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To cite this article: Ganesh Prasad M.A. D.Sc. (1918) LIII. On a peculiarity of the normal component of the attraction due to certain surface distributions , Philosophical Magazine Series 6, 36:216, 475-476, DOI: [10.1080/14786441208635856](https://doi.org/10.1080/14786441208635856)

To link to this article: <http://dx.doi.org/10.1080/14786441208635856>

Published online: 08 Apr 2009.

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LIII. *On a Peculiarity of the Normal Component of the Attraction due to certain Surface Distributions.* By GANESH PRASAD, M.A., D.Sc., Professor of Mathematics and Principal in the Hindu University of Benares*.

THE object of this paper is to point out certain surface distributions for each of which the component N of the Newtonian attraction at a point P along the normal, which passes through P and meets the surface at a point O , tends to *no* limit as P approaches O along the normal. It is believed that such surface distributions have not been pointed out by any previous writer.

1. At P , let N be equal to $N_1 + N_2$, where N_1 corresponds to a small area S round O , and N_2 to the remaining part of the surface. Then it is obvious that the limit of N_2 is existent; we have to consider the limit of N_1 . For the sake of simplicity, the surface may be taken to be regular in the neighbourhood of O and, consequently, S may be taken to be a circle of centre O , radius a , and density σ .

$$\text{Case 1. } \sigma = \cos \log \frac{1}{r}.$$

2. First let $\sigma = \cos \log \frac{1}{r}$, where r is the distance between O and the point Q where the density is σ . Then it will be shown that the limit of N_1 is non-existent.

Divide the circle S into thin concentric rings. Then, taking the origin at O and the axis of z as the normal at O , we have

$$N_1 = -2\pi \int_0^a \frac{z\sigma r dr}{(z^2 + r^2)^{3/2}}.$$

Thus we have to investigate

$$\lim_{z=0} N_1, \text{ i. e., } -2\pi \lim_{z=0} \int_0^{a/z} \frac{t\sigma dt}{(1+t^2)^{3/2}},$$

where $zt = r$.

3. Now let C be a sufficiently large quantity independent of z . Then

$$\int_0^{a/z} \frac{t\sigma dt}{(1+t^2)^{3/2}} = \int_0^C + \int_C^{a/z} \frac{t\sigma dt}{(1+t^2)^{3/2}}.$$

* Communicated by the Author.

But

$$\sigma = \cos \log \frac{1}{r} = \cos \left\{ \log \frac{1}{z} + \log \frac{1}{t} \right\}.$$

Therefore

$$\int_0^c \frac{t \sigma dt}{(1+t^2)^{3/2}} = R \cos \left\{ \log \frac{1}{z} + \gamma \right\}$$

approximately, where

$$R \cos \gamma = \int_0^\infty \frac{t \cos \log \frac{1}{t}}{(1+t^2)^{3/2}} dt,$$

$$R \sin \gamma = \int_0^\infty \frac{t \sin \log \frac{1}{t}}{(1+t^2)^{3/2}} dt.$$

Again,

$$\left| \int_C^{a/z} \frac{t \sigma dt}{(1+t^2)^{3/2}} \right| < \int_C^{a/z} \frac{t dt}{(1+t^2)^{3/2}}, \text{ i. e. } \left\{ \frac{1}{\sqrt{1+C^2}} - \frac{1}{\sqrt{1+\frac{a^2}{z^2}}} \right\}$$

which can be made as small as we please by choosing z to be sufficiently small and C to be sufficiently large. Thus it is proved that N_1 behaves as

$$-2\pi R \cos \left\{ \log \frac{1}{z} + \gamma \right\},$$

as z tends to 0.

Therefore the limit of N_1 , and, consequently, that of N are non-existent.

Case II. $\sigma = \cos \chi(r)$.

4. Take the general case in which $\sigma = \cos \chi(r)$, where $\lim_{r=0} \chi(r)$ is infinite. Then the same peculiarity is noticed as in Case I, if

$$\lim_{r=0} \frac{\chi(r)}{\log \frac{1}{r}}$$

is zero or a *finite* quantity different from zero. For the proof of this statement, see a paper of mine which will appear shortly in the 'Bulletin of the Calcutta Mathematical Society,' vol. ix.