Review: Cajori's History of Mathematics

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$\therefore$ The reverse conic is a circle.

namely, the way of scientific history, which masters real connection by living over again actual growth. In the sense of being a short general history of mathematics carried up to the present day, Cajori's book may be called unique in the world. Its greatest strength and originality lies in this modern part, but the whole treatment is conservative and pleasing.

It is a pity the author wholly misses one of the most interesting and instructive points in the history of mathematical thought, the question of the ratio of a piece of a curve to a piece of a straight. Did Archimedes consciously come down from Euclid's high critical standpoint when he made the paradoxical assumptions that the perimeter of an inscribed polygon is less, of a circumscribed greater, than the circle?

Again, the question of the parentage of our modern so-called Arabic numerals has been so extraordinarily fruitful, even in unexpected results about the political history of the Mohammedan peoples, that more than only a single sentence should be given to the true theory.

That the way of historical development is usually the most intelligible and interesting way of expounding any difficult part of a subject, is not only of highest pedagogic importance, but finds especial application in the teaching of the Calculus. But legend should be distinguished from history. The Introduction should not say: "After the class . . . tell them . . . how Pythagoras, jubilant over his great accomplishment, sacrificed a hecatomb to the Muses who inspired him," when we now know this not only unauthentic, but wholly out of accord with the character of the great master. Indeed, the pupil need only turn to page 21 of this very book to see the story discredited.

GEORGE BRUCE HALSTED.

NOTES.

1. On an extension of VI. 3.

Theorem.—If AQ, AR be two chords of a triangle ABC equally inclined to the sides AB, AC ; then

$$BQ \cdot BR : CR \cdot CQ :: BA^2 : CA^2 \text{ (Fig. 7).}$$

For $BQ : CR :: \triangle ABQ : \triangle ACR$ (vi. 1)

$$:: AB \cdot AQ : AC \cdot AR \text{ (vi. 23 Cor.),}$$

since $\hat{BAQ} = \hat{CAR}$.

$$\text{So } BR : CQ :: \triangle ABR : \triangle ACQ, \\ :: AB \cdot AR : AC \cdot AQ.$$

Compounding these ratios we have

$$BQ \cdot BR : CR \cdot CQ :: AB^2 : AC^2.$$

Cor.—If B, Q, R, C be given, the locus of A at which BQ, RC subtend equal angles is the same as the locus of points whose distances from B and C are in a given ratio—that is, a circle. R. W. GENESE.

2. A theorem connected with homography.

ABC is a triangle having $AB = AC$. AL is drawn parallel to CB . The angle LAB now rotates round A , its arms, in any position, meeting BC in Q, R . $BR \cdot QC$ is constant and $= BA^2$ (Fig. 8).

From construction $\hat{BAR} = \hat{LAQ} = \hat{AQC}$,
and $\hat{ABR} = \hat{ACQ}$.

$\therefore \triangle ABR$ is similar to $\triangle QCA$,

$$\therefore AB : BR :: QC : CA,$$

or

$$BR \cdot QC = AB \cdot CA = AB^2.$$

(If $\hat{LAC}$, i.e. the supplement of $\angle LAB$, be the rotating angle, we shall obtain $BQ \cdot RC = AB^2$.)

Conversely, if B, C being fixed, Q, R be variable points on BC , such that $BR \cdot QC = K^2$, then (provided $K > \frac{1}{2}BC$) we can find the point A so that $\angle QAR$ is a constant angle.

For the case in which $K < \frac{1}{2}BC$ a method (which is, too, applicable in all cases) of studying the relation of Q, R is to erect ordinates BH, CK each equal to K (and in the proper directions for the different cases); then (Fig. 9) HR, KQ are at right angles. Therefore, on the variation of Q, R , HR, KQ turn through equal angles.

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3. On the fundamental equations in spherical trigonometry.

The following is suggested as an alternative investigation of the fifteen equations involving four elements of a spherical triangle:—

ABC (Fig. 10) is the triangle, O is the centre, EOK is the diametral plane perpendicular to the plane AOB . AC produced meets EOK in K . Then angle $mOn = A$, and taking radius as unity, $Om = \sin c$, $Bm = \cos c$, $On = \sin b$, $Cn = \cos b$.

$$\therefore mn^2 = \sin^2 b + \sin^2 c - 2 \sin b \sin c \cos A.$$

$$\text{Now } (\text{chord of } BC)^2 = 4 \sin^2 \frac{a}{2} = mn^2 + (Cn - Bm)^2,$$

$$\therefore 4 \sin^2 \frac{a}{2} = \sin^2 b + \sin^2 c - 2 \sin b \sin c \cos A + (\cos b - \cos c)^2,$$

which reduces to

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \quad (i).$$

This holds good for all values of sides and angles, whether greater or less than 90° .

The group of three such equations being found by symmetry, the group $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$ may be deduced as in the text books.

Join BK . Then from $\triangle ABK$, $\cos BK = \sin c \cos A$; and from $\triangle CBK$, $\cos BK = \sin b \cos a - \cos b \sin a \cos C$; this second equation being found to hold good whether b is greater or less than 90° . Therefore in all cases

$$\sin c \cos A = \sin b \cos a - \cos b \sin a \cos C \quad (ii).$$

$$\therefore \frac{\sin a \sin C}{\sin A} \cos A = \sin b \cos a - \cos b \sin a \cos C.$$

$$\therefore \cos b \cos C = \cot a \sin b - \cot A \sin C.$$

Interchanging a, b ; A, B —

$$\cos a \cos C = \cot b \sin a - \cot B \sin C.$$

From this pair of equations, by symmetry, the entire group of six may be deduced.

From (ii) $\sin C \cos A = \sin B \cos a - \cos b \sin A \cos C$.

Interchanging, $\sin c \cos B = \sin A \cos b - \cos a \sin B \cos C$; eliminating $\cos b$ from these—

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a.$$