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UNIPOLAR INDUCTION AND ELECTRON THEORY.

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THE most simply constructed apparatus for showing unipolar, homopolar or acyclic induction of electromotive force is a cylindrical permanent bar magnet spinning about its axis with a stationary loop of wire terminating in brushes which make contact with the rotating magnet at two points, one nearer the end of the magnet than the other. The old unipolar question is as to the seat of the electromotive force, whether in the moving magnet or in the stationary wire; or, as sometimes put from the standpoint of the "cutting of lines of force" view, does the magnetic field rotate with the magnet and by cutting the stationary loop generate an electromotive force in it; or does it remain stationary and cut the moving magnet? In terms of the electron theory the question is whether electrons in the conducting material of the magnet, and rotating with the magnet, are acted on by a force arising from this rotation or whether it is the electrons in the stationary loop of wire that are immediately influenced by the spinning of the magnet. Of late certain questions involving the theory of relativity have also been brought into discussions. The most recent articles on the subject are by Barnett,[1] Kennard[2] and Howe.[3]

While a very simply constructed apparatus for showing unipolar induction results from using a cylindrical permanent magnet, a permanent magnet is a complex thing, and for easier analysis we may well

[1] S. J. Barnett, Phys. Rev., 35, 1912, p. 324 (2), 2, 1913, p. 323; Phys. Zeitz., 14, 1913, p. 251.

[2] E. H. Kennard, Phys. Rev. (2), 1, 1913, p. 355; (2), 7, 1916, p. 399. Another article by Dr. Kennard describing experimental results like those described in this paper, has appeared in the Phil. Mag. for February, 1917, but the theoretical treatment of Mr. Kennard is so different from the simple method of treatment herein attempted that I venture to publish this as it stands and as it was presented before the American Physical Society in October, 1916. G. B. P.

[3] G. Howe, Electrician, LXXVI., p. 169, Nov. 5, 1915, and subsequent discussion.

substitute a long helical solenoid carrying a steady current. In the end the assimilation of a permanent magnct to a solenoid will be fairly obvious. Making use of such a rotating solenoid a unipolar induction current can be obtained by having a conducting disc fastened coaxially to the solenoid, and letting brushes from a stationary conducting loop bear on this rotating disc at different distances from the axis of rotation. The question then becomes in essence this: In which case will an electron near a rotating long solenoid, with steady current through it, experience a radial force, in case the electron is rotating with the solenoid as if rigidly connected with it, or in case the electron is stationary?

Barnett, by studying open rather than closed circuits, that is, by observing the displacement of charges on conductors in the field of a rotating solenoid or magnet, made the first direct experimental attack on the question, which has been followed up by Kennard. Barnett used a condenser of concentric conducting cylinders with the outer cylinder, closed at the ends, held coaxially in a solenoid which could be magnetized and rotated. He found that the inside cylinder of the condenser did not become charged if while the magnetized solenoid was rotating a radial conductor made connection for a time between the inner and outer cylinders of the condenser. This he proved by breaking the connection between the inner and outer cylinders of the condenser while the magnetized solenoid was rotating, stopping the current through the solenoid, or bringing the solenoid to rest, and then testing the inner cylinder for charge by connecting to an electrometer. Barnett varied the experiment by arranging the cylindrical condenser coaxially with two large round electromagnets, which with their magnetizing coils were rotated in place of the solenoid. The result was the same as when the solenoid was used, the inner cylinder did not become charged when a radial connection was made between it and the outer cylinder. Both these experiments therefore showed that when the system which produces a magnetic field symmetrical about an axis is rotating about that axis, it does not establish an E.M.F. in a stationary conductor such as the radial connecting wire between the two cylinders used in the experiment. If the solenoid or magnet used in the experiment were kept stationary and the condenser with the radial connection between the cylinders were rotated, the inner cylinder would undoubtedly become charged. On this point no one has raised any question, but Mr. Kennard has gone so far with the experiment as to obtain observations showing the existence of the charge on the inner cylinder in this case.

In each of the two cases just cited the *relative* motion between the solenoid and the condenser with the radial connection is just the same, consequently the different results prove that the generation of an electromotive force in a conductor is not simply a question of the *relative* motion of the conductor and the solenoid which furnishes the magnetic field. There is indeed no good reason for expecting the observed effect of the electromotive force to depend simply on the relative motion of the conductor and solenoid, for the observer with his electrometer and other apparatus is an equally important third system to be considered in specifying the motions, and so there is no conflict with relativity theory.

Experiments.

There is still another variation of the experiment, namely, to test whether or not the inner cylinder becomes charged when the cylindrical condenser with radial connection is rigidly connected with the solenoid and the whole system rotated. I have recently completed an experiment begun some time ago which confirms Professor Barnett's negative result with a stationary condenser and rotating solenoid, and confirms and gives more exact results on the experiment of Mr. Kennard with both solenoid and condenser rotating. The apparatus used was the following: a solenoid A 29 cm. inside diameter, 60 cm. long, 55 turns per cm. of length, mounted to rotate about a vertical axis at speeds up to 1000 R.P.M.; a cylindrical condenser BC of sheet copper mounted coaxially with the solenoid, outer cylinder B of condenser 25 cm. diam., 60 cm. length, with closed ends, except that shielded connection to the electrometer ran through a central hole in top end; inner cylinder C 10 cm. diam., 33 cm. length, supported by hard rubber blocks; a copper strip DE, running diametrically across the inner cylinder and out nearly to the outer cylinder, by means of which the inner cylinder could be connected at will with either the outer cylinder, by pushing down the rod EF, or connected with the electrometer by pushing down the electrometer connection DG. The electrometer used was one made for this purpose with small quadrants and a very light silvered mica needle, sensitiveness $.87 \times 10^{-4}$ volts per division. The capacity of the shielded wire leading to the electrometer, which was placed across the room from the rotating apparatus, was considerable, and so the capacity of the cylindrical condenser was only .125 the capacity of the whole system when the electrometer was connected.

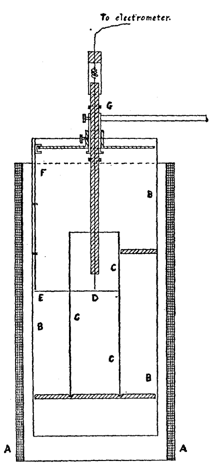

Fig. 1.

1. *Experiment with Cylindrical Condenser Attached to Solenoid and*

Rotating with It.—The solenoid was kept rotating steadily at a speed of about 900 R.P.M. With no current in the solenoid the connection was made between the two rotating cylinders by pushing down the rod BC to touch the strip AB, and the whole was grounded through the central wire AD. Then in turn the current was switched on the solenoid; the central connection AD raised; the rod BC raised to break connection between the outer cylinder and the now insulated inner cylinder; the current switched off the solenoid; connection of the inner cylinder with the electrometer made by lowering the central connection AD; and the electrometer deflection observed. The same cycle of operations was then performed with the solenoid current reversed, and finally as a check the same cycle, but without any solenoid current. The results in a set of 10 measurements varied, for the double deflection, solenoid current direct and reversed, from 47 to 50, mean 48.6, electrometer scale divisions, or .00424 volt for double deflection, .00212 volt for deflection from one cycle of operations. As the capacity of the inner cylinder was only .125 that of the whole system when connected with electrometer, the potential to which the inner cylinder was charged by rotating in the field of the solenoid was .0170 volt. That no appreciable deflection of the electrometer was obtained when the cycle of connections was performed with no current in the solenoid simply proved that the inner cylinder and electrometer connections were well shielded electrostatically. In all the experiments the outer cylinder was constantly earthed.

To determine the E.M.F. that might be expected in a conductor, such as the strip AB, rotating at the speed used in the field of the solenoid with the current used, a copper brush was held against the outer cylinder near the level of the strip AB connecting the two cylinders and with the electrometer the potential difference was measured between this brush and the central connection AD. This was .0206 volt. Assuming the field in the solenoid at this level to be uniform and subtracting the E.M.F. induced in the part of the strip AB inside the inner cylinder, there is left $\frac{3}{4}$ of $.0206 = .0161$ as the E.M.F. in the part of the strip between the two cylinders, as against the .017 volt measured as the potential to which the inner cylinder was charged. Allowance for non-uniformity of field in the solenoid would bring a still better agreement for the two results.

2. *Experiment with Cylindrical Condenser and Connections Stationary, Solenoid Rotating.*—Confirming Barnett's result, on carrying out the cycle of connections described above with the cylindrical condenser stationary, the electrometer indicated no charge at all on the inner cylinder.

The answer given by experiment to the question of the seat of the electromotive force in unipolar induction is therefore that it is in the moving conductor and that without a moving conductor there is no such E.M.F., regardless of whether the system which produces the magnetic field is rotating or not.

Theory of Unipolar Induction.

The same answer to the question, without need of recourse to such open circuit experiments as described, is given by even the crudest electron theory of conduction. For on an electron theory the current in a stationary solenoid would be viewed as a steady circular transport of electrons around the solenoid, and the rotation of the solenoid would amount simply to superposing a similar steady circular transport of all the electrons, positive and negative, in the material of the solenoid. But a steady current in a fixed circuit certainly does not affect a neighboring stationary charge or electron. The solenoid with its current, whether stationary or rotating, can therefore not have any action on a neighboring stationary electron; but the current in the solenoid, through its magnetic field, does act on neighboring moving electrons, hence in a unipolar circuit it must necessarily be the moving electrons, *i. e.*, the electrons of the moving conductor, which are acted upon by the E.M.F., whether the solenoid be spinning or at rest.

In the early days of electron theory Sir Joseph Larmor[1] stated the same result as an application of his theory. In the article referred to above Professor Howe arrived at the same conclusion by reasoning based wholly on the fact that the mechanical force on an element of length of wire carrying a current across a magnetic field does not depend on the motion of the magnetic field or the source of the field, but only on the magnitude and direction of the field at the element considered. It is to be remarked, however, that the accepted facts as to the mechanical force acting on a conductor in a magnetic field do not of themselves entirely justify Professor Howe's argument. For let us imagine a horseshoe magnet carried along with its poles either side of a long straight wire through which a current runs. Let us assume for the sake of simplicity that conduction in the wire is by convection of the negative electrons only and that the magnet is moving with the same speed as these electrons. We might claim on the one hand that the transverse force on the wire in the field does come from the negative electrons moving (with respect to the observer) across the magnetic field as it exists at the instant, without regard to the motion of the magnet that produces the field. Or

[1] Larmor, Royal Society Transactions, 1895*A*, p. 727.

on the other hand, conceiving the relative motion of the electrons in the conductor with respect to the magnet to be the cause of the force on the conductor, we might very well claim that when the magnet is stationary the force on the conductor comes from the force on the negative current electrons arising from their motion with respect to the magnet, and that when the magnet moves along as fast as the negative electrons there is no longer any force on the negative electrons of the current, but that there is and should be just the same force on the conductor, arising from a force acting on the positive electrons of the conductor, which now have relative to the magnet the same velocity as the negative electrons in the first case, except in the opposite direction. The real trouble with adopting the second line of argument, which would be compatible with the localization of the electromotive force in the stationary part of a unipolar induction circuit, is that we should be adopting a too naïve relativity principle, which misleads by not taking account of the fact that we are supposing the force on the wire to be that manifest to the observer, who is not at rest with respect to either the magnet or the negative electrons in the current. Such a relativity theory would for example teach that the force between two electrons moving abreast with identical velocities in parallel lines is, to a stationary observer, just the same as if both electrons were stationary. This conclusion is at variance with the Lorentz-Einstein relativity theory and with every theory of the electrodynamics of moving charges. According to accepted relativity theory two electrons stationary with respect to the observer have only the electrostatic repulsion, but if they are moving with respect to the observer the force between them appears to the observer to be something different from the electrostatic force. Nothing in the experiments on unipolar induction is at all at variance with the Lorentz-Einstein relativity theory.

The Lorentz electron theory may be readily applied to a more complete analysis of the unipolar problem and connected questions, and may make clearer certain points.

The two fundamental phenomena of electromagnetic induction may be given the following expression in terms of the electron theory:—

(*a*) A force may be exerted on a *stationary* electron by suitable motions or variations of magnets or currents in the vicinity; that is, electronically interpreted, by suitable motions of electrons in the vicinity.

(*b*) A force acts in general on an electron *moving* in a magnetic field, which force is perpendicular both to the instantaneous magnetic field intensity and to the velocity of the electron.

It is the essence of the Lorentz theory that the phenomena (*a*), (*b*) and (c, electrostatic phenomena) are assumed to be independent of and superposable upon one another. Therefore the total force on any electron is the vector sum of three parts: (a) the force arising from the velocities and accelerations of neighboring electrons, which force is independent of the motion of the electron under consideration, + (b) the force arising from the motion of the electron under consideration in a magnetic field, + (c) the force arising from the electrostatic action of neighboring electrons.

By well-known mathematical development from the Maxwell field equations in the Lorentz form, the quantitative expression for the effect (a) of moving charges on a given electron of charge e comes out $e/c(-(d\boldsymbol{A}/dt))$, $\boldsymbol{A}$ being the vector potential at the momentary position of e; for the effect (b) of the motion of e with velocity $\boldsymbol{v}$ in a magnetic field $\boldsymbol{H}$ it is $e/c[\boldsymbol{v} \times \boldsymbol{H}]$; for the electrostatic effect $-e$ grad ϕ, ϕ being the electrostatic potential. Hence

$$\text{force on electron} = -\frac{e}{c}\frac{d\boldsymbol{A}}{dt} + \frac{e}{c}[\boldsymbol{v} \times \boldsymbol{H}] - e \operatorname{grad} \phi.$$

Applying this to finding the force on an electron in the vicinity of a spinning solenoid, we may at once conclude that the first term, which is a force not dependent on the velocity of the electron under consideration, vanishes, for, as reasoned above, the transport of electrons in a rotating solenoid merely adds to the transport of electrons in the current when the solenoid is stationary a similar circular transport, by the rotation, of equal numbers of positive and negative electrons, and so the whole effect is just that of the current in the stationary solenoid, which is nil on a stationary electron. The second term obviously vanishes for stationary electrons, and we may also suppose the third term, referring to the static field, to vanish. Hence, there is no force on a stationary electron, therefore no E.M.F. in stationary conductors in the vicinity of a steadily spinning solenoid carrying a constant current. On the other hand, since the second term does not vanish when the electron is moving, there is an E.M.F. on electrons in moving conductors, which is easily seen to be quantitatively just what would be computed on the "rate of cutting magnetic lines" scheme, supposing the lines of the magnetic field to remain stationary with the conductors rotating.

Although the conclusion that no electromotive force is set up in stationary parts of the circuit in the unipolar induction experiment follows so immediately from electron theory, many well-trained physicists and engineers at first are inclined to disagree with the conclusion. They are accustomed to the experience that in general the motion of the source of a magnetic field sets up an electromotive force in neighboring conductors, and they have not examined the rate of variation of the vector potential, which is the only function adequate to express the electromotive force at a point in a stationary conductor. It is not sufficient to know the magnetic field intensity at the point and its variation in time and space. The analysis must be carried back to an expression of the effect at the given point of each neighboring moving element of charge or electron, that is the vector potential

$$A = \frac{1}{4\pi c} \text{vector } \Sigma \frac{deu}{r},$$

the vector summation being for all the moving charge (summation for all moving electrons), r the distance from the point for which the vector potential is calculated to the position where the element of charge was at time r/c earlier, and u the velocity of the element of charge at that time.

Since the vector potential at a point is a function of the positions and velocities of all the neighboring electrons, the reason why there is no rate of change of the vector potential in the neighborhood of the rotating solenoid or magnet used in a unipolar induction machine is that, statistically considered, the configuration of positions and velocities of the electrons of the solenoid or magnet remains constant. As electrons of the solenoid or magnet move out one side of a stationary element of volume as many more move with the same velocity into the same element of volume.

An example may be cited of a case in which the magnetic field intensity and its time and space variations are known at a point, yet from these nothing can be said as to the electromotive force in a conductor (or the force on an electron) at that point. Imagine two long solenoids one inside the other with axes parallel and currents through them so their magnetic fields just neutralize each other inside the inner one. Now suppose a conductor inside the inner solenoid and suppose this solenoid is moved a little transversely. Where the conductor is the magnetic field intensity is constantly zero, and its time variation and its space variation are therefore also zero, hence these give us no indication of any probability of a force on the electrons of the conductor as the inner solenoid is being displaced transversely. Consideration of the rate of variation of vector potential in this case determines at once that it is not zero as the solenoid is moved transversely and that the E.M.F. in the conductor is just the same as though the outer solenoid, which neutralizes the magnetic field, were not present. Of course we may adhere to the "cutting of line of force" computation of the electromotive force if we say that we must treat the fields of the two solenoids as entirely separate and distinct in their effects and say that when the solenoid is moved transversely all its lines of force move with it, even though the experimental facts of unipolar induction preclude our saying that the lines move with the solenoid when it rotates. If, however, we once begin this analysis of a magnetic field at a point into discrete constituents, we should logically continue it down to the magnetic fields of the individual electrons, which amounts to just the same thing as the vector potential analysis.

A still more familiar case in which we have an induced E.M.F. in a region where the magnetic field is constantly zero is that of a point near a transformer, say with a toroidal core and closely wound primary through which an alternating current flows. The moving electrons in the primary coil and the core give a varying vector potential at points in the surrounding space, although there is never any magnetic field there. The usual explanation on the "cutting lines of force" basis is to say the lines of force spring out and in, but if they do so, and yet have at no moment a density different from zero at points outside the core and winding, where there is no magnetic field, they must be springing in and out with infinite velocity; which makes an unsatisfactory representation.

One more case of unipolar induction may be referred to. Suppose an insulated copper wire runs through a hole along the axis of a cylindrical bar magnet and out through a radial hole to a collector ring near the middle of the magnet. If the magnet be set in rotation and a stationary loop of wire have its ends brought in contact with the axial end of the copper wire and the collector ring respectively a current will flow around the circuit. Neglecting for the sake of the argument the small magnetic field in the axial and radial parts of the hole in which the copper wire lies, we may say that in this arrangement there is no conductor moving across a magnetic field, so the induction of E.M.F. is not to be explained as in the unipolar induction cases already discussed. But here the vector potential is varying at the position momentarily occupied by an electron in the radial copper wire, in a manner quite analogous to the variation at a point near a solenoid in transverse motion, and so again the seat of the E.M.F. is in the moving wire, although it is now to be referred to the first term in the Lorentz expression for force on an electron, instead of to the second. Of course the vector potential at all points in the stationary part of this circuit is in general varying on account of the asymmetry of configuration of electrons and velocities resulting from the radial hole in the magnet, but this variation integrates out for a whole turn of the magnet. The shift in this example from the second term of the Lorentz expression for force to the first term as the cause of the unipolar or acyclic electromotive force is suggestive of the close relation between the two terms, of, in fact, the relative nature of the two. An observer stationed on a transversely moving solenoid observing an E.M.F. in a "stationary" but to him apparently moving conductor, would attribute the electromotive force to motion of the conductor in a magnetic field, the second term in the Lorentz expression, while a stationary observer seeing the solenoid move would refer the separation of the charges in the stationary conductor to the variation of the vector potential with the motion of the solenoid; which is to say what is now generally accepted, that the quantities involved in all electromagnetic induction are the positions and motions of the electrons relative to the observer.

Summary.—Experiments confirm the results of Barnett and of Kennard showing that in unipolar induction the "seat of the electromotive force" is in a moving conductor and is entirely independent of the rotation of the magnetic field.

The facts of unipolar induction are in accord with the theory of relativity. The theory of unipolar induction emphasizes the importance of electron theory and the vector potential function in the discussion of such questions.