LXXII. On the mutual induction between two circular currents

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LXXII. *On the Mutual Induction between two Circular Currents.* By Dr. D. N. MALLIK, F.R.S.E.*.

1. **T**HE mutual induction (M) between two circular currents i, i' in any relative position to each other is

$$4\pi^2 i i' c_1 (1 - \mu^2) (1 - \mu'^2) \Sigma \left(\frac{c_1}{c_2} \right)^2 \frac{P_n \cos \theta}{n(n+1)} \frac{dP_n(\mu)}{d\mu} \frac{dP_n(\mu')}{d\mu'},$$

where c_1, c_2 are radii of the concentric spheres of which the circles are the cross-sections, $c_1 < c_2$ and θ = the inclination between the normals to the two circles.

2. If the circles are coplanar, then $\mu = 0, \mu' = 0, \theta = 0$.

Also

$$\frac{dP_{2m+1}(\mu)}{d\mu} \Big|_{\mu=0} = \left(\frac{-1}{2} \right)^m \cdot \frac{3 \cdot 5 \cdot 7 \cdot (2m+1)}{2 \cdot 4 \cdot 6 \dots 2m}.$$

If M_0 is the value of M in this case ($i = 1 = i'$), then

$$\begin{aligned} M_0 &= 4\pi^2 c_2 \left[\left(\frac{c_1}{c_2} \right)^2 \cdot \frac{1^2}{1 \cdot 2} + \frac{1}{3 \cdot 4} \left(\frac{c_1}{c_2} \right)^4 \cdot \frac{3^2}{2^2} + \dots \right] \\ &= 4\pi^2 c_2 \left[2 \cdot \frac{c_1^2}{c_2^2} \cdot \frac{1^2}{2^2} + \frac{4}{3} \cdot \left(\frac{c_1}{c_2} \right)^4 \cdot \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2} + \frac{6}{5} \cdot \left(\frac{c_1}{c_2} \right)^6 \cdot \frac{(1 \cdot 3 \cdot 5)^2}{(2 \cdot 4 \cdot 6)^2} + \dots \right]. \end{aligned}$$

But if V = gravitational potential of a wire of radius c_2 at a distance c_1 from the centre in the plane of the wire (density = 1),

$$V = 2\pi \left[1 + \frac{1^2}{2^2} \left(\frac{c_1}{c_2} \right)^2 + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2} \left(\frac{c_1}{c_2} \right)^4 + \dots \right];$$

$$\therefore 2\pi c_2 \frac{1}{c_1} \cdot \frac{\partial V}{\partial c_1} = \frac{\partial}{\partial c_1} \left(\frac{M_0}{c_1} \right). \quad \dots \dots (1)$$

$$\begin{aligned} \text{Now } V &= \int \frac{c_2 d\theta}{\sqrt{c_2^2 + c_1^2 - 2c_1c_2 \cos \theta}} \\ &= \frac{4c_2}{c_1 + c_2} \int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}} \\ &= 2(k' + 1) \log \frac{4}{k'} \left(1 + \frac{1^2}{2^2} k'^2 + \frac{1^2 \cdot 3^2}{2^2 \cdot 4^2} k'^4 + \dots \right) \\ &\quad - \frac{1}{2^2} k'^2 \cdot \frac{2}{1 \cdot 2} - \dots, \end{aligned}$$

$$\text{where } k^2 = \frac{4c_1c_2}{(c_1 + c_2)^2}$$

$$\text{and } k' = \frac{c_2 - c_1}{c_1 + c_2} \text{ (assumed to be small).}$$

* Communicated by the Author.

From (1) we get

$$2\pi \frac{k'+1}{1-k'} \frac{\partial V}{\partial k'} = \frac{\partial \left(\frac{M_0}{c_1} \right)}{\partial k'};$$

$\therefore$ we have

$$A + \frac{M_0}{c_1} = 2\pi \left[\frac{k'+1}{1-k'} V - \int 2V \frac{\partial k'}{(1-k')^2} \right],$$

where A is the constant of integration.

Rejecting k'^2 and higher terms in the expression for V ,

$$\begin{aligned} A + \frac{M_0}{c_1} &= 2\pi \frac{c_2}{c_1 c_1 + c_2} \log \frac{4}{k'} - 8\pi \int \left(\log \frac{4}{k'} \right) \frac{k'+1}{(1-k')^2} dk' \\ &= \frac{8\pi c_2^2}{c_1(c_1+c_2)} \log \frac{4}{k'} + 8\pi \int \log \frac{4}{k'} \cdot (1+3k') dk' \\ &= 8\pi \frac{c_2^2}{c_1(c_1+c_2)} \log \frac{4}{k'} + 8\pi \left[k' \log \frac{4}{k'} + k' \right. \\ &\quad \left. + \frac{3}{2} k'^2 \log \frac{4}{k'} + \frac{3}{4} k'^2 \right]. \end{aligned}$$

To determine the constant, we observe that the quantity independent of c_1 on the right-hand side is 8π . Hence, in view of the actual value of M_0 , we get $A=8\pi$.

Accordingly

$$\frac{M}{c_1} = 8\pi \left[\frac{c_2^2}{c_1+c_2} \log \frac{4}{k'} - 1 \right] + 8\pi \left[k' \log \frac{4}{k'} + k' + \dots \right].$$

It will be seen that if we put $c_2-c_1=x$, and reject powers about x^2 , the first two terms reduce to $4\pi \left[c_2 \log \frac{4}{k_1} - 2 \right]$, which agrees with Maxwell's result.

3. In the case of two parallel wires at a small distance apart, $\mu=0$, and we have

$$\begin{aligned} M &= 4\pi^2 c_1 \Sigma \left(\frac{c_1}{c_2} \right)^2 \frac{1}{n(n+1)} \left(\frac{dP_n}{d\mu} \right)_0 \left[\left(\frac{dP_n}{d\mu} \right)_0 \right. \\ &\quad \left. + \frac{\partial}{\partial \mu'} \left[(1-\mu'^2) \frac{dP_n}{d\mu'} \right] d\mu' + \frac{1}{2!} \frac{\partial^2}{\partial \mu'^2} \left[(1-\mu'^2) \frac{dP_n}{d\mu'} \right] d\mu'^2 + \dots \right] \\ &= M_0 - 4\pi^2 c_1 \Sigma \left(\frac{c_1}{c_2} \right)^n \cdot \left(\frac{dP_n}{d\mu_0} \right) \left[\frac{1}{2!} \left(\frac{dP_n}{d\mu'} \right)_0 d\mu'^2 \right. \\ &\quad \left. + \frac{1}{4!} \frac{d^3 P_n}{d\mu_0^3} d\mu'^4 + \dots \right], \end{aligned}$$

since

$$\left(\frac{dP_n}{d\mu} P_n \right)_0 = 0,$$

$$\left(\frac{dP_n}{d\mu} \frac{d^2 P_n}{d\mu^2} \right)_0 = 0, \text{ \&c.}$$

Moreover, $d\mu' = \frac{r}{c_1},$

where r is the shortest distance between the wires.

The expression for μ can accordingly be expressed in the following symbolical form:—

We have

$$\left[\int_1^{\rho} e^{\rho \frac{d}{d\mu}} d\rho \right] P_n(\mu) = \left(\rho + \frac{\rho^2}{2!} \frac{d}{d\mu} + \dots \right) P_n(\mu)$$

But
$$\int_1^{\rho} e^{\rho \frac{d}{d\mu}} P_n(\mu) d\rho = \int_1^{\rho} P_n(\mu + \rho) d\rho;$$

i. e.,
$$\int_1^{\rho} P_n(\rho) d\rho = \left(\rho + \frac{\rho^2}{2!} \frac{d}{d\mu_0} + \dots \right) P_n(0)$$

or
$$\frac{1}{n(n+1)} \cdot \rho^2 \frac{dP_n(\rho)}{d\rho} = \left[P_n(0) + \frac{\rho^2}{2!} \frac{dP_n}{d\mu_0} + \dots \right].$$

Hence

$$M = M_0 - 4\pi^2 c_1 \Sigma \left(\frac{c_1}{c_2} \right)^n \frac{1}{n(n+1)} \frac{dP_n}{d\mu_0} \cdot \frac{r^2}{c_1^2} \cdot \frac{dP_n \left(\frac{r}{c_1} \right)}{d \left(\frac{r}{c_1} \right)},$$

since
$$\left(P_n \frac{dP_n}{d\mu} \right)_0 = 0, \text{ \&c.}$$

LXXIII. *Molecular Thermodynamics. I.* By BERNARD A. M. CAVANAGH, B.A., Balliol College, Oxford*.

I. THE GENERAL CONDENSED PHASE.

THE treatment of the thermodynamics of solutions given by Planck † for the simple limiting case of extreme dilution appears to be capable of extension to the general case. Planck himself, indeed, suggested ‡ the expansion of the specific Total-Energy and Volume of the solution, in integral powers of the various concentrations. But for general applicability this assumption of integral powers would appear to be quite unnecessarily arbitrary and narrow.

We have, in general, for the Total-Energy and Volume per unit quantity of solution, functions F_1 and F_2 (say) of the various concentrations, which will involve parameters dependent on Temperature and Pressure.

Now it would seem that the *minimum* assumption required

* Communicated by Dr. J. W. Nicholson, F.R.S.

† 'Thermodynamics' (Trans. Ogg) 1917, Chap. v.

‡ *Ibid.* p. 225.