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II. *A Resonance Method of Measuring Energy dissipated in Condensers.* By EDWARD B. ROSA and ARTHUR W. SMITH*.

THAT the dielectric of a condenser becomes warmed when an alternating electromotive force is applied to the terminals of the condenser has long been known, and the study of this heating effect has been undertaken by numerous observers. Kleiner† used a thermo-electric couple imbedded in the dielectric to measure the rise of temperature, and noted a considerable heating effect in ebonite, gutta percha, glass, and mica, but none at all in paraffin and kolophonium. He reports that in spite of all attempts by variations of the conditions of the experiment, no heating could be detected in these two last-named substances. On the other hand, Boucherot‡ has made paraffin-paper condensers for use on the 3200-volt commercial circuits of Paris, some of which became so hot in use that they were obliged to be cut out. Boucherot says of the heating effect that if a condenser rises as much as 30° C., it should be rejected. That this is good advice is evident from the fact that paraffin melts at 54° C.; and hence when 30° C. above the temperature of a summer's day (say, 25° C. or 77° F.), the paper would be floating in melted paraffin.

Bedell, Ballantyne, and Williamson§ report experiments upon a paraffin-paper condenser of 1.5 microfarad capacity, the efficiency of which was found to be 95.6 %, or 4.4 % lost in heat. The loss was determined by a three-voltmeter method, similarly to measurements on a transformer. It was put upon a 500-volt circuit at a frequency of 160, and the current was therefore about 0.7 ampere, the apparent watts about 350, and the heating effect 15.4 watts. The temperature rose one degree per hour.

Threlfall|| reports a test on a paraffin-paper condenser of his own construction which had a capacity of 0.123 microfarad, and on a circuit of 3000 volts and a frequency of 60 the rise of temperature was less than one-fifth of a degree per hour. The apparent watts would be about the same as in the experiment of Bedell, Ballantyne, and Williamson, although the capacity was only one-twelfth as much. Threlfall concludes, apparently, that since his condenser had only one-twelfth the

* Communicated by the Authors.

† Wied. Ann. vol. i. p. 138.

‡ *L'Eclairage Electrique*, Feb. 12, 1898.

§ 'Physical Review,' October 1893, vol. i. p. 81

|| 'Physical Review,' vol. iv. p. 458 (1897).

electric capacity, it also had only one-twelfth the capacity for heat of the other condenser; and since the temperature rose five times more slowly, therefore that the percentage loss of energy was only one-sixtieth as great, that is, $\frac{1}{60}$ of 3 %, or 0.05 %, giving his condenser an efficiency of 99.95 %! The reasoning is, however, quite inconclusive, for nothing is said concerning the thickness of the paraffin-paper dielectric and the volume of the condenser. Suppose the dielectric of the smaller condenser to be 0.0129 inch thick, which is three times the thickness of the other. (It might have been as thick as that, seeing it sustained for a considerable period an alternating voltage of 3000.) Then the volume per unit of capacity would be nine times as great as the other, and the heat required to raise its temperature as rapidly would be nine times as great, assuming the capacity for heat and rate of radiation the same for both. The loss would then have been 0.45 % instead of 0.05 %. This illustrates how entirely inconclusive any determination merely of rise of temperature is, unless all such circumstances as heat capacity, rate of cooling, thickness of dielectric (or intensity of the electrostatic induction) are specified.

The enormous discrepancy between the results above referred to as to the quantity of the heating effect in condensers, and the almost entire lack of precise statements as to its numerical value, led us to undertake, more than a year ago, to measure this energy loss in such a way that it could be expressed absolutely.

We proposed to measure by means of a wattmeter the energy dissipated in a condenser when it is subjected to an alternating electromotive force. In order that the frequency of charge and discharge be perfectly definite, the electromotive force should be a simple harmonic one, that is, the upper harmonics of the fundamental should be absent. This is most easily effected by inserting in the circuit in series with the condenser a coil of wire having large self-induction, but without an iron core. The variable permeability of the iron will give rise to upper harmonics, especially if the magnetic induction of the core attains large values; and hence a coil without an iron core is necessary. Moreover, if the self-induction is not large enough it will reinforce some of the upper harmonics instead of quenching them; and the presence of the coil will be detrimental rather than beneficial. The best value of the self-induction of the coil is such that the fundamental is reinforced to a maximum degree; in other words, it is that value of L given by the equation

$$t = 2\pi \sqrt{LC},$$

where t is the period of the fundamental component of the impressed electromotive force and C the capacity of the condenser.

There is another practical advantage resulting from this arrangement, namely, that the resulting resonance raises the electromotive force at the terminals of the condenser very greatly, saving the necessity of raising the voltage by transformers. And a third advantage now appears in the fact that the wattmeter may be inserted across the low-voltage supply wires to measure the total power expended upon coil and condenser. Then subtracting the I^2r loss of the coil, the remainder will be the power expended upon the condenser. This supposes, of course, that there is no iron core and no eddy-current loss in the copper coil itself.

The Resonance Method.

Fig. 1 shows the connexions for this method with the coil in series with the condenser. M, N are the low-voltage supply-wires of an alternating circuit, S is an adjustable resistance, E is a dynamometer, F a wattmeter, R a non-inductive resistance in shunt with the coil and condenser,

Fig. 1.

joining the points A and D . The wattmeter therefore measures the power expended between the points A and D , including the I^2r_w loss in the fixed coil of the wattmeter, but not including the I^2R loss in the shunt-resistance R . The

energy expended in the resonance-coil and the fixed coil of the wattmeter, $I^2(r_c + r_w)$, subtracted from the total power measured leaves the condenser loss. This remainder is found to be also proportional to the square of the current. Hence we may write it $I^2 r_s$, and r_s is the "equivalent resistance" of the condenser, which it is desired to find. This does not indicate the nature of the process by which energy is dissipated in a condenser, but simply that for a given condenser made of a given dielectric at a given temperature and frequency, the heating effect is the same as though there were a certain resistance r_s in series with a perfect condenser of the same capacity. For the same dielectric r_s changes with changes in the temperature or the frequency; and for another dielectric with the same capacity, temperature, and frequency r_s would be different.

If $t = 2\pi\sqrt{LC}$, there is complete resonance; $t = \frac{1}{n}$, and $p = 2\pi n$. Therefore $\frac{1}{p} = \sqrt{LC}$, or $C = \frac{1}{p^2 L}$. That is, for complete resonance the capacity is inversely proportional to the square of the frequency for a given self-induction. If

Fig. 2.

the frequency is fixed, either the capacity or the inductance may be varied until the current is a maximum; but if the frequency can be varied, the maximum resonance may be attained without varying C or L.

In fig. 2, let $Aa=r_e$, the ohmic resistance of the coil;
 $aB=pL$, the reactance of the coil;
 $Bb=r_c$, the equivalent resistance of the con-
 denser;
 $bC=\frac{1}{pC}$, the condensance of the condenser;
 $CD=r_w$, the resistance of the fixed coil of the
 wattmeter, its induction being ne-
 gligible;
 AD =the resultant impedance of the circuit.

Then if $pL = \frac{1}{Cp}$, the reactance is equal and opposite to the condensance (fig. 2 c), the resonance is complete, and the impressed electromotive force e is expended in overcoming the resultant resistance $AD = r_c + r_s + r_w$, and $I = \frac{e}{r_c + r_s + r_w}$.

In fig. 3, similar to fig. 2 c, the several lines represent electromotive forces. $Aa = Ir_c = e_c$ = that part of the electromotive force expended in overcoming the ohmic resistance of the coil ; similarly, $Bb = e_s$, $CD = e_w$. Ba is the electromotive force (due to resonance) which overcomes the reactance, and bC is the electromotive force which overcomes the condensance. The potential of the point B varies through a wide range; whereas the points A, C, and D, and the instruments suffer only small changes of potential.

Fig. 3.

Advantages of the Method.

Herein lies one of the chief advantages of the method, that voltages below a hundred have to be dealt with at the instruments, whereas upon the condenser there may be an active electromotive force of several thousand volts. The noninductive resistance R is at most a few hundred ohms. On the contrary, if the shunt-resistance were applied directly at the terminals of the condenser, it would necessarily be several thousands of ohms, it must be capable of carrying the entire shunt-current of the wattmeter, it must be strictly non-inductive, and be of known value—conditions difficult to fulfil. In the resonance method a small inductance in the shunt-resistance or the movable coil of the wattmeter produces no appreciable error; whereas in the simple wattmeter method it produces a large error.

To illustrate this point let us take a special case. Suppose the resistance R is 500 ohms and the inductance of the shunt-circuit, including both the resistance R and the movable coil of the wattmeter, is 0.003 henry, $2\pi n$ being 800. Then $pL = 2.4$ and $\tan \phi_1 = 0.0048$, $\phi_1 = 16' 30''$, the angle of lag of the shunt-current behind the electromotive force. Suppose the true angle of the condenser-current, ϕ_2 (fig. 4), is $89^\circ 40'$ ahead of the electromotive force. Then the difference of phase of the two currents in the wattmeter will be $\phi = \phi_1 + \phi_2 = 89^\circ 56' 30''$, and the power factor, $\cos \phi$, of the expression watts = $EI \cos \phi$ will be $\cos 89^\circ 56' 30''$ instead of $\cos 89^\circ 40'$, that is .00102 instead of .00582; thus the wattmeter would indicate that the power absorbed in the condenser was only about one-sixth of what it really is. If the lag of the shunt-current were more than $20'$ (a not improbable value in many

cases), the deflexion of the wattmeter would be *negative*! It is possible that this explains why it has often been claimed that the loss in certain condensers is too small to be measured by a wattmeter. For example, Swinburne*, speaking of some of his own condensers, says "a condenser that takes 2000 volts and 10 amperes has a loss that is too low to measure—that is to say, it is less than 5 or 10 watts."

On the other hand, if by means of a resonance-coil the current and electromotive force have been brought very nearly, if not exactly, into phase, any small lag of the shunt-current will make no appreciable error. Thus, the cosine of 10° is .9848, and of $10^\circ 16' 30''$ is .9840, a difference of less than one part in a thousand.

In order to determine the precise values of r_c and r_w a Wheatstone bridge is joined to A D and the condenser short-circuited, so that the resonance-coil and the fixed coil of the wattmeter and the lead-wires form the fourth arm of the bridge. The resistance is then quickly measured just after the wattmeter has been read and the alternating circuit broken, and changes due to temperature are included.

* 'Electrician,' Jan. 1 (1892).

Method of Measuring Energy dissipated in Condensers.

Difficulty of the Method.

We have seen that the presence of a resonance-coil in series with the condenser (1) quenches, to a large extent at least, the upper harmonics, (2) raises the voltage upon the condenser, thus avoiding transforming up, (3) enables measurements to be made more safely and more conveniently upon a low voltage, and (4) transfers the wattmeter problem from the most unfavourable case (where the angle of phase-difference is nearly 90°) to the most favourable case where the current and electromotive force are nearly in phase. There is, however, one serious difficulty in the method. If the resonance-coil is made of small wire, it has a great resistance, and of the total power measured only a small part is expended on the condenser. Thus the condenser loss is the difference between two relatively large quantities, and cannot be determined as accurately as would be desired. If, on the other hand, a large coil of larger wire is used so that its resistance is small, there will be eddy-currents in the copper of the coil, and the power expended on the coil will be greater than $I^2 r_c$. This excess will go into the remainder as condenser loss, and may give rise to a considerable error. If the wire is of large cross-section, but stranded, so that its resistance is small and the eddy-currents negligible, then a large coil will have a large inductance, and no difficulty appears. The method is then accurate as well as quick and convenient.

The Resonance Ratio.

As the condenser is alternately charged and discharged energy is handed to and fro between the coil and the condenser. When the condenser is charged to its maximum extent the current is zero and all the energy is potential and residing in the condenser. A quarter of a period later the condenser is discharged and the current is a maximum; the energy is now kinetic, and resides in the magnetic field of the resonance-coil. At other instants the energy is partly potential (in the condenser) and partly kinetic (in the coil). As this transfer of energy to and fro continues, the dynamo supplying the current furnishes just enough energy to make good the losses, that is, the heating effect in the wires and the dielectric of the condenser. The losses due to electromagnetic radiation and mechanical vibrations are usually negligible.

For the condenser alone,

$$I = \frac{E}{\sqrt{r_c^2 + \frac{1}{C^2 p^2}}},$$

I and E being the square root of the mean square values as indicated by an electro-dynamometer and electrometer.

For the combined circuit

$$I = \frac{e}{\sqrt{(r_w + r_c + r_s)^2 + \left(pL - \frac{1}{Cp}\right)^2}},$$

where e is the small impressed electromotive force and the denominator is the combined impedance of the circuit.

For complete resonance,

$$pL = \frac{1}{Cp}, \text{ and hence } I = \frac{e}{r_w + r_c + r_s}.$$

Hence

$$\frac{E}{e} = \frac{\sqrt{r_c^2 + \frac{1}{C^2 p^2}}}{r_c + r_s + r_w} = \frac{\text{Impedance of the condenser}}{\text{Total resistance}} = \text{Resonance ratio.}$$

In one case e was 50 volts and E was 2250, giving a resonance ratio of 45. The impedance was 51 ohms, $r_c + r_w$ was .38, r_s was .72. Hence $\frac{51}{.38 + .72} = 46.4$, agreeing very nearly with the ratio of the voltages. In this case the coil was of large wire (No. 5 B & S), and had considerable eddy-current loss. Hence the value .72 for r_s was too large, and the degree of resonance was lower than it would have been in the absence of eddy-currents. In another case, using a coil of No. 10 wire, the impressed electromotive force was 29.5 volts, the voltage on the coil or the condenser was 1808, and the resonance ratio $\frac{E}{e}$ was therefore 61.9.

The Resonance-Coil in Parallel.

A second arrangement of the resonance-coil is to put it in parallel with the condenser (fig. 5), and impress upon both a high electromotive force. Each of the two parallel circuits from B to C takes its own current, independently of the other, but being nearly opposite in phase they nearly cancel each other in the supply wires. Hence a small transformer is sufficient to supply the small current needed, although without the resonance-coil a large transformer would be necessary. If, as before,

$$pL = \frac{1}{Cp},$$

there is complete resonance. The two parallel circuits having the same impedance take the same current; one current is nearly 90° ahead in phase of the electromotive force (see

Fig. 5.

curve 2, fig. 6), the other is nearly 90° behind (curve 3), the sum of the two being relatively very small (curve 4) and in phase with the impressed electromotive force (curve 1). The shunt resistance must be applied at the high voltage terminals, but as a small amount of self-induction produces no appreciable error in the wattmeter, the movable coil may be long enough to make the wattmeter quite sensitive, and so a quite small shunt current may be used. This requires a larger resistance, but with much smaller carrying capacity, since a much smaller shunt current will suffice than when the main current differs largely in phase from the shunt current.

To illustrate this point, suppose as before that for a given condenser the angle of advance of the current is $89^\circ 40'$. The power factor, $\cos \phi$, in the expression watts = $EI \cos \phi$ is in this case .00582. If now a resonance-coil be placed in parallel with the condenser and the current in the fixed coil of the wattmeter brought into phase with the electromotive force, then $\cos \phi = 1$. To get a certain deflexion of the wattmeter, therefore, we must have the product of the two currents in the wattmeter nearly 200 times as great in the first case

Fig. 6.

as in the second, and this requires a relatively large shunt current.

A modification of the method, if a second small transformer is available, is to transform down again to a low voltage, and put the shunt circuit of the wattmeter on the low voltage secondary of this second transformer. The current will now be almost exactly opposite in phase to the high electromotive force at the terminals of the condenser, and by interchanging the terminals the wattmeter deflexion will be the same as before, if the shunt resistance is reduced in the ratio of transformation. The currents in the two coils of the wattmeter are so nearly in phase with one another that a small change in the phase of the shunt current will produce no appreciable error.

The Efficiency of a Condenser.

Having thus determined the energy, w , dissipated in a condenser, by wattmeter measurements, we readily find r_s , the equivalent resistance of the condenser, from the expression

$$w = I^2 r_s.$$

The ratio of the equivalent resistance to $\frac{1}{Cp}$ is $\cot \phi$ (fig. 7), ϕ being the angle of advance of the current ahead of the electromotive force. It remains to calculate the efficiency of a condenser.

In fig. 8 I is the current flowing into and out of the condenser, assuming both current and electromotive force to be simply harmonic. The dotted curve is the power curve.

$$e = E_1 \sin pt,$$

where e = the instantaneous E.M.F. acting on the condenser, and E_1 is its maximum value.

$$i = \frac{E_1}{\text{Impedance}} \sin (pt + \phi) = I_1 \sin (pt + \phi).$$

$$ei = E_1 I_1 \sin pt \sin (pt + \phi)$$

$$= E_1 I_1 [\sin^2 pt \cos \phi + \sin pt \cos pt \sin \phi].$$

The area of the power curve for one half-period, that is the area of the positive loop from B to C, minus the area of the

Fig. 7.

Fig. 8.

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negative loop from C to D, is given by the integral

$$\begin{aligned}\int e i dt &= E_1 I_1 \cos \phi \int_0^\pi \sin^2 pt dt + E_1 I_1 \sin \phi \int_0^\pi \sin pt \cos pt dt, \\ &= \frac{E_1 I_1}{p} \cos \phi \left[\frac{pt}{2} - \frac{\sin 2pt}{4} \right]_0^\pi - \frac{E_1 I_1}{4p} \sin \phi \left[\cos 2pt \right]_0^\pi (1) \\ &= \frac{1}{2} \frac{E_1 I_1}{2\pi} \cos \phi,\end{aligned}$$

and this is the work done in $\frac{1}{2n}$ of a second.

Hence for one second the work done, and therefore the power, is the well-known expression

$$\frac{1}{2} E_1 I_1 \cos \phi = EI \cos \phi \quad (E = E_1 \sqrt{\frac{1}{2}}),$$

E and I being the effective values of the electromotive force and current.

The area of the positive loop, which represents the work done upon the condenser in charging it, is

$$\int_0^{\pi-\phi} e i dt = \frac{E_1 I_1}{p} \cos \phi \left[\frac{pt}{2} - \frac{\sin 2pt}{4} \right]_0^{\pi-\phi} - \frac{E_1 I_1}{4p} \sin \phi \left[\cos 2pt \right]_0^{\pi-\phi},$$

and the area of the negative loop, which represents the work done by the condenser in its discharge, is the value of the same integral between the limits $\pi - \phi$, and π , for pt .

The efficiency is the ratio of the work done by the condenser in its discharge to the work done upon the condenser in charging it.

This is the *gross efficiency*, η .

$$\begin{aligned}\text{Taking } \phi = 89^\circ, \eta &= \frac{\text{Area of the Negative loop}}{\text{Area of Positive loop}} = \frac{1/p E_1 I_1 \times .48638}{1/p E_1 I_1 \times .51378} = 94.67 \% \\ \text{,, } \phi = 88^\circ, \eta &= \text{,,} = \frac{.47287}{.52769} = 89.68 \% \\ \text{,, } \phi = 87^\circ, \eta &= \text{,,} = \frac{.45684}{.53905} = 84.75 \% \\ \text{,, } \phi = 45^\circ, \eta &= \text{,,} = 6.38 \%\end{aligned}$$

Having found the angle ϕ by the wattmeter measurements the gross efficiency of the condenser may be found from the above equations, or taken from fig. 9, which is drawn from them.

Fig. 9.

Second Definition of Efficiency.—Net Efficiency.

Regarding the condenser as an instrument for storing electrical energy, and one in which a certain amount of energy is dissipated in the process, we may define the percentage of loss as the ratio of the energy dissipated to the energy stored. The efficiency is then unity minus the loss, or, in per cent., the efficiency is 100 minus the per cent. of energy dissipated. This we may call the net efficiency, and represent it by ϵ . Then

$$\epsilon = \frac{\text{Energy Stored} - \text{Energy Dissipated}}{\text{Energy Stored}}, \text{ or } 1 - \frac{\text{Energy Dissipated}}{\text{Energy Stored}}.$$

In fig. 10,

$$I_1 r_s = E_1 \cos \phi = \text{active electromotive force};$$

$$\frac{I_1}{Cp} = E_1 \sin \phi = \text{wattless electromotive force, or the E.M.F.}$$

which charges the condenser.

The maximum charge of the condenser is

$$CE_1 \sin \phi,$$

and its energy

$$W = \frac{1}{2} C E_1^2 \sin^2 \phi.$$

The expression for the power is

$$\frac{1}{2} I_1 E_1 \cos \phi.$$

But

$$I_1 = C E_1 \sin \phi \cdot p.$$

Hence the energy dissipated per second is

$$\frac{1}{2} C E_1^2 \sin \phi \cos \phi \cdot p,$$

and the energy dissipated per half-period (that is during the time of a single charge and discharge) is

$$w = \frac{1}{2} C E_1^2 \sin \phi \cos \phi \cdot \pi.$$

The relative loss is therefore

$$\frac{w}{W} = \pi \cot \phi,$$

and the net efficiency is

$$\epsilon = 1 - \pi \cot \phi.$$

For

$$\phi = 89^\circ, \epsilon = 94.52 \%,$$

$$\phi = 88^\circ, \epsilon = 89.03 \%,$$

$$\phi = 87^\circ, \epsilon = 83.54 \%,$$

$$\phi = 72^\circ 20' 30'', \epsilon = 0.$$

The net efficiency, ϵ , is therefore slightly less than the gross efficiency, η , for values of ϕ nearly 90° ; but, as the angle ϕ diminishes, ϵ falls rapidly below η , and for $\phi = 72^\circ 20' 30''$ the energy dissipated is equal to the energy stored, and the net efficiency is therefore zero, while the gross efficiency is about 38 per cent. For greater angles of lag the loss is greater than the maximum energy stored, and ϵ becomes negative (see fig. 9).

For ordinary cases the angle ϕ is greater than 88° , and ϵ and η are nearly equal. Since the wattmeter method gives directly the value of ϕ it is much easier to express the value of the net efficiency ϵ (namely, $1 - \pi \cot \phi$) than the value of η . For small values of $\cot \phi$ this is sufficiently exact to write it $\epsilon = 1 - \pi \cos \phi$.

Suppose that instead of assuming the effective resistance r_s of the condenser to be in series with the condenser, as we have done in figs. 2, 3, 7, and 10, we consider that it is in

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D

Fig. 10.

parallel, as in fig. 11. Of course there is a slight leakage current in every case, if the resistance of the dielectric is not infinite. Boucherot* says of his paraffin-paper condensers

Fig. 11.

that the "heating is chiefly due to the Joule effect, that is, to leakage current; the action of dielectric hysteresis, if it exists at all, being very slight." We shall give reasons in a subsequent paper for believing that this is seldom, if ever, true of good condensers, but at present let us assume it to be true. Then the condenser current is 90° ahead of, and the leakage current in phase with, the impressed electromotive force. I being the total current, the condenser current is $I \sin \phi$ and the leakage current is $I \cos \phi$. The energy stored is

$$W = \frac{1}{2} C E_1^2,$$

and the energy dissipated per second is

$$\frac{1}{2} E_1 I_1 \cos \phi,$$

or per half-period

$$w = \frac{1}{4n} E_1 I_1 \cos \phi.$$

The maximum condenser current, $I_1 \sin \phi = p C E_1$.

$$\therefore w = \frac{E_1^2 \cdot p C \cos \phi}{4n \cdot \sin \phi} = \frac{1}{2} C E_1^2 \pi \cot \phi.$$

$$\therefore \epsilon = 1 - \frac{w}{W} = 1 - \pi \cot \phi, \quad \text{as before.}$$

Referring to fig. 13, we can derive anew the value of the net efficiency. Curve 1 is the electromotive force, curve 2 is the current, in advance in phase by the angle ϕ , nearly 90° ; curve 3 is the power, the positive and larger loop being the work done on the condenser, and the negative and smaller

Fig. 12.

* *L'Eclairage Electrique*, Feb. 12, 1898.

Fig. 13.

D 2

loop being the work done by the condenser upon its discharge. Equation (1), p. 31, shows the area of this power-curve to consist of two terms, the coefficient of one containing $I \cos \phi$ and of the other $I \sin \phi$. Now $I \cos \phi$ is the component of the total current which is in phase with the E.M.F., and is represented by curve 5. $I \sin \phi$ is the condenser current, 90° ahead of the E.M.F., and is represented by curve 4. The power-curve for 4 is 6; its positive and negative loops are equal, and it is the power-curve for a perfect condenser. The power-curve for 5 is 7, and is the total work done, or the energy dissipated. One loop of 6 is energy stored, W , one loop of 7 is energy dissipated, w , and the ratio $\frac{w}{W}$ is the relative loss, or $1 - \frac{w}{W}$ is the net efficiency.

The area of one loop of 7 is

$$w = \frac{E_1 I_1 \cos \phi}{p} \left[\frac{pt}{2} - \frac{\sin 2pt}{4} \right]_0^\pi = \frac{E_1 I_1 \cos \phi \pi}{p \cdot 2} = \text{Energy Dissipated}$$

The area of one loop of 6 is

$$W = \frac{E_1 I_1 \sin \phi}{4p} \left[\cos 2pt \right]_0^{\pi \text{ for } pt} = \frac{E_1 I_1 \sin \phi}{4p} \cdot 2 = \text{Energy Stored.}$$

$$\therefore \epsilon = 1 - \frac{w}{W} = 1 - \pi \cot \phi, \quad \text{as before.}$$

If the equivalent resistance of the condenser is taken to be a series resistance then we have $E \cos \phi$ for the active E.M.F., $E \sin \phi$ for the condenser E.M.F., and the same result follows.

Example of the Resonance Method.

The condenser used was one which we had made ourselves, and consisted of paraffined paper and tinfoil. The paper was 12×17 centim. and .0038 centim. thick; the tinfoil was .0025 centim. thick, and its effective area was 10×15 centim. approximately. The paper and tinfoil were piled up dry and clamped between brass plates. It was then placed in melted paraffin and maintained for some hours at 100° to 150° C. This condenser then had a volume of about 300 cub. centim. and a capacity of about .8 microfarad.

The resonance-coil consisted of 3000 metres of No. 10 wire (B and S gauge, .259 centim. diameter) wound into a coil of 40 centim. internal diameter, 56 centim. external diameter, and 17 centim. axial length. Its resistance was about 10 ohms and its inductance 1.60 henrys. This coil was wound in ten

sections, so that by choosing different sections or combinations of sections, a wide range of inductance could be secured. In this particular case the entire coil was used. The frequency of the alternating electromotive force was varied by varying the speed of the generator, complete resonance being attained at a speed of 2175, for which the frequency is 145. The current was 1.20 amperes, the resistance $r_c + r_w$ was 9.82 ohms, $I^2(r_c + r_w) = 14.15$ watts. The wattmeter gave a deflexion of 188, corresponding to 37.6 watts. This leaves 23.45 watts for condenser loss, or $EI \cos \phi$.

E being 1808, $I = 1.20$, $EI = 2169$, and $\cos \phi = \frac{23.45}{2169} = .0108$;

$$\pi \cot \phi = 3.39 \text{ per cent.}, \quad \epsilon = 96.61 \text{ per cent.}$$

The quantity of the eddy-current loss in the coil does not of course appear. From subsequent experiments we became satisfied that it was large enough to cause a serious error in the above value of the condenser loss. Hence we shall not give any of the other values found using this coil. The results obtained over a range of from 400 to 2250 volts showed that the loss is sensibly proportional to the square of the electromotive force. This conclusion is not seriously affected by the presence of eddy-currents, since the latter are themselves proportional to the square of the E.M.F. and yet are not large enough to swamp the condenser loss.

We therefore wound up a coil of nearly 2000 metres No. 14 wire (B and S gauge, diam. .160 centim.), in 41 layers of 45 turns each, external diameter of the coil being 37 centim. The eddy-current loss in this coil is less, owing to the smaller diameter of the wire and the smaller quantity; a subsequent measurement by an independent method gave 3.2 per cent. as the increase of the effective resistance by the eddy-currents at a frequency of 120. At a lower frequency it would, of course, be less. Its use will therefore illustrate the method and give a fairly accurate value of the condenser efficiency.

Measurements on Beeswax and Rosin Condensers.

We give below a series of measurements on the efficiency of a set of commercial condensers made of tinfoil and paper, the latter being saturated with melted beeswax and rosin. We understand that they are piled up dry, as we have done with condensers made in our laboratory, and while immersed in the melted beeswax and rosin placed in a receiver from which the air is exhausted, to free them from air and moisture. With the details of the process we are not, however,

Table of Measurements on Beeswax and Rosin Condensers.

a.	b.	c.	d.	e.	f.	g.	h.	i.	j.	k.	l.	m.
	Tempera- ture of the Condenser.	Current through the Coil and Condenser = I.	Resistance of Coil and Wattmeter = $r_c + r_w$ (ohms).	$I^2(r_c + r_w)$ = W_1 (watts).	Wattmeter Readings.	Total energy in Watts. W.	Watts in Condenser. $W_2 =$ $W - W_1$.	Voltage on Condenser, E.	Apparent Watts, EL.	$\cos \phi =$ W_2 / EL .	Per cent. loss $= 100\pi \cot \phi$.	Net Efficiency in per cent. $= 100$ $(1 - \pi \cot \phi)$.
1	30	1.53	17.18	40.20	129½	66.70	26.50	910	1392	.0190	6.0	94.0
2	36	1.16	17.92	24.10	90	46.35	22.25	669.5	776.5	.0287	9.0	91.0
3	39	1.34	17.96	32.22	122	62.83	30.61	752	1007	.0304	9.5	90.5
4	39	1.27	17.96	28.97	108	55.62	26.65	712	904	.0295	9.3	90.7
5	47	1.31	18.22	31.26	108	55.62	24.36	731	958	.0253	8.0	92.0
6	49½	1.50	18.42	41.44	131	67.46	26.02	839	1259	.0207	6.5	93.5
7	59	0.80	18.34	11.74	173	89.10	77.36	433	346	.2230	72.0	28.0

acquainted, and cannot say whether it is something in the method of manufacture or the nature of the dielectric which makes the dissipation of energy so large; we presume, however, that it is the latter. There were six condensers joined together, each being a solid slab of about 11 centim. $\times$ 15 centim. $\times$ 1.5 centim., thus having a volume of about 250 cub. centim., and a capacity of one third of a microfarad.

The six slabs were placed on a table, joined together in parallel, and in series with the resonance-coil (which was at a distance from them and from the measuring instruments), loosely covered with a woollen cloth, and coil and condenser subjected to an alternating electromotive force of about 50 volts, and a frequency of 120. No effort was made to secure the maximum degree of resonance, and the voltage on the condensers was found to be about 900. In a short time the temperature of the condensers had risen to 30°, as indicated by a thermometer inserted between two of them, and the first set of readings was taken. The loss of energy in the condensers was greater than it had been at lower temperatures, and continued to increase as the temperature rose.

At the same time, owing to this increase in the equivalent resistance of the condenser, the resonance ratio decreased and the current and voltage on the condenser decreased. The loss at 36° C. is 50 per cent. greater than at 30° C., and is approaching a maximum. At 39° it is 9.5 per cent., and the fourth reading at sensibly the same temperature (but which doubtless was a little higher, at least in some of the condensers) showed a slightly less loss. At 47° C. the loss had decreased to 8.0 per cent., and at 49° 5 C. to 6.5 per cent., only two-thirds its maximum value. The condensers were not all at the same temperature, and the indicated temperatures are therefore not exact. But they show unmistakably a maximum value of the condenser loss, or energy converted into heat, at about 39° C., and beyond that a very considerable diminution. No further readings were taken until the condensers had risen several degrees, when it was suddenly noticed that one pair was hotter than the others and getting soft. The thermometer in a cooler pair registered 59°, but the warmest pair was considerably higher. The loss was astonishingly large, but the condenser had not broken down. Moreover, the "leakage current" had not greatly increased, for while 839 volts gave 1.50 amperes at 49° 5, 433 volts gave .80 ampere at 59° C. To be in exact proportion to the voltage the current should have been .774 instead of .80 at the higher temperature, a comparatively small difference.

To find so large a loss in commercial condensers of good

repute was a surprise to us. To find a well marked maximum as the temperature rose, beyond which the loss decreased as the beeswax and rosin composition softened was a second surprise. To find so large a loss as the last observation shows without the condensers giving way, and without any very large leakage current, was a third surprise.

In order therefore to verify these results by a totally different method, and to determine as accurately as possible the losses in some paraffin-paper condensers which we possessed which showed relatively very small heating effects, we built a special form of calorimeter, into which the condensers could be placed and the heat directly measured. The calorimeter was copied after the large respiration calorimeter which one of us designed for experiments under the patronage of the U. S. Government, and which is located at Wesleyan University. The description of the calorimeter and the results obtained with it are reserved for a subsequent communication. We will only add that they fully confirm the unexpected results obtained by the resonance method given above concerning the dissipation of energy in beeswax and rosin condensers.

Wesleyan University,
Middletown, Conn., July 1, 1898.

III. *Experiments with the Brush Discharge.*

By E. H. COOK, *D.Sc. (Lond.)*, Clifton Laboratory, Bristol*.

[Plate I.]

THE ordinary phenomena which accompany the brush-discharge are well-known, but in view of the recent extension of our knowledge of electric discharges in high vacua, it seemed desirable to study the subject a little more closely. The following experiments have been made with this object.

Most of the results have been obtained with an ordinary Wimshurst machine with 15-inch plates, but they have also been produced with the discharge from an induction-coil, as well as, though less readily, with a plate frictional machine.

In experiments requiring the production of the brush for a short period the machine was turned by hand, but where a long-continued effect was desired, motion was obtained by the use of one of Henrici's hot-air motors. The number of revolutions of the plates was counted by means of a tachometer, and the number of volts was taken as being about equal

* Communicated by the Author, having been read before the British Association at Bristol, 1898.