.44327×10^{-6}

TABLE VI.
Corrected Data from Impedance Measurements Made at Fort Lawton End. Check Test with Known Value of Z_0 .

Length of cable, 14.294 km.

Remote End Closed Through 100.01 Ohms Resistance ($=Z_0$).

No.	Bridge type	Shunt resist.	L_f henrys	C_f farads	R_1 ohms	R_2 ohms	R ohms	C_0 farads
36	CAP	Inf.	0.39895	0.95447×10^{-6}	499.90	99.92	777.90	0.36732×10^{-6}
37	CAP	Inf.	$f = 199.3$		499.90	99.92	806.91	0.36931×10^{-6}
38	CAP	Inf.	0.31320	0.50387×10^{-6}	499.90	99.92	691.37	0.35527×10^{-6}
39	CAP	Inf.	0.14140	0.50387×10^{-6}	499.90	99.92	567.48	0.32793×10^{-6}
40	CAP	Inf.	0.42433	0.10000×10^{-6}	499.90	99.92	470.87	0.29272×10^{-6}
41	CAP	Inf.	0.24900	0.10000×10^{-6}	499.90	99.92	379.88	0.23276×10^{-6}
42	CAP	Inf.	0.30151	0.050051×10^{-6}	499.90	99.92	320.52	0.15494×10^{-6}

After reduction by equations (1) and (2) the foregoing data yield the apparent impedance (or admittance) components given in Tables VII and VIII. Only four significant figures are tabulated, although all computations, up to and including the final results, were carried out to five.

TABLE VII.
Components of Apparent Impedances from Measurements at Seattle End.

Cable short circuited				Cable open circuited		
f	No.	r' ohms	L' henrys	No.	g'' mhos	C'' farads
200	7	80.63	0.009389	12	0.0008929	3.888×10^{-6}
256	..	..	..	20	0.0014259	3.855×10^{-6}
320	6	87.76	0.006854	13	0.002197	3.819×10^{-6}
384	5	92.61	0.005692	..	..	..
400	4	93.75	0.005349	11	0.003438	3.764×10^{-6}
435	..	..	..	19	0.004104	3.727×10^{-6}
512	8	101.63	0.003228	14	0.005840	3.612×10^{-6}
634	..	..	..	17	0.009228	3.310×10^{-6}
640	..	..	..	16	0.010082	3.215×10^{-6}
		g' mhos	C' farads			
652	10	0.009007	0.00000×10^{-4}	..	..	..
824	9	0.008530	0.2198×10^{-6}	15 and 18	0.015308	2.463×10^{-6}

Observations 15 and 18 agree so well that the average of their values is tabulated above. The components r' , L' or g' , C' and g'' , C'' , go to make up the impedances Z' and Z'' , respectively.

TABLE VIII.
Components of Apparent Impedances from Measurements at Fort Lawton End.

Cable short circuited				Cable open circuited		
f	No.	r' ohms	L' henrys	No.	g'' mhos	C'' farads
199.3	26	81.22	0.009270	30	0.0008899	3.873×10^{-6}
257.9	28	84.49	0.007845	29	0.0014006	3.850×10^{-6}
400.6	21	95.82	0.004978	31	0.003365	3.769×10^{-6}
596.3	22	108.72	0.0009305	32	0.007879	3.469×10^{-6}
		g' mhos	C' farads			
772.6	23	0.008691	0.2174×10^{-6}	33	0.013405	2.853×10^{-6}
1008.6	24	0.008501	0.5028×10^{-6}	34	0.01971	1.617×10^{-6}
1295.6	25	0.009346	0.7212×10^{-6}	35	0.02092	0.4428×10^{-6}

Observation 30 in the above table is the average of the values obtained from observations 30a and 30b in Table V. Similarly with observation 35.

From the components of Z' and Z'' given in the two foregoing tables, the fundamental line properties per unit of length are to be calculated in accordance with equations (7), (8), (9), and (10). Although these equations appear simple, the fact that the quantities entering therein are complex makes the carrying out of the numerical work rather laborious. A sample computation of the kind required is to be found in Section 106, "Electrical Phenomena in Parallel Conductors."

Table IX gives the final results of the computations based upon the data in Tables VII and VIII. In addition to the four fundamental electrical properties, their combinations into impedances and admittances per unit of length are given also.

TABLE IX.
Fundamental Cable Properties per Kilometre.

f	r ohms/km.	L henrys/km.	$10^9 g$ mhos/km.	$10^9 C$ farads/km.	Z ohms	$10^4 Y$ mhos
Measurements from Seattle End.						
0	4.660		0.0030		4.660/0°	0.0030/0°
200	5.388	0.001215	17.02	0.2712	5.600/15.82°	341.3/87.14°
320	5.557	0.001100	29.80	0.2670	5.981/21.71°	537.7/86.82°
400	5.686	0.001057	36.02	0.2650	6.276/25.04°	666.9/86.90°
512	5.813	0.001006	50.43	0.2628	6.653/29.10°	847.0/86.59°
824	6.147	0.000928	107.31	0.2575	7.801/38.00°	1337.5/85.40°
Measurements from Fort Lawton End.						
0	4.660		0.0030		4.660/0°	0.0030/0°
199.3	5.432	0.001214	17.09	0.2704	5.641/15.64°	339.1/87.11°
257.9	5.513	0.001141	20.23	0.2683	5.815/18.54°	435.2/87.34°
400.6	5.810	0.0010585	27.24	0.2643	6.392/24.64°	665.9/87.66°
596.3	6.023	0.0009588	38.46	0.2613	7.013/30.81°	979.8/87.75°
772.6	6.191	0.0008850	52.74	0.2617	7.535/34.76°	1272/87.62°
1008.6	6.404	0.0008314	73.88	0.2636	8.293/39.44°	1672/87.47°
1295.6	6.551	0.0007947	129.6	0.2645	9.206/44.64°	2157/86.56°

The complex values of Z and Y are given in the above table in the form of a scalar value and angle.

The values of r , L , g , C , and θ_y as given in the above table are shown plotted as functions of the frequency in Figs. 4, 5, 6, 7, and 8. The dotted lines connect the observations made at the Seattle end and the full lines connect the observations made at the Fort Lawton end. In estimating the consistency of the results from the appearance of the curves, due regard must be paid to the greatly magnified scales used in some cases.

The quantities which enter more directly into the equations determining the manner of transmission of alternating currents than do the fundamental electrical properties themselves, are:

$$\left. \begin{aligned} u &= \sqrt{\frac{Z}{Y}} = \text{infinite-line impedance, ohms;} \\ \theta_u &= \frac{1}{2}(\theta_z - \theta_y) = \text{angle of infinite-line impedance;} \end{aligned} \right\} U = u / \theta_u$$

α = real portion of V = attenuation constant; numeric per km.

β = imaginary portion of V = wave-length constant; radians per km.

$S = \frac{\omega}{\beta}$ = velocity of propagation; km. per sec.

$\lambda = \frac{2\pi}{\beta}$ = wave length; km.

The values of the above quantities as determined from the measurements and in the units specified above are given in Table X.

TABLE X.
Transmission Constants for Alternating Currents.

f	α km. $^{-1}$	β km. $^{-1}$	z ohms	Θz	S km./sec.	λ km.
Measurements from Seattle End.						
0	0.00011	0.00000	39000	00000		
200	0.02723	0.03421	128.10	-35.66°	36740	183.7
320	0.03312	0.04604	105.47	-32.55°	43680	136.5
400	0.03620	0.05362	97.01	-30.93°	46870	117.2
512	0.03995	0.06356	88.63	-28.74°	50620	98.86
824	0.04843	0.08094	76.37	-23.70°	57560	69.86
Measurements from Fort Lawton End.						
0	0.00011	0.00000	39000	00000		
199.3	0.02730	0.03417	129.0	-35.73°	36650	183.9
257.9	0.03032	0.04015	115.6	-34.40°	40360	156.5
400.6	0.03635	0.05418	97.97	-31.51°	46460	115.96
596.3	0.04234	0.07126	84.60	-28.47°	52580	88.17
772.6	0.04717	0.08577	76.98	-26.43°	56600	73.25
1008.6	0.05263	0.10535	70.42	-24.01°	60150	59.64
1295.6	0.05822	0.12834	65.33	-20.96°	63430	48.96

TABLE XI.
Infinite-line Impedance of Sitka-Seattle Cable.
Measured at Sitka.

f cyc.	Conductance, mhos	Susceptance, mhos	U ohms and angle
22.18	0.002234	0.001873	343.0/39.97°
33.59	0.002678	0.002235	286.6/39.85°
39.80	0.002893	0.002410	265.6/39.79°
46.58	0.003108	0.002585	247.4/39.75°
56.41	0.003394	0.002816	226.8/39.69°
81.44	0.004031	0.003306	191.8/39.36°
95.47	0.004359	0.003532	178.2/39.02°
131.09	0.005152	0.003994	153.4/37.78°
161.78	0.005742	0.004262	139.8/36.50°
203.1	0.006450	0.004597	126.2/35.48°
245.8	0.007140	0.004892	115.5/34.42°
296.0	0.007694	0.005025	108.8/33.15°
384.3	0.008696	0.005315	98.12/31.43°
476.5	0.009593	0.005492	90.47/29.78°
613.8	0.01079	0.005491	82.60/26.97°
739.2	0.01164	0.005382	77.98/24.82°
859.0	0.01228	0.005195	74.99/22.93°
1039.0	0.01296	0.004935	72.09/20.84°
1184.0	0.01334	0.004794	70.53/19.76°

In the above table the conductance and susceptance as tabulated are the two components of the reciprocal of U . The angle of U is to be counted as negative.

The attenuation constant α is shown as a function of the frequency in Fig. 9. The data obtained from the two sets of measurements are in such close agreement that a single curve suffices to represent all of the points.

FIG. 4.

Fig. 10 shows the scalar value of the infinite-line impedance u . In addition to the values given in Table X it contains the values determined by measurements made at Sitka, Alaska, on the long cable (approximately 1700 kilometres) between there and Seattle.

As far as the tabulated frequencies are concerned, this constitutes an infinite length, or at any rate, a length sufficient to prevent conditions at the remote end from affecting the measured impedances

FIG. 5.

at the input end. The data obtained in these measurements at Sitka are given in Table XI. Because of the great length, a single determination with the impedance bridge gave the desired result immediately. Both in the case of u and of θ_u the agreement

between the values determined by the two impedance measurements on the short Seattle-Fort Lawton cable and by the single impedance measurements on the long Sitka-Seattle cable is excel-

FIG. 6.

lent, indicating that the measurements are reliable and that the cables are similar electrically.

Fig. 11 shows the angle of the infinite-line impedance as a function of the frequency.

A further check on the accuracy of the results is obtained from the data in Table VI. From this data values of Z_0 as determined by direct measurement when $Z_0 = 100.01 + j 0.00000$ are

FIG. 7.

obtained. Using equation (11) and the transmission constants given in the second portion of Table X for the same frequencies as those at which Z_0 was measured, the values of Z_0 can be computed. A comparison of the computed and the directly measured

values can then be made. Table XII contains the results obtained by the two methods. In place of Z_n itself, the two components of its reciprocal are tabulated.

$$\frac{1}{Z_n} = g_n + j \omega C_n.$$

The greatest deviation between observed and computed values is slightly over two per cent., which is an amount at least four times what might reasonably be expected to be the possible error

TABLE XII.

*Comparison of Measured and Computed Admittances when $Z_0 = 100.01 + j 0.000$.
From data obtained at Fort Lawton end.*

f	g_n measured	g_n computed	$10^8 C_n$ measured	$10^8 C_n$ computed
199.3	0.006200	0.006155	1.848	1.822
257.9	0.006431	0.006382	1.838	1.816
400.6	0.007236	0.007106	1.777	1.744
596.3	0.008816	0.008668	1.641	1.623
772.6	0.01062	0.01051	1.464	1.477
1008.6	0.01317	0.01319	1.164	1.192
1295.6	0.01561	0.01577	0.7752	0.7788

in the observations. This discrepancy, as well as the small, consistent deviations between the curves representing the line properties as determined from the two ends of the cable, undoubtedly arise from the fact that the cable is not homogeneous throughout its length. It is made up of two pieces, each having the same specifications as the other, but the two parts undoubtedly having slightly different properties because they were manufactured at different times. Such an explanation is sufficient to account for the differences observed, in view of the fact that the formulæ for the reduction of the observations presuppose complete uniformity. As they stand, the experimental results are undoubtedly of greater accuracy than can be hoped for in any analysis relating the observed electrical properties of the system to the fundamental physical constants and the dimensions.

ANALYSIS OF RESULTS AND TRANSMISSION CALCULATIONS.

An examination of the results given in the foregoing reveals important general facts. Without going into a detailed, rigorous

analysis of the whole problem, some things can be found out which are of general approximate application.

The resistance, as expected, increases with the frequency.

FIG. 8.

At the frequencies employed, only a very small increase in resistance should result from the so-called skin effect in the main conductor itself, so that the observed increase, which is considerable, must arise from causes associated with the steel armor and the

surrounding water. The total resistance as observed is expressible as

$$r = r_o + \phi(\omega) \quad (11)$$

in which

r_o = direct-current resistance

and

$\phi(\omega)$ = an increasing function of ω whose value is zero at zero frequency.

The direct-current resistance depends only upon the conductor itself, while the increase in resistance, $\phi(\omega)$, depends only upon the surroundings of the conductor. All submarine cables are subjected to similar water immersions, and also are similarly-arranged combinations of dielectric and steel armor. The increase in resistance, $\phi(\omega)$, should then be approximately the same for most cables, regardless of what r_o may be.

When it is assumed that the increase in resistance is accounted for by the presence of a definitely localized short-circuited system in the vicinity of the main conductor, the equation for the apparent resistance takes the form (see Univ. of Calif. Publ. in Engineering, vol. i, "Alternating and Transient Phenomena in Coupled Electric Circuits," p. 171).

$$r = r_o + \frac{1}{a + \frac{b}{\omega^2}}$$

where a and b are constants depending upon the second system and the electromagnetic coupling between the two systems. To this expression should be added a term representing the hysteresis loss in the steel armor. This term is proportional to the frequency. Another term proportional to the square of the frequency may be added to represent the first term in the series expansion for the eddy-current or skin-effect phenomena. The final approximate form is then

$$r = r_o + \frac{1}{a + \frac{b}{\omega^2}} + c\omega + d\omega^2 \quad (12)$$

The observed data can be very well represented by this equation when the constants a , b , c , and d are determined by least-square methods. This shows that as far as the one quantity r is concerned, the hypothesis just made offers a satisfactory explanation

for the phenomena, but even so, this information does not aid materially in predetermining the resistance change for different sizes and kinds of cables. This latter requires that the effective

FIG. 9.

resistance be expressed in terms of the fundamental properties of the different components of the system. Without such detailed analysis, about the best that can be done is to accept equation (11), with $\phi(\omega)$ as determined from the data submitted. The

observed increase is approximately 1.7 ohms per kilometre, at a frequency of 1300 cycles per second.

The total apparent self-inductance is determined by those factors which determine the change in resistance. The percentage

FIG. 10.

error in estimating the total self-inductance for any cable will then be much greater than that in estimating the total resistance, because the total resistance contains a constant component which is capable of accurate determination at the time of manufacture.

If the surroundings of the main conductor be again considered as constituting a definite closed electrical path, the equation representing the total self-inductance as a function of the frequency will be of the form (see the above-named reference),

$$L = L_0 - \frac{1}{a' + \frac{b'}{\omega^2}} \quad (13)$$

in which a' and b' are constants depending as before upon the electrical relations of the main conductor and the surrounding closed path. When these constants are determined empirically from the data given, the observations are represented very well, but relations which necessarily exist between a , b , a' and b' are not fulfilled. The non-fulfillment of these relations can indicate only what is obviously the truth—namely, that the combination of armor and surrounding water does not constitute a system equivalent in all respects to some simple closed circuit coupled electromagnetically to the main conductor. At zero frequency the return current distributes itself throughout the entire surrounding liquid and armor, but as the frequency increases, the return current becomes more and more concentrated in the immediate neighborhood of the cable. The same kind of analysis required to determine r accurately is also required to predetermine L .

For the same reasons that the increase in resistance with increasing frequency may be considered as approximately constant for different cables, the total self-inductance may be considered as approximately the same. This is so particularly in view of the fact that the observed self-inductance is much greater than that calculated on the basis of the return current being concentrated in the armor.

Little is to be said concerning the components of the admittance Y , other than to call attention to the fact that their variations with the frequency very closely meet the predictions given in the opening paragraphs. The effective alternating-current conductance, or leakage, is scarcely related to the simple dielectric resistance for direct current. It increases in approximate proportion to the frequency so that, with a practically constant capacity, the phase angle of Y is very nearly constant. For frequencies up to 1300 cycles per second, the change in the total effective capacity is only a few per cent.

When the four fundamental electrical constants are combined into the transmission constants given in Table X, marked discrepancies are to be noted between the experimental results and

FIG. 11.

those given by the ordinary cable theory. The usual analyses of the performances of cables acted upon by alternating currents have neglected the self-inductance and leakage (effective alternating-current) and have also assumed r and C to be constant and

equal to their respective zero-frequency values. Such theory leads to the simple expressions

$$\left. \begin{aligned} \beta = \alpha &= \sqrt{\frac{1}{2} \omega r_o C_o} \\ u &= \sqrt{\frac{r_o}{\omega C_o}}; \theta_u = -45^\circ. \end{aligned} \right\} \quad (14)$$

If, from the curves in Figs. 4 and 7, the zero-frequency values be taken as

$$r_o = 4.66, C_o = 0.277 \times 10^{-6}$$

the approximate equations (14) become

$$\left. \begin{aligned} \beta = \alpha &= 0.002014 f^{0.5000} \\ u &= 1636 f^{-0.5000}; \theta_u = -45^\circ. \end{aligned} \right\} \quad (15)$$

From the values of α , in Table X, an empirical equation as derived by least-square methods is

$$\alpha = 0.00320 f^{0.405} \quad (16)$$

It is from this equation that the smooth curve in Fig. 9 is drawn. The very satisfactory nature of the measurements is indicated by the way the observations lie on the curve. It is to be noted that the empirical equation makes α proportional to the 0.405 power of the frequency instead of the 0.5000 power as given by the approximate theory. It is rather remarkable that the empirical equation represents the observations with such accuracy, in view of the complicated way in which α truly depends upon the frequency. Its true value is

$$\alpha = \text{real portion of } \sqrt{(r + j\omega L)(g + j\omega C)}$$

where, r , L , g , and C are functions of the frequency (or ω) as shown by the curves in Figs. 4 to 7, inclusive.

Table XIII shows for comparison the measured values of α and also the values computed by equation (15). For all of the frequencies shown, the observed values of α are smaller than the values computed by the approximate theory on the basis of the zero-frequency values of r and C . This can only be caused by the self-inductance, L , since the per cent. increase of resistance with frequency is much greater than the per cent. decrease in capacity at any frequency. This fact is readily appreciated after inspecting Table IX, where the angle of the impedance Z is seen

to reach 45 degrees at a frequency of 1300 cycles per second, instead of being 0 as given by the approximate theory. Table XIII also contains the observed and the calculated values of the infinite-line impedance. Because of the increase in r with frequency and also because of the self-inductance, the observed values of u are much greater than the approximate calculated values. Since the amount of power delivered to the cable at a given voltage is inversely proportional to u , and since the safety of the cable requires that the impressed voltage be definitely limited, it appears impossible to safely supply as much power to the cable as is indicated by the approximate calculations. If the impressed voltage be e , then the power input to the long cable is

$$P = \frac{e^2}{u} \cos \theta_u \quad (17)$$

The approximate theory gives $\theta_u = -45$ degrees, a constant, but the experiments show that θ_u is very much less numerically than 45 degrees when the frequency reaches 1000 cycles per second, or beyond. As an illustration, if the impressed emf. be limited to 50 volts, the power inputs, as based upon the observed and calculated values of U , are, for a frequency of 1295.6 cycles per second,

$$P_{obs} = \frac{50^2}{65.33} \cos (-20.96^\circ) = 35.7 \text{ watts}$$

$$P_{cal} = \frac{50^2}{45.5} \cos (-45.0^\circ) = 38.9 \text{ watts.}$$

TABLE XIII.

Comparison of the Measured Values of the Transmission Constants and the Values Calculated by the Approximate Theory which Neglects L and G.

f	α obs.	α cal.	u obs.	u cal.	θ_u obs.
199.3	0.02730	0.02843	129.0	115.9	-35.7°
200	0.02723	0.02848	128.1	115.7	-35.7°
257.9	0.03032	0.03234	115.6	101.9	-34.4°
320	0.03312	0.03602	105.5	91.4	-32.6°
400	0.03620	0.04027	97.0	81.8	-30.9°
400.6	0.03635	0.04031	98.0	81.7	-31.5°
512	0.03995	0.04556	88.6	72.3	-28.7°
596.3	0.04234	0.04917	84.6	67.0	-28.5°
772.6	0.04717	0.05598	77.0	58.9	-26.4°
824	0.04843	0.05781	76.4	57.0	-23.7°
1008.6	0.05263	0.06394	70.4	51.5	-24.0°
1295.6	0.05822	0.07248	65.3	45.5	-21.0°

Even though the calculated impedance is much lower than the observed, the calculated power is not much larger than the observed because of the fact that the true power factor ($\cos 20.96^\circ$) is considerably greater than the calculated power factor ($\cos 45^\circ$).

As an illustration of the use of the foregoing data, calculations relative to the possibilities of signal transmission by alternating currents will be shown. The receiving-end condition which insures the reception of maximum power is that the load impedance Z_o be equal in scalar value to u , and also that it have an angle equal to, and opposite in sign, to θ_u . (See page 221, "Electrical Phenomena in Parallel Conductors," vol. i.) The calculation of the transmission phenomena is much simplified, however, when it is assumed that $Z_o = U$. When this is the case, the received power is slightly less than when the above optimum relation is fulfilled, but in a system with such an extremely low efficiency as a cable, this small difference is negligible. The general formula relating input voltage E and received voltage E_o is

$$E = E_o \left(\cosh V l + \frac{U}{Z_o} \sinh V l \right) \quad (18)$$

When $Z_o = U$, this reduces to

$$E = E_o (\cosh V l + \sinh V l) = E_o e^{V l} \quad (19)$$

Since, in this problem, it is only the effective scalar values of voltage which are of interest, the rotation of the vectors as produced by the imaginary portion of $V l$ may be neglected, giving

$$e = e_o e^{a l} \text{ or } e_o = e e^{-a l} \quad (20)$$

as the equation for scalar values when there is no reflection at the load end ($Z_o = U$).

As stated previously, e is limited by considerations of safety to 50 volts. Present possibilities in the construction of vacuum-tube amplifiers are such that an electromotive force of 10^{-7} volts is about the minimum value which will produce satisfactory signals at frequencies between 50 and 1500 cycles per second. By taking $e = 50$ and $e_o = 10^{-7}$, and using equation (20), the greatest distance over which signals can be transmitted satisfactorily can be calculated when a is known. The equation is

$$l = \frac{1}{a} \ln \frac{e}{e_o} = \frac{1}{M a} \log \frac{e}{e_o} \quad (21)$$

where M = modulus.

The power input may be computed by equation (17).

Table XIV contains the results of calculations made for the Alaskan cable whose properties have been given in the foregoing. For this cable, since

$$a = 0.00320 f^{0.405},$$

equation (21), after substituting 5×10^8 for $\frac{e}{e_0}$, becomes

$$l = \frac{6259.4}{f^{0.405}}$$

$$\log l = 3.79653 - 0.405 \log f \quad (22)$$

The last column contains values of the distance for which $\epsilon^{al} = 10$, or those distances in which the voltage decreases by a factor $\frac{1}{10}$. If l_1 designate this distance, then

$$l_1 = \frac{1}{Ma} \quad (23)$$

TABLE XIV.

Transmission Data for Alaskan Cable.

Calculated from observed properties for $e=50$ and $e_0=10^{-7}$ volts, u and θ_u from Figs. 10 and 11.

f	α per km.	u ohms	θ_u deg.	P watts	l km.	l_1 km.
50	0.01561	237	-39.7°	8.1	1284	147.6
100	0.02066	172	-38.8°	11.3	969	111.4
200	0.02736	126	-35.5°	16.1	732	84.2
300	0.03224	107	-33.0°	19.6	621	71.4
400	0.03622	96	-30.8°	22.4	553	63.6
600	0.04269	83	-27.1°	26.8	469	53.9
800	0.04796	76	-23.8°	30.1	418	48.0
1000	0.05250	72	-21.2°	32.4	382	43.9
1200	0.05652	70	-19.3°	33.7	354	40.7

It is hardly desirable to use frequencies below 50 cycles per second, for then in rapid signaling, each signal element would contain too few complete cycles to permit of selectivity in reception. For this particular cable, if 50 cycles per second be considered as the lowest usable frequency, then the greatest distance over which this method of signaling will produce 10^{-7} volts at the receiving end is 1284 kilometres (approximately 800 miles). In all cases the amount of power required for sending is small.

Description of actual trials of this method of signaling is given later.

As a further illustration of the foregoing analyses, calculations for a typical Atlantic cable will be attempted. For this illustration the cable described by Squier, "On an Unbroken Alternating Current for Cable Telegraphy," Proceedings of the Physical Society of London, vol. xxvii, Part V, page 553, has been chosen. The data given for it are

Length = 2164 nautical miles.
 Total resistance = 4895 ohms.
 Total capacity = 914.1 microfarads.
 The dielectric is gutta percha.

Taking from Bowditch, 1918, the value 6080 feet per nautical mile, the above data gives for the zero-frequency properties

$r_0 = 1.2206$ ohms per kilometre.
 $C_0 = 0.22794$ microfarads per kilometre.
 Total length = 4010.3 kilometres.

This is all of the data available. Approximate values of the four fundamental electrical properties at different frequencies must be found in order to calculate the transmission constants.

The capacity may be considered as constant, independent of the frequency, since this was found to be very nearly the case for the Alaskan cable. For the Alaskan cable the angle of the admittance, θ_y , was found to be practically constant, with a mean value of 87.0 degrees. This is for a rubber insulation, which undoubtedly is poorer than a gutta-percha insulation. It is probable that for the latter an angle of 88 degrees would be nearer the truth, but this cannot be definitely asserted without some comparative study of the two kinds of dielectric. The value of 88.0 degrees will, however, be accepted as the basis for this calculation. The admittance then is

$$Y = (\omega C \operatorname{cosec} 88^\circ) / 88.0^\circ \\ = \omega C \cot 88^\circ + j \omega C \quad (24)$$

since $C = C_0$.

For the resistance, the direct-current value r_0 increased by $\phi(\omega)$, as determined for the Alaskan cable, is the best approximation possible. $\phi(\omega)$ can be found for any frequency from Fig. 4 by subtracting the direct-current value (4.660 ohms) from the

appropriate ordinate of the curve. For the self-inductance the values given by Fig. 5 are the only ones available to apply to the problem in hand.

Table XV gives the quantities required in the calculation of the transmission constants. In it

$\phi(\omega)$ was taken from Fig. 4.

$r = 1.2206 + \phi(\omega)$.

L was taken from Fig. 5.

The table also contains the scalar values and angles of the derived quantities Z and Y .

TABLE XV.

Approximate Values of Cable Properties for Atlantic Cable, Based upon its Direct-current Resistance and Capacity and the Alaskan Cable Data.

$C = 0.22794 \times 10^{-6}$ farads per km.

$r_0 = 1.2206$ ohms per km.

$\theta_u = 88.0$ degrees.

f	$\phi(\omega)$	r	L	Z	Y
0	0	1.22		1.22/0°	
50	0.07	1.29	0.00144	1.37/19.3°	0.0000717/88.0°
100	0.30	1.52	0.00134	1.74/29.0°	0.0001433/88.0°
200	0.73	1.95	0.00121	2.47/38.0°	0.0002866/88.0°
300	0.95	2.17	0.00112	3.03/44.2°	0.0004299/88.0°
400	1.11	2.33	0.00105	3.52/48.6°	0.0005733/88.0°
600	1.35	2.57	0.00095	4.41/54.3°	0.0008600/88.0°
800	1.54	2.76	0.00088	5.21/58.0°	0.001146/88.0°
1000	1.71	2.93	0.00083	5.98/60.7°	0.001433/88.0°
1200	1.86	3.08	0.00080	6.77/63.0°	0.001720/88.0°

By using equation (6) in conjunction with equation (24) and the values given in Table XV, the transmission constants a , u , and θu are found to be as given in Table XVI. The distance l is calculated by equation (21) for $e = 50$ and $e_0 = 10^{-7}$ volts. The power P is calculated by equation (17). The last column contains the quantity l_1 , the distance over which the voltage and current change by a factor $\frac{1}{10}$. l_1 is given by equation (23).

Fig. 12 shows the quantities given in the table. The values of a are represented with sufficient accuracy by the equation shown on the curve. The curves, naturally, are similar to those for the Alaskan cable. The attenuation is much less and, on

TABLE XVI.

*Calculated Transmission Data for Atlantic Cable.*From data in Table XV and for $\epsilon = 50$ and $e_0 = 10^{-7}$ volts.

f cyc.	α per km.	u ohms.	θu deg.	P watts	l km.	l_1 km.
50	0.00587	138	-34.3°	14.9	3415	393
100	0.00825	110	-29.5°	19.8	2429	279
200	0.01210	93	-25.0°	24.4	1656	190
300	0.01461	84	-21.9°	27.6	1371	158
400	0.01663	78	-19.7°	30.0	1205	138
600	0.01988	72	-16.8°	33.4	1008	116
800	0.02258	67	-15.0°	35.8	887	102
1000	0.02499	65	-13.7°	37.6	802	92
1200	0.02706	63	-12.5°	38.9	740	85

account of the lower resistance and consequent greater proportionate effect of the self-inductance, the angle of the infinite-line

FIG. 12.

impedance is more nearly zero. At 50 cycles per second, the maximum workable distance is found to be 3415 kilometres. This

is on a basis of 10^{-7} volts at the receiving end. The total length of this cable across the Atlantic is 4010 kilometres, so that it appears that the scheme of signaling with alternating currents is just possible for this cable. A slight decrease in frequency would decrease the attenuation sufficiently to still leave 10^{-7} volts at the full distance of 4010 kilometres. Since $h = 393$ km. for $f = 50$ cycles per second (this being the distance for a 10-to-1 ratio of voltage or current), an improvement in the amplification at the receiving end by

$$\log^{-1} \left(\frac{4010 - 3415}{393} \right) = 33$$

times would also suffice to make transmission at a frequency of 50 cycles per second possible.

The above results should be highly significant in suggesting improvements in the already overworked Atlantic cables. The errors arising from the approximate manner of finding the cable properties should not be sufficient to vitiate the above conclusions.

TRIALS OF ALTERNATING CURRENT TRANSMISSION.

In addition to the measurements of the cable properties as described in the foregoing, a number of trials were made of the transmission of alternating currents over cables, both over the long cable between Sitka, Alaska, and Seattle, Washington, and over shorter cables between various towns in southeastern Alaska. The results which could be obtained in tests of this kind were, of course, entirely dependent upon the degree of amplification which was obtained in them. As it naturally was impossible, in the limited time available for this work, to do a great deal towards the development of powerful low-frequency amplifiers, the results here given cannot be considered as the best possible.

The first trials were made over the long cable and, as a preliminary step, calculations based upon the fundamental cable properties as previously determined, were made in order to ascertain approximate values of received voltage at the different frequencies. The total cable length between Sitka and Seattle is 1751 kilometres (1088 miles). The impressed electromotive force, both as used in the tests and in the calculations, was 50 volts. The results of the calculations are given in Table XVII, in which

the received voltage e_0 is that which occurs when the load impedance equals the infinite-line impedance U . The empirical equation for the attenuation constant a was used. At these very low frequencies no observations had been made to determine the cable properties, so that the empirical formula serves as a means of interpolating between the zero-frequency value zero and the value of a found at 200 cycles per second. For such low frequencies as are involved, the empirical equation for a gives values greater than does the approximate equation (15), whereas the reverse is true at the higher frequencies shown in Table XIV. The following data shows the differences between the two equations for the frequencies under consideration.

f cycles per second	$a = 0.00320 f^{0.405}$ Empirical	$a = 0.00201 f^{0.500}$ Approximate
10	0.0081	0.0064
15	0.0096	0.0078
20	0.0108	0.0090
25	0.0118	0.0101
30	0.0127	0.0110
35	0.0135	0.0119
50	0.0156	0.0142

The difference between the two values is as much as 20 per cent. at the lowest frequency. By equating equations (15) and (16) and solving for f , the frequency at which both equations give the same result is found to be 65 cycles per second. It is probable that at the lowest frequencies the true value of a is nearer to that given by the approximate equation than to that given by the empirical equation, because at the low frequencies the self-inductance does not play an important part. The true value of a and the empirical equation become asymptotic before a frequency of 200 cycles is reached, because at this frequency the measurements began and the empirical equation was deduced so as to fit the facts. The conclusion is certain that, at frequencies below 50 cycles per second, the empirical equation gives values of a which are too high. It is equally certain then, that the conditions represented in Table XVII are at least as bad, if not worse, than the true state of affairs. The true values of e_0 may be several times those shown.

For the trials of transmission over the long cable, the generator and sending apparatus was installed at Sitka and the amplify-

ing and receiving apparatus at Seattle. For amplification a tuned, vacuum-tube amplifier with any desired number of stages up to six was developed. It consisted of the standard type (V.T.-2) Signal Corps transmitting tubes interconnected by transformers which were arranged to be tuned by adjustable condensers connected to their secondary windings. A voltage amplification of from 6 to 8 per stage was obtained at frequencies of from 20 to 40 cycles per second. For six stages a total voltage amplification of from 50,000 to 250,000 was thus obtained. Because of the fact that no success attended any of the trials made over this long cable, a detailed description of the amplifier and receiving apparatus is not necessary here.

TABLE XVII.

Calculated Attenuation Constant and Received Voltage for Seattle-Sitka Cable.

$$\alpha = 0.00320 f^{0.405}, e_o = e e^{-\alpha l}, l = 1751, e = 50.$$

f	αl	e_o
10	14.2	33.0×10^{-6} volts
15	16.8	2.61×10^{-6} volts
20	18.8	0.328×10^{-6} volts
25	20.6	0.0553×10^{-6} volts
30	22.2	0.0114×10^{-6} volts
35	23.6	0.0027×10^{-6} volts
50	27.3	0.00007×10^{-6} volts

It was at first planned to use a telephone receiver to detect signals of audible frequency, but a consideration of the results given in Table XVII showed conclusively that it would be hopeless to detect received currents of frequencies above 35 cycles per second and that, therefore, a signal of audible frequency could not be obtained. For use over the range of frequencies between 15 and 35 cycles per second a vibration galvanometer was selected. This instrument was kindly loaned for the duration of the experiments by the Electrical Engineering Department of the University of Washington. In addition to high sensitivity at the low frequencies it had the additional advantage of being very selective as to frequency, so that it should be an aid in minimizing the effect of "stray" disturbances. The galvanometer resistance was 49 ohms; excellent adjustment of sensitivity was obtained by shunting its terminals with a known resistance.

A typical record of one of the numerous trials is given here :

Date of trial, August 23, 1919.

Sending station, Sitka, Alaska, cable office.

Impressed electromotive force, 50 volts.

Frequency, 20 cycles per second.

Approximate input current, 0.15 amperes.

At Seattle, with the amplifier and galvanometer placed in a room about 15 metres distant from the cable office, it was found necessary to have all sounders and relays shut off because of the disturbances transmitted electromagnetically from them. When these instruments (sounders and relays) were not in operation, the amplifier by itself was entirely quiet, with no tendency to oscillate, as indicated by the galvanometer remaining steadily at zero. The amplifier and galvanometer were both tuned to 20 cycles per second.

With the cable connected to the input terminals of the amplifier, and when voltage was not impressed upon the cable at Sitka, the galvanometer gave a very unsteady deflection, averaging 2 centimetres on the scale when the galvanometer terminals were shunted by 0.2 ohms. This shunt reduced the galvanometer sensitivity to about $\frac{1}{250}$ of normal. No change in the unsteady deflection could be detected when alternating current was sent from Sitka. The observed deflection could only have been caused by "stray" electromotive forces induced in the cable system by street-car and power circuits, or by some natural, but obscure, magnetic or electric effects in the long cable itself. The cable office being in the midst of a busy portion of Seattle, it is known that the former influences were present, while it is not likely that natural magnetic or electrical variations, of such abruptness as to affect a tuned vibration galvanometer, connected to the cable through several stages of transformers, are to be found. Accurate information concerning this matter is needed, however. Calibration of the amplifier and galvanometer at the time of this test showed that at 20 cycles per second an input to the amplifier of 10^{-3} volts was necessary to produce the observed 2-centimetre deflection. The stray phenomena at the office cable terminals were, then, of this same order of magnitude, *viz.*, 10^{-3} volts, and probably were several times this amount, since in the calibration the frequency supplied was steady and equal to that

for which the galvanometer was tuned, which is not the case with the strays. From Table XVII it is seen that the received voltage to be expected is only 0.328×10^{-6} volts, a value only $\frac{1}{3000}$ of that arising from strays. It would obviously be impossible, therefore, to detect the small steady deflection in the midst of erratic stray deflections 3000 times its magnitude. With the galvanometer unshunted, or at normal sensitivity, the deflection for the steady alternating voltage of the above amount would have been

$$250 \times \frac{0.328 \times 10^{-6}}{10^{-3}} \times 2 \text{ cm.} = 0.16 \text{ cm.}$$

In the absence of stray disturbances, this amount could easily have been detected.

Since so much stray disturbance was encountered in the Seattle office building, it was decided to remove the amplifier and galvanometer to the shore-end of the sea cable, thereby eliminating about 400 metres of double-conductor, lead-covered cable between the shore-end and the Seattle office. Fig. 13 shows the

arrangement of the cable and ground connections at the Seattle end. Ground connection was made through approximately 13 kilometres of the standard type of submarine cable running out from Seattle into Puget Sound. The outer end of this cable and the attached ground contact is about 1 kilometre distant from the main Sitka-Seattle cable. Between the University Avenue shore terminal of the sea cable and the water, both the Sitka and the ground cables pass for approximately 100 metres beneath pavements and wharves.

With the amplifier and galvanometer adjusted as in the preceding test it was found that, when connection was made to the University Avenue cable terminals, a shunt resistance to the

galvanometer of 12 ohms was required to reduce the deflections caused by stray disturbances to 2 centimetres. Since for this same deflection a shunt of 0.2 ohms was required in the Seattle office building, it appears that the disturbances coming from the cables are only about $\frac{1}{50}$ of those reaching the office. According to the calibration cited, the above corresponds to about 20×10^{-6} volts, which, as stray disturbance, is still about 60 times the alternating voltage of this same frequency which could be received from Sitka.

A series of results similar to those given above were obtained at different times, after which it was decided that it would be useless to attempt reception at the Seattle end. On account of the limited time available it was not possible to interchange apparatus and thus attempt transmission in the opposite direction. The amplifier and galvanometer were taken to Sitka, however, and there it was found that, because of the absence of extensive city power systems, very little disturbance could be detected. The small disturbances which were observed could very well have been caused by a low-power alternating-current lighting circuit which passed within 50 or 75 metres of the testing room.

In any practical application of this method of alternating-current signaling, great care will be required to avoid any local sources of disturbances in those cases where, on account of great cable length, a high degree of amplification becomes necessary.

Because of the failure to transmit alternating currents over the long Sitka-Seattle cable, and because other work required the presence of the entire party in Alaska, arrangements were made to conduct further experiments with alternating currents over the shorter lengths of cable extending from Sitka to other points in southeastern Alaska. These tests occupied the first two weeks in October, 1919, and in them alternating currents of frequencies from 50 to 700 cycles per second were satisfactorily transmitted and received over distances from 300 to 700 kilometres.

For these trials the amplifier was altered slightly in order to meet the conditions imposed by the higher frequencies which could be transmitted over these shorter lengths of cable. In its modified form an amplification of approximately 14.5 per stage, throughout the above range of frequencies, was obtained. With

the four stages available, a total voltage amplification of 40,000, or a corresponding power amplification of 16×10^8 , was thus available. This is the maximum amplification possible when the load, cable output, and amplifier input and output impedances are so related as to give the best results. During the tests it is probable that the total voltage amplification actually secured was only about 20,000 or 30,000 instead of 40,000, because the several conditions required for maximum effectiveness could not be met exactly.

The cable network over which the experiments were made is as shown in Fig. 14, where the distances in kilometres between the various stations are shown. In the tests the generator was at

FIG. 14.

Sitka, while the amplifier and receiving apparatus were taken to the various points. At Cape Fanshaw the receiving set was connected to the Sitka cable after the other two branch cables had been disconnected. During all subsequent tests at the other points the three cables were joined at Cape Fanshaw, so that the received voltage was somewhat decreased because of the additional load imposed on the system by the branch cable. When the separate branches are of such length that reflected waves are negligible in comparison with the main waves, then the exact effect of this branching is to reduce all currents and voltages in the two branches to $\frac{2}{3}$ of the values they would have in the case of transmission over a single cable of a length equal to the distance between sending and receiving stations via the junction point. This reduc-

tion in voltage for all tests except that at Cape Fanshaw is taken account of in Table XVIII, which contains values of received voltages as calculated for the different points and different frequencies included in the experiments.

TABLE XVIII.
Calculated Values of Received Voltage.

Frequency	Station and distance				
	Cape Fanshaw 330.1	Petersberg Beach 398.5	Juncieu 486.2	Haines 657.4	Skagway 686.4
50	290,000	66,000	17,000	1,200	740
75	116,000	22,000	4,400	190	110
100	55,000	8,900	1,400	42	23
125	29,000	4,100	560	12	6.0
150	16,000	2,000	240	3.7	1.8
200	6,000	610	56	0.52	0.23
300	1,200	88	5.2	0.021	0.0082
400	320	18	0.75	0.0015	0.00053
500	100	4.6	0.14	0.00016	0.000051
700	12	0.46	0.0085	0.000004	0.00000095

In Table XVIII the values corresponding to the highest frequencies satisfactorily received at the several stations are printed in heavy type. Were the amplifier and telephone receiver used in connection therewith of the same sensitivity at the different frequencies, these limiting values of voltage would all be equal provided the empirical equation for the attenuation constant α truly represents the facts. Since this equation was determined so as to agree with the facts as determined by experiment, no error of consequence need be expected in the calculations. As a matter of fact, all of those limiting voltages shown in the table are nearly constant. Had time permitted of a greater number of observations and a calibration of the amplifier for each, it is likely that a still greater uniformity in these results would have been obtained. The average of the several figures shown gives

Average received voltage for good signals with 4-stage
amplifier = 24 microvolts.

For receiving the signals a standard, high-impedance Baldwin radio head set with double receivers was used. By good signals are meant those which were distinctly audible with the telephone receivers lying on a table at a distance of one metre from the observer. With the telephones on the head in the ordinary manner, this gives very good signals, indeed.

With the receiving apparatus as used, it is unquestionable that a received electromotive force of 5 microvolts would in all cases produce signals of ample intensity. It is, further, entirely possible to construct an amplifier 50 or 100 times as powerful as the one used throughout these tests, because, for the one used, such apparatus as was available had to be used instead of apparatus especially designed for the purpose. In addition, the amplifier had to be carried about from place to place and set up hurriedly in whatever place could be found in the various cable offices. All of these things conspire to prevent the maximum effectiveness in the use of such an apparatus.

The only thing which could prevent the use of a considerably greater degree of amplification than that employed is excessive extraneous, or stray, disturbances. The experiences met with in these tests showed that trouble of this nature can be avoided by taking the proper precautions. The amplifier cannot be used in the vicinity of power systems, and the cable must not pass close to such systems. It is considered quite certain that most of the disturbances are produced by electromagnetic induction and earth currents arising from grounded systems; conditions which make effective shielding practically impossible. With a cable in the open sea or everywhere at a distance of at least 2 or 3 kilometres from overhead power systems or of 10 or 12 kilometres from earth-return power systems, no difficulties should be expected.

Accepting, then, 5 microvolts as a sufficient amount to produce good signals with the amplifier used, and further, accepting the possibility of 50 or 100 times the amplification actually attained, it becomes possible to produce good signals with a received electromotive force of from 5×10^{-8} to 10^{-7} volts. It is in this way that the basic value of 10^{-7} volts was obtained for use in the calculation of the data given in Tables XIV and XVI.

A summary of the notes taken during the several trials of alternating-current transmission and reception is given in the

following table. The data contained in Table XVIII should be considered along with these notes:

TABLE XIX.

Notes on Observations Taken in Trials of Alternating-Current Transmission.

Date of tests, October 1 to October 15, 1919.

Sending station, Sitka, Alaska.

Generator electromotive force, 50 volts.

AT CAPE FANSHAW (330.1 km.).

Juneau and Petersburg cables disconnected.

Using a standard, low-impedance, bi-polar telephone receiver, readable signals were obtained at 300 cycles per second without amplification. Very loud signals of this frequency were obtained with one or more stages of amplification.

At 500 cycles per second the signals were just audible without amplification. Good signals were received with 2 and 3 stages of amplification, and very loud signals with 4 stages.

At 700 cycles a very steady, clear note which produced excellent signals was obtained with 4 stages of amplification.

AT PETERSBURG OFFICE (402.4 km.).

All cables interconnected at Cape Fanshaw.

Amplifier installed at cable office, which connects with the beach end of the cable through 3.9 km. of overhead line. Ground connection at the office.

Excessive inductive interference from surrounding power circuits was encountered, which made tests at this location impossible.

AT PETERSBURG BEACH (398.5 km.).

Amplifier mounted in open air at the cable terminal.

Excellent reception with 2 stages at 150 cycles.

At 200 cycles very loud signals were obtained using 3 and 4 stages; strong signals at 300 cycles with 3 stages.

With 4 stages good signals were received at 400 cycles.

All of the tests were satisfactory in every respect. No trouble from stray disturbances was encountered.

AT JUNEAU (486.2 km.).

Amplifier installed in cable office. Some interference from power circuit, which produced a distinct hum in the receiver.

The 100-cycle note could be heard all over the room with 4 stages of amplification, was very good with 3 stages, and produced good readable signals with 2 stages.

At 150 cycles the power-line interference was disturbing, but very good signals were obtained with 4 stages of amplification.

At 200 cycles satisfactory signals were obtained, but the interference still persisted.

The 250-cycle note could be heard distinctly, but beats between it and some harmonic in the power system prevented satisfactory reading of signals. (In Table XVIII the limiting voltage for the Juneau test should, therefore, be considerably less than the 56 microvolts as given for 200 cycles.)

During the experiments at Juneau a satisfactory trial was made of receiving direct-current signals from Petersberg on the regular cable instruments and at the same time receiving 100-cycle signals from Sitka,

AT HAINES (657.4 km.).

At Haines direct connection with Sitka was established by connecting through the Juneau office, but the two cables are not normally operated in this way.

Very loud 50-cycle signals were received with 4 stages. For a duration of several minutes the regular station operator there read Continental Morse code signals at the rate of 15 words per minute, and in a further test of several minutes, at the rate of 25 words per minute with the greatest ease.

Excellent signals, both with 3 and 4 stages of amplification, were received at 100 cycles, and sending in Continental Morse code was easily read at a rate of 25 words per minute.

At 125 cycles the signals were distinctly audible with 4 stages of amplification, but too much interference from power sources was present to permit of clear reading. The power interference was of the same character as that observed at Juneau, and since the cables were interconnected through the office there, it is thought that the interference originated in the Juneau office and exposed cable. The attenuation of these interfering currents over the distance of 70 km. between Juneau and Haines would not be sufficient to make them negligible, because the incoming signals are attenuated in the same way.

Signals at 150 cycles were not readable on account of the interference.

AT SKAGWAY (686.4 km.).

Excellent signals at 50 cycles per second were received using 4 stages of amplifier. Whereas it had previously been somewhat doubtful if frequencies as low as 50 cycles per second would produce satisfactory telegraphic signals, the actual tests both here and at Haines showed that for a signaling rate as high as 25 words per minute, at least, this frequency serves as well as frequencies in the more sensitive region of audibility.

At 75 cycles the signals would have been perfectly satisfactory except for the interference from power circuits.

In the same way, 100-cycle signals were clearly audible with 4 stages of amplifier, but on account of the interference were not readable.

The signals were just audible at 150 cycles, and at 200 cycles nothing at all could be heard.

At Skagway, the cable comes in the sheath directly from the shore to the office through a trench of approximately 0.5 km. in length, passing near to power lines and transformers in the vicinity of the office.

Disturbances created at this point undoubtedly affected both the tests here as well as at Haines, in addition to disturbances arising at Juneau.

The data contained in the above notes serve to establish the limiting values of voltage shown in Table XVIII.

The results of the trials as described above, and particularly when considered in conjunction with the mathematical analysis and calculations given are such as to fully justify the statement that a very great increase is possible in the usefulness of a great majority of existing cable systems. Although it is something that had not been hoped for, the calculations nevertheless show that this method of signaling is applicable even to an Atlantic cable. If it should prove capable of superposing even one message on the existing duplex working of an Atlantic cable, the method would increase the working value of such a cable by 50 per cent. On cables shorter than this, of which there are very many, the increase in working effectiveness would be still greater, because of the greater range in frequencies which can be utilized.

ACKNOWLEDGMENTS.

In conclusion it is desired to render acknowledgment to a number of people through whose effective aid the work described in the foregoing was made possible.

First, to the Chief Signal Officer of the Army, Major-General George O. Squier, must be given the credit for recognizing that work of the kind described was desirable, and even necessary, and for authorizing the Signal Corps Research Laboratory to proceed with the experimental work.

In addition to the writer, the experimental party consisted of

Dr. Sylvan J. Crooker, Radio Engineer,
Mr. Lester J. Rich, Radio Laboratory Assistant,
Mr. Chance E. Böhner, M.S.E., U. S. A.,

and for a portion of the time,

Mr. George L. Greves, Assistant Professor of Electrical Engineering, University of California.

Needless to say, to them is due the credit for the expeditious accomplishment of the work undertaken.

Without the active interest and coöperation of Lieut.-Col. B. O. Lenoir, Officer-in-charge, Washington-Alaska Military Cable and Telegraph System, the work could not have been carried out. Although every effort was made not to interfere with the regular operation of the system, the experimental work could not but cause considerable inconvenience. Much of the burden of keeping this disturbance at a minimum naturally fell on the chief operators, John A. Perry, M.S.E., at Seattle, and Phil McQuillan, M.S.E., at Sitka. Their efforts are very much appreciated.

For the trials of alternating-current transmission in southeastern Alaska, the Forest Service very kindly placed their gasoline cruiser, "Ranger V.," at the disposal of the testing party, and at the same time authorized the Forester at Sitka, Mr. Geo. A. Peterson, to accompany the expedition. Mr. Peterson's inimitable qualities as a host made the two weeks spent aboard his vessel, going from place to place in the waters of southeastern Alaska, a delightful experience long to be remembered.

Phosphorus in Butter.—According to J. T. CUSICK (*Cornell University Agricultural Experiment Station Memoir* 30, pp. 153-187, 1920), approximately one-fourth of the total phosphorus of the cream is retained by the butter; the remaining three-fourths are lost in the buttermilk during churning, in the wash waters, and in the exudates which form during the process of salting. During storage, the organic phosphorus compounds are broken down into inorganic phosphorus compounds. The treatment of the milk and cream prior to churning influence the amount and the form of the phosphorus retained in the butter, and also determine how soon after storage the organic phosphorus compounds are transformed into inorganic compounds. Under certain conditions, bacteria are the controlling factors in producing chemical changes in the phosphorus compounds. Fishy flavor in butter is due to the decomposition of lecithin and the formation of trimethylamine; development of this flavor is always accompanied by an appreciable decrease in soluble organic phosphorus. Salt in butter has a marked effect in bringing about decomposition of the proteins during storage, even at a temperature of -10° C. The recently discovered, alcohol-soluble protein of milk also occurs in butter.

J. S. H.