



LXXVI. Analytical development of Fresnel's optical theory of crystals

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exposed to the influence of the sun under water, the disengagement of the oxygen from the divided air-ducts soon ceased; it began however anew if the leaves were again placed in their natural position.

Dutrochet finally infers from his various experiments that plants at night absorb the oxygen from the air, and that this is only an auxiliary respiration, while the true process of respiration of vegetables consists in the disengagement and diffusion of the oxygen in the interior of the plant caused by the solar light.

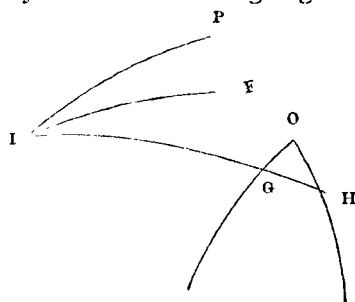
Morren*, who made several experiments in the botanical garden at Louvaine on the respiration of plants, observed on the 18th of May, during the great solar eclipse, that the respiration of the green parts of plants, that is the expiration of oxygen, entirely ceased at that time. We can observe something very similar to this on very warm summer days, when for instance the respiration of oxygen is very considerable from the action of solar light, and the sun all at once disappears under dark clouds; I have noticed several times how soon the disengagement of gas bubbles diminished, and at last ceased more or less completely.

[To be continued.]

LXXVI. *Analytical Development of Fresnel's Optical Theory of Crystals.* By J. J. SYLVESTER, Member of St. John's College, Cambridge.

[Continued from p. 469.]

Cor.—Hence we may reduce the discovery of the two fronts into which a plane front is refracted on entering a crystal to the following trigonometrical problem.



Let a sphere be described about any point in the line in which the air front intersects the plane of incidence. Let the great \odot^e P I denote the latter plane, I F the former, O A, O C also great circles, the planes of single velocity. Suppose I G H to be one of the refracted fronts inter-

secting O A, O C in G and H, then

$$\frac{(a^2 + c^2) - (a^2 - c^2) \cos(G + H)}{2 (\text{vel. in air})^2} = \frac{\sin(P I F)^2}{(\sin P I G H)^2}.$$

* *L'Institut* 1836, p. 416.

The double sign will give rise to two positions of the refracted front I G H.

The propositions which follow are perhaps more curious than immediately useful.

PROPOSITION 10.

To determine the portion of a line of vibration in terms of the two velocities of its corresponding front.

We have here to determine the quantities $\frac{y_l}{x_l} \frac{z_l}{x_l}$ (of Prop. 1) in terms of v_l, v_{ll} , or on putting $x_l^2 + y_l^2 + z_l^2 = 1$, $x_l y_l z_l$ are to be found in terms of v_l, v_{ll}

$$\begin{aligned} \text{By Prop. (3.) } x_l : y_l : z_l &:: \frac{l}{a^2 - v_l^2} : \frac{m}{b^2 - v_l^2} : \frac{n}{c^2 - v_l^2} \\ \text{and by Prop. (5.) } l^2 : m^2 : n^2 &:: \frac{l^2}{b^2 - c^2} \cdot \frac{a^2 - v_l^2}{a^2 - v_l^2} \cdot \frac{a^2 - v_{ll}^2}{b^2 - v_{ll}^2} \\ &:: \frac{l^2}{c^2 - a^2} \cdot \frac{b^2 - v_l^2}{b^2 - v_l^2} \cdot \frac{b^2 - v_{ll}^2}{c^2 - v_{ll}^2} \\ &:: \frac{l^2}{a^2 - b^2} \cdot \frac{c^2 - v_l^2}{c^2 - v_l^2} \cdot \frac{c^2 - v_{ll}^2}{a^2 - v_{ll}^2} \\ \therefore x_l^2 &: y_l^2 : z_l^2 \\ :: (b^2 - c^2) \frac{a^2 - v_{ll}^2}{a^2 - v_l^2} (c^2 - a^2) \frac{b^2 - v_{ll}^2}{b^2 - v_l^2} &: (a^2 - b^2) \frac{c^2 - v_{ll}^2}{c^2 - v_l^2} \end{aligned}$$

Let α, β, γ be the \angle s made by the given line of vibration with the elastic axes, then

$$\begin{aligned} (\cos \alpha)^2 &= \frac{x_l^2}{x_l^2 + y_l^2 + z_l^2} \\ &= \frac{(b^2 - c^2) (a^2 - v_{ll}^2) (b^2 - v_l^2) (c^2 - v_l^2)}{\text{divided by}} \\ &\quad \left\{ \begin{aligned} &(b^2 - c^2) (a^2 - v_{ll}^2) (b^2 - v_l^2) (c^2 - v_l^2) \\ &+ (c^2 - a^2) (b^2 - v_{ll}^2) (c^2 - v_l^2) (a^2 - v_l^2) \\ &+ (a^2 - b^2) (c^2 - v_l^2) (a^2 - v_l^2) (b^2 - v_l^2) \end{aligned} \right\} \end{aligned}$$

and therefore

$$= \frac{(b^2 - c^2) (a^2 - v_{ll}^2) (b^2 - v_l^2) (c^2 - v_l^2)}{(v_l^2 - v_{ll}^2) (a^2 - b^2) (b^2 - c^2) (c^2 - a^2)}$$

(where it is to be observed that the reduction of the denominator is simply the effect of a vast heap of terms disappearing under the influence of contact with the magic circuit $a^2 - b^2$,

$\overline{b^2 - c^2}, \overline{c^2 - a^2}$, a simpler instance of which was seen in proposition 5.)

In fact the coefficient of $v^4 \cdot v^2$

$$= (b^2 - c^2) + (c^2 - a^2) + (a^2 - b^2)$$

$$= 0$$

that of $v_l^2 \cdot v_{ll}^2 = (c^2 + b^2) \cdot (c^2 - b^2)$

$$+ (a^2 + c^2) \cdot (a^2 - c^2)$$

$$+ (b^2 + a^2) \cdot (b^2 - a^2)$$

$$= (c^4 - b^4) + (a^4 - c^4) + (b^4 - a^4)$$

$$= 0.$$

The term in which neither v_l nor v_{ll} enters

$$= a^2 b^2 c^2 \{ (b^2 - c^2) + (c^2 - a^2) + (a^2 - b^2) \}$$

$$= 0.$$

The coefficient of

$$- v_l^2 = a^2 \cdot (b^4 - b^4) + b^2 \cdot (c^4 - a^4) + c^2 \cdot (a^4 - b^4)$$

and that of

$$v_{ll}^2 = b^2 c^2 \cdot (c^2 - b^2) + c^2 a^2 \cdot (a^2 - c^2) + a^2 b^2 \cdot (b^2 - a^2)$$

each of which $= (a^3 - b^2) \cdot (b^2 - c^2) \cdot (c^2 - a^2)$

Hence,

$$(\cos \alpha)^2 = \frac{v_l^2 - b^2}{v_l^2 - v_{ll}^2} \cdot \frac{(a^2 - v_{ll}^2) (c^2 - v_l^2)}{(a^2 - b^2) (a^2 - c^2)},$$

in like manner $(\cos \beta)^2 = \&c.$

$$\text{and } (\cos \gamma)^2 = \frac{v_l^2 - b^2}{v_l^2 - v_{ll}^2} \cdot \frac{(c^2 - v_{ll}^2) (a^2 - v_l^2)}{(c^2 - b^2) (c^2 - a^2)}.$$

PROPOSITION II.

$\epsilon_l, \epsilon_{ll}$ being the \angle s between any line of vibration and the optic axes, required the velocity due to that line in terms of $\epsilon_l, \epsilon_{ll}$.

By analytical geometry,

$$\cos \epsilon_l = \cos \alpha \cdot \cos \phi_l + \cos \gamma \cdot \cos \psi_l$$

$$\cos \epsilon_{ll} = \cos \alpha \cdot \cos \phi_l - \cos \gamma \cdot \cos \psi_l$$

$$\therefore \cos \epsilon_l \cdot \cos \epsilon_{ll} = (\cos \alpha)^2 (\cos \phi_l)^2 - (\cos \gamma)^2 (\cos \psi_l)^2$$

$$= \frac{v_l^2 - b^2}{v_l^2 - v_{ll}^2} \cdot \left\{ \frac{a^2 - v_{ll}^2 \cdot c^2 - v_l^2 - c^2 - v_{ll}^2 \cdot a^2 - v_l^2}{(a^2 - c^2)^2} \right\}$$

$$= \frac{v_l^2 - b^2}{v_l^2 - v_{ll}^2} \cdot \frac{(a^2 - c^2) (v_{ll}^2 - v_l^2)}{(a^2 - c^2)^2}$$

$$= \frac{b^2 - v_l^2}{a^2 - c^2}$$

Hence $v_l^2 = b^2 - \overline{a^2 - c^2} \cdot \cos \varepsilon_l \cos \varepsilon_{ll}$

and in like manner, for the *conjugate* line of vibration.

$$v_{ll}^2 = b^2 - (a^2 - c^2) \cos \varepsilon_l' \cdot \cos \varepsilon_{ll}'.$$

PROPOSITION 12.

To find $\varepsilon_l \varepsilon_{ll}$ in terms of $\iota_l \iota_{ll}$

$$\begin{aligned} & (\cos \varepsilon_l)^2 + (\cos \varepsilon_{ll})^2 \\ &= 2 (\cos \alpha)^2 \cdot (\cos \phi_l)^2 + (2 \cos \gamma)^2 \cdot (\cos \psi_l)^2 \\ &= \frac{v_l^2 - b^2}{v_l^2 - v_{ll}^2} 2 \left\{ \frac{a^2 - v_{ll}^2 \cdot c^2 - v_l^2 + c^2 - v_{ll}^2 \cdot a^2 - v_l^2}{(a^2 - c^2)^2} \right\} \end{aligned}$$

but by Prop. (9)

$$v_l^2 = a^2 \left(\sin \frac{\iota_l - \iota}{2} \right)^2 + c^2 \left(\cos \frac{\iota_l - \iota}{2} \right)^2$$

$$v_{ll}^2 = a^2 \left(\sin \frac{\iota_l + \iota}{2} \right)^2 + c^2 \left(\cos \frac{\iota_l + \iota}{2} \right)^2$$

$$\therefore (\cos \varepsilon_l)^2 + (\cos \varepsilon_{ll})^2 = \frac{b^2 - v_l^2}{(a^2 - c^2) \sin \iota_l \cdot \sin \iota_{ll}}$$

multiplied by

$$\begin{aligned} & 2 \left\{ \frac{a^2 - c^2}{(a^2 - c^2)^2} \left(\left(\cos \frac{\iota_l + \iota_{ll}}{2} \right)^2 \left(\sin \frac{\iota_l - \iota_{ll}}{2} \right)^2 + \left(\cos \frac{\iota_l - \iota_{ll}}{2} \right)^2 \left(\sin \frac{\iota_l + \iota_{ll}}{2} \right)^2 \right) \right\} \\ &= \frac{b^2 - v_l^2}{(a^2 - c^2) \sin \iota_l \cdot \sin \iota_{ll}} \left\{ (\sin \iota_l)^2 + (\sin \iota_{ll})^2 \right\} \end{aligned}$$

and we have seen that

$$\cos \varepsilon_l \cos \varepsilon_{ll} = \frac{b^2 - v_l^2}{a^2 - c^2}$$

$$\therefore \cos \varepsilon_l + \cos \varepsilon_{ll} = \sqrt{\left(\frac{b^2 - v_l^2}{a^2 - c^2} \right)} \cdot \frac{\sin \iota_l + \sin \iota_{ll}}{\sqrt{\sin \iota_l \cdot \sin \iota_{ll}}}$$

$$\cos \varepsilon_l - \cos \varepsilon_{ll} = \sqrt{\left(\frac{b^2 - v_l^2}{a^2 - c^2} \right)} \cdot \frac{\sin \iota_l - \sin \iota_{ll}}{\sqrt{\sin \iota_l \cdot \sin \iota_{ll}}}$$

$$\therefore (\cos \varepsilon_l) = \sqrt{\left\{ \frac{b^2 - v_l^2}{a^2 - c^2} \cdot \frac{\sin \iota_l}{\sin \iota_{ll}} \right\}}$$

$$(\cos \varepsilon_{ll}) = \sqrt{\left\{ \frac{b^2 - v_l^2}{a^2 - c^2} \cdot \frac{\sin \iota_{ll}}{\sin \iota_l} \right\}}$$

and in like manner

$$(\cos \epsilon'_I) = \sqrt{\left\{ \frac{b^2 - v_{II}^2}{a^2 - c^2} \cdot \frac{\sin \iota_I}{\sin \iota_{II}} \right\}}$$

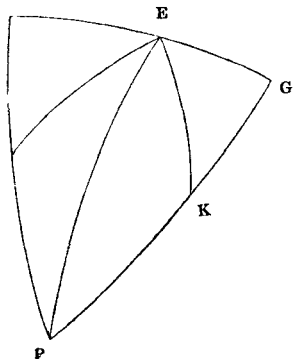
$$(\cos \epsilon'_{II}) = \sqrt{\left\{ \frac{b^2 - v_{II}^2}{a^2 - c^2} \cdot \frac{\sin \iota_I}{\sin \iota_{II}} \right\}}$$

$v_I v_{II}$ for the sake of neatness are left *unexpressed* in terms of $\iota_I \iota_{II}$. This is the simplest form by which the position of the lines of vibration can be denoted.

Corollary.

From the last proposition it appears that

$$\frac{\cos \epsilon'_I}{\cos \epsilon'_{II}} = \frac{\sin \iota_I}{\sin \iota_{II}}.$$



Hence we may construct geometrically for the two planes of polarization.

Let I K be the projections of the two optic axes on a sphere, E the projection of the normal to the front, P the projection of one line of vibration; then

$$\frac{\cos P K}{\cos P I} = \frac{\sin K E}{\sin I K}$$

Draw F E G the \odot^e of which P is the pole, meeting P K, P I

produced in G and F.

Then $\cos P K = \sin K G$, and $\cos P I = \sin I F$,

$$\therefore \frac{\sin K G}{\sin F I} = \frac{\sin K E}{\sin I E}$$

$$\therefore \frac{\sin K G}{\sin K E} = \frac{\sin I F}{\sin I E}$$

$$\therefore \sin K E G = \sin I E F$$

$$\therefore K E G = I E F \text{ or } 180 - I E F. \text{ But } P E F = P E G$$

\therefore E P bisects either the \angle I E K or the supplement to it.

These two portions of E P give the two planes of polarization. The construction is the same as that given in Mr. Airy's tracts, and originally proposed, I believe, by Mr. Maccullagh.

[To be continued.]