

### III.—THE PRINCIPLE OF LEAST ACTION AS A PSYCHOLOGICAL PRINCIPLE.

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#### I.—THE PHYSICAL PRINCIPLE OF LEAST ACTION.

THE principle of Least Action is no doubt best known as a leading generalisation of Mechanical Science. As such it can of course have no obvious connexion with any psychological principle. A principle, to have psychological value, must be a principle for the explanation of psychical facts, and not a principle imported from another science on the ground that it has proved effective in explaining the facts of that science. The tendency to extend the explanatory office of a principle or category beyond the realm of facts for which it was originally designed is however so strong that one almost feels called upon to justify one's self for not indulging in so prevalent a weakness. This I propose to do at the outset by a short critical estimate of the meaning of the Principle of Least Action in Mechanical Science.

I consider it has been the misfortune of Mechanical Science that its principles should not have been given names having associations of a strictly mechanical kind. It has sometimes seemed to me as though the illusions produced by the psychical associations of the name were to a certain extent responsible for the subsequent appropriation of these mechanical principles by Philosophy for the elucidation of mental phenomena. A law which explains a number of material phenomena is given a name suggesting some psychical activity; it is called a law of Inertia, of Attraction, of Least Constraint, of Repose, of Least Action; and then Philosophy, brooding over the name without an expert knowledge of the facts designated by the name, has sight of some profound cosmological principle and does not see why what has cosmological significance should not apply to movements of mind as well as to the movements of matter; are not the phenomena of mind expressive also of inertia, of a dislike to constraint, of a preference for least action? It

is significant that where names have been judiciously given, as *e.g.*, the law of Conservation of Areas, the principle of virtual velocities, D'Alembert's principle, names which all agree in suggesting nothing psychical, there has been no tendency to extend their application outside the realm of the Science they represent. How could one apply the Law of Least Squares to the facts of mental life?

The principle of Least Action has been peculiarly unfortunate in this respect. Maupertuis, who first publicly enunciated it, proclaimed it as a universal teleological principle, and in this he was supported by Euler, its real discoverer,<sup>1</sup> who first presented the principle in a serviceable form. Lagrange was the first to see clearly that far from being a principle from which the designs of the Creator could be inferred, it could itself be deduced as a necessary consequence from the ordinary laws of motion.<sup>2</sup>

The principle is now a century and a half old, but has in that time been expressed in so many different ways that it is not easy to say in a few words what is exactly meant by it. Its general meaning is simply the expression of the fact that in moving from one point to another a body will follow the path which involves the least sum total of action, the Action of a body during any time being a term adopted by Leibnitz to express the continued product of the mass, velocity and space traversed by the body during that time.

What then is the significance of this mechanical principle? We may say that the value of a mechanical principle depends on three considerations: (1) On its generality, *i.e.*, on the number of other mechanical principles deducible from it; (2) on its being a good working principle, a principle easily applied to the solution of mechanical problems<sup>3</sup>; (3) on the simplicity of its physical import. Now, the principle of Least Action possesses great generality, and two great mathematicians, Lagrange<sup>4</sup> and Helmholtz,<sup>5</sup> have made it the funda-

<sup>1</sup> Cf. Herr Adolph Mayer, *Geschichte des Princips der Kleinsten Action*, Leipzig, 1877.

<sup>2</sup> *Mécanique Analytique*, p. 246.

<sup>3</sup> In this respect the Principle of Least Action is found wanting; cf. Bartholomew Price, *Infinitesimal Calculus*, vol. iv., p. 150.

<sup>4</sup> Lagrange, (*Œuvres*, ed. Serret, vol. i., p. 865), in a sequel to a paper of his *Essai d'une nouvelle méthode pour déterminer les maxima et les minima des formules intégrales indéfinies*.

<sup>5</sup> H. v. Helmholtz, "Über die physikalische Bedeutung des Princips der Kleinsten Wirkung," *Journal für die reine und angewandte Mathematik* (usually known as *Crelle's Journal*), Berlin 1886. Hundertster Band, Zweites Heft, pages 187-166, cf. especially pages 142, 148. For a good general review of the various treatises in which Helmholtz attempts to

mental principle from which they have attempted to deduce all others. It is still a vexed question whether the principle of Least Action has a right to this supreme position in the hierarchy of mechanical principles, Mach contending that no general mechanical principle has any claim to priority over any other, inasmuch as they are all different forms or aspects of one and the same fundamental physical fact—the only true prior—and can all be deduced one from the other; and Hertz, contending that in addition to the presence of certain refractory facts which in his opinion argued strongly against Helmholtz's apotheosis of the principle of Least Action, the principle had not a sufficiently simple physical import to justify its standing at the head of an entirely deductive Mechanical Science. In his own attempt at elaborating the principles of mechanics into a single deductive system Hertz has enunciated a principle—a composite of the principle of Inertia and of Gauss' law of Least Constraint—which has apparently the double merit of possessing supreme generality and a relatively intelligible physical import.<sup>1</sup>

As regards the physical import of the principle of Least Action, I cannot see that anything at once certain and satisfactory can be said at present. Lagrange speaks of it as expressing 'a remarkable property of the movements of bodies,'<sup>2</sup> but does not attempt to make its import really clear. Helmholtz has a whole treatise on 'The Physical Meaning of the Principle of Least Action,' but he does not succeed in displaying this remarkable property as a natural consequence of properties less remarkable but more intelligible. This most desirable reduction of the remarkable to the obvious is definitely attempted by Mach, but alas! with a similar result. The same writer, however, gives casually, in other parts of the same work, certain indications of the direction in which the solution must be sought. He most tantalisingly points out in the first place that all mechanical principles, being deducible each from the other, are only different forms of one and the same physical fact, but leaves us uncertain as to what this interesting fact may be. Probably if Mach were pressed to state it in a word, he would answer 'Work,' work being, as he puts it, the factor that determines motion, motion taking place only where there is work to be done.

found the Science of Mechanics on the principle of Least Action see *Herman von Helmholtz's Untersuchungen über die Grundlagen der Mathematik und Mechanik*, von. Dr. Leo Königsberger, especially p. 50.

<sup>1</sup> *Gesammelte Werke von Heinrich Hertz*, Band iii., *Die Prinzipien der Mechanik* (with a preface by Helmholtz himself).

<sup>2</sup> Lagrange, *Mécanique Analytique*, p. 299.

From this we would gather that the property of least action has its intelligible physical import in some obvious proposition as to the conditions under which mechanical work is done.

This inference is strengthened by the following extract from the same work: 'Often the phenomena of Nature exhibit maximal or minimal properties, because when these greatest or least properties have been established the causes of all further alteration are removed. The catenary gives the lowest point of the centre of gravity, for the simple reason that when that point has been reached all further descent of the system's parts is impossible. Liquids exclusively subjected to the action of molecular forces exhibit a minimum of superficial area, because stable equilibrium can only subsist when the molecular forces are able to effect no further diminution of superficial area. The important thing, therefore, is not the maximum or minimum, but the removal of work; work being the factor determinative of the alteration. It sounds much less imposing, but is much more elucidatory, much more correct and comprehensive, instead of speaking of the economical tendencies of nature, to say, "So much and so much only occurs as in virtue of the forces and circumstances involved can occur".'<sup>1</sup>

Mach again explicitly points out that the physical import of the principle of virtual velocities—one of the principles from which the principle of least action can be deduced—is simply this same result, that 'motion can never take place except where work can be performed'. Taking this in conjunction with the terms of the extract quoted above it would seem as though this were perhaps the fundamental physical fact of which all mechanical facts are merely differing forms. If so, it would be a most meritorious action on the part of some physicist to point out clearly and without the use of calculus how to deduce from this simple fact that motion never takes place except where work can be performed, that remarkable property of bodies expressed in the law of Least Action. This would completely solve the question of the physical import of the principle.<sup>2</sup>

<sup>1</sup> Cf. also Mach, *Science of Mechanics*, pp. 74-77.

<sup>2</sup> It may be that the call for this deduction is a fanciful one. The so-called remarkable property was discovered by Euler as follows: He sought an expression whose variation equated to zero would give the ordinary equations of motion. This expression is, however, as Mach points out, only one of various devisable expressions whose variations equated to zero give the ordinary equations of motion. It does not follow that all these mere mathematical formulæ have a direct physical

The fact that the principle of Least Action can be deduced from the principle of Virtual Velocities with strict logical necessity suffices, as Hertz ingeniously points out,<sup>1</sup> to dispose of the fiction that the action in question involves any occult economic activity on the part of the body concerned. The most deanimistic physicist will not grudge to a material body any tendency to control its own motion economically, which can be shown to be a necessary consequence of the fact that motion can never take place except where work can be performed. We may take it then, provisionally, that the principle of Least Action owes its importance as a mechanical principle merely to this, that it is one of the many mutually deducible forms for expressing some fundamental, obvious, instinctively understood physical fact. 'In the case of *all* principles,' writes Mach,<sup>2</sup> 'we have to deal merely with the ascertainment and establishment of a fact.'<sup>3</sup> This one main result of our inquiry into the meaning of the mechanical principle of Least Action will serve us as a clue in the further inquiry with which we are now confronted as to the validity of the principle as a psychological principle.

We put the question to ourselves as follows: What is the main fact or facts with which the Science of Psychology has to deal? What are the principles that embody this fact or facts in the most general and appropriate form? Can some principle of Least Action be said to be among these principles? If so, what is the psychological import of the principle? If not, can such a principle be allowed a secondary place in psychological theory, or must it be banished altogether from Psychology?

## II.—THE PRINCIPLES OF PSYCHOLOGY.

The main fact with which the Science of Psychology deals is, as I take it, the activity of the individual Consciousness. The aim of Psychology is to analyse and mentally reconstruct in an intelligible way the incessant change which character-

import. Jacobi's form of the Hamiltonian principle of Least Action abounds in square roots to which it is impossible to give a direct physical meaning. It is mainly the simplicity of Euler's form of the principle which has led physicists to inquire so persistently into its physical import.

<sup>1</sup> *Prinzipien der Mechanik*, p. 178, cf. also p. 272.

<sup>2</sup> Mach, *Science of Mechanics*, p. 76.

<sup>3</sup> Throughout this inquiry I use the word 'fact' as a fact of Physics or Psychology, as the case may be, not as a fact in the eyes of Metaphysics.

ises the process of Consciousness. Now this change is by no means a mere Heraclitean flux; in so far as it is, it has only a subordinate interest for the psychologist. The change which is of primary concern to Psychology is the change known as mental development, a change possessing definite continuity and direction. The fundamental fact within this ceaseless activity of the individual Consciousness is thus, for Psychology, the fact which gives intelligible unity to this activity. The ultimate psychological principle would then seem to be a principle expressive of this unity—in a word, the principle of the Unity of Consciousness.

What then is this Unity, this fact of the mental life, the presence of which serves to distinguish mental development from mere mental change? In answer to this question I should like first of all to emphasise this, that what we are in search of is a certain fact required by Psychology as a basis for some theory of mental development. It is thus imperative that we should make our own statement as to the scope of Psychology, for the scope determines the data. I propose then, with a view to fixing the fundamental fact, to restrict the scope of Psychology to the study of the development of Consciousness in so far as it is determined, directly or indirectly, by normal attentive processes. A larger conception of the scope of Psychology would of course include subattentive processes and the attentive processes of the multiple Consciousness, but this widening of the scope enormously enhances the difficulty of finding as an experiential fact what we have called the Unity of Consciousness. At the same time the restriction should be estimated at its due value, for all actual Consciousness is attentive, attentive Consciousness being by no means synonymous with the reflective Consciousness. In the case of Consciousness that is reflective, Unity of Consciousness is practically the same thing as Consciousness of Unity, but the Unity of Consciousness we are considering is a fact that is given with—and indeed makes possible—not only Consciousness of Unity, whatever that may mean, but the very Consciousness of an object. It is a form of Consciousness that is as characteristic of animals and of savages as it is of civilised adults.

Having limited our inquiry in one direction let us now limit it in another. The concept of the Unity of Consciousness has had so many meanings given to it that it may be in the interests of clearness to repudiate the more obvious of these as lying outside what we are in search of. In the first place the Unity of Consciousness, as a fundamental fact in mental development is not the simple indestructible

self-identity of the uncritical metaphysician. It is not the metaphysico-theological Unity of Consciousness. Nor again is it the more subtle logico-methodological Unity of Consciousness. The unity we are considering is not of that shadowy sort whose existence is purely ideal, and is variously known as a postulate, precondition or presupposition of conscious activity. 'Unity' is in all cases a conception, but what we want is not 'a conception apart from which the activity of Consciousness is meaningless,' but a conception which embodies an actual fact. What we want is not a precondition, but an actual condition, fact or a constituent of Consciousness. Finally by the Unity of Consciousness as a basis for mental development is not meant what we may call the Coherency of Consciousness. That which serves as a *basis* for mental development makes growing coherency possible; it is not in itself coherency.

To make these eliminations more intelligible I must add that they are made with special regard to the place Psychology is commonly supposed to hold relatively to other sciences. They are made in fact on the assumption that a study of Psychology follows naturally on a study of Biology and is prefatory to a study of Logic and Metaphysics. This point of view once taken it seems most reasonable (1) to entirely exclude from psychological inquiry the two conceptions of the Unity of Consciousness which I have called the logico-methodological and the metaphysico-theological; (2) to take as the fundamental working conception of Psychology that conception of the Unity of Consciousness which may suitably be called the biological conception, the conception namely of the Unity as a vital<sup>1</sup> Unity, the Unity of a conative and developing Consciousness; (3) to take as the true function of the Science of Psychology the discussion of the processes whereby on the basis of this vital Unity, a rational coherent Unity of meaning and purposive movement is built up. In so far as the coherency of motor ideation implies the control of conduct by reference to some good, we have reached a conception of the Unity of Consciousness which might constitute a suitable starting-point for Ethics.

We can now approach with some confidence the *fact* of which we are in search. Let us take any attentive mental process and inquire into the conditions of its unity. The most obvious condition of Unity is that the object of attention,

<sup>1</sup> By *vital* Unity I mean nothing more than Unity of Interest or Conative Unity. Cf. The "dependent vital series" of A. Arsenius, series of a purely psychical kind

whatever it may be, must remain one and the same throughout, *i.e.*, the discriminative activity of attention must move within one and the same sphere of Interest. For it is only in being related to one and the same interest that the discriminations of attention find their meaning; apart from this reference they are mere disconnected observations, the well-known products of inattention. Hence there must be oneness of interest, and it is this oneness of interest that gives oneness to the object of attention and hence oneness to the consciousness attending. We may say then that the Unity of Consciousness, as the fundamental experiential fact in attentive mental process consists in a continuous identity of interest or object aimed at.

Summing up we may say that the conative unity with which we are concerned is a unity whose main characteristic is not the coherency of parts within a whole but the persistency of one conative attitude, and further that it is just this presence in attentive consciousness of a relatively abiding element of sameness which makes possible mental retentiveness and reproduction, the factors most vitally concerned in the formation of that coherent unity of Experience which gives to Consciousness what we may call its unity of meaning<sup>1</sup>

It is an inevitable result of trying to seize and to name the fundamental facts of a science that one does the fact injustice. To name it is but to name an aspect. What the fact really is can perhaps be best stated by stating the condition it must satisfy in order to be fundamental. That condition, as I take it, is simply this: it must be that feature of the subject-matter of the science which makes the subject-matter in all its diversity amenable to scientific treatment. But this feature which saves diversity from becoming a chaos of isolated fragments cannot be exhaustively embodied in the term 'unity' unless we conceive of this unity as indissolubly involving other aspects to which we often give other names. In the case of mental process, or indeed of any time-process, the unity, as we have attempted to define it, involves indissolubly at least one other aspect, that of 'continuity'. It is of no use attempting to deduce the continuity of mental process from its unity, or *vice-versa*. The deduction can of course be made, but as it can be made either way, the making of it does not prove the primacy of either factor. In precisely the same way the principle of virtual velocities may be deduced from the principle of least action and the principle of

<sup>1</sup> Cf. Stout's *Manual of Psychology*, bk. i., ch. ii.



least action from the principle of virtual velocities, in fact all mechanical principles can be deduced indifferently one from the other, all being mere varying forms for expressing some one fundamental physical fact. No one of them can therefore be said to be more ultimate than the rest, except from the point of view of convenience, *i.e.*, of theoretical simplicity, that principle being the most ultimate from this point of view which entails the simplest deductive superstructure and has the simplest real import.

But here we must distinguish. Mental process is not a fact of the same order as mechanical work.<sup>1</sup> The unity and continuity of mental process are facts which derive their richest meaning from considerations totally inapplicable to mechanical work. Mental process is a vital, rational process, a conscious striving.<sup>2</sup> The unity of mental process, its fundamental fact, involves, therefore, not merely a certain abstract continuity such as is given in the unity of a movement in space from one point to another, but a continuity proper to a something that grows, and grows by thinking. The Unity of Consciousness is abstractly one simply in so far as it is continuous or persistent. It is a vital unity from the beginning in virtue of the fact that whatever meaning or skill is acquired is, from the moment we begin to learn by experience, *i.e.*, to develop, utilised for the further acquisition of meaning or skill. It is a vital unity in virtue of the *cumulative* nature of its activity. It is a rational unity, at the outset, only in the sense that its vital unity is not mere unity or continuity of life as in the case of a plant or diatom, but unity and continuity of interest and attention. In all that follows we shall understand by the vital Unity of Consciousness that primary unity which is not only the basis of growth generally, but of that specific form of growth which is known as mental development. We presuppose the rational activities whereby unity of experience is acquired.

By the cumulative activity of Consciousness, as displayed in

<sup>1</sup> And yet Mach's fundamental principle of mechanics, that work is the factor determinative of motion and that where there is no longer any work to be done, there can no longer be any motion, is interestingly like an abstract statement of the fundamental principles of psychical activity which connects effort with the impulse to satisfy felt needs, stating that effort variously adapts itself so long as the impulse still remains unsatisfied, but ceases to be, so soon as the impulse is satisfied.

<sup>2</sup> Or, as we might have put it, though less aptly for the purpose we have in view, mental process is a process made up of impulses and their controlling, and mental development, the gradual acquisition of a more determinate and organised control over impulse. Cf. Lloyd Morgan's *Comparative Psychology*, p. 182.

attentive mental process, I mean not only an activity which in virtue of the continuity of interest shown succeeds in bringing into relation all the successive discriminations made during the process, but an activity which operates in such a way that its later discriminations could not be made unless the earlier ones had been previously made. This is a first cumulative factor in the process of mental growth. Each new acquisition of meaning becomes incorporated into the interpreting context of acquired experience by the help of which new acquisitions of meaning can alone be made. Meaning once acquired is instrumental in acquiring new meaning.

A second cumulative factor is associated with the fact that the more attentively an interest is fed, as above described, the more effective does it become in diverting all fresh knowledge to itself. Attention, as we say, becomes expectant on its behalf, sensitive, that is, to the presence of anything that in any way concerns it. In a word, there is a cumulative effect due not only to an increase in the number of feelers engaged in apprehending the new material, but due also to an increase in the sensitiveness of these feelers.

The cumulative activity of consciousness is most effective when it works continuously within one and the same sphere of interest, the greater, that is, the vital unity of Consciousness. For in readopting a temporarily forsaken interest our first duty is always of a purely restorative character: an interest withers through neglect and in order to revive it to its former efficacy we have first to reassimilate a mass of half-forgotten material. Moreover, if this work of re-assimilation is done too rapidly, the subtle associations of thought and fancy that gave the interest much of its previous force will not be won back. Where there is dissipation of interests there is always a dissipation of the results of previous activity going on in all the spheres of interest except one. This presupposes that the interests are alien to each other. In so far as they are co-operative they come within one and the same enlarged sphere of interest.

The question before us now is whether the activity characteristic of the vital Unity of Consciousness can be said to involve a principle of Least Action. A principle of Least Action as expressive of psychical facts must mean, in the main, one of three things:—<sup>1</sup>

<sup>1</sup> It will be noticed that I make no attempt to deal with the Principle of Least Resistance, except by implication, though such a well-worn notion no doubt requires and would no doubt repay a direct attempt to elucidate it.

- (1) A principle of least exertion possible ;
- (2) A principle of lessening exertion ;
- (3) A principle of the most effective exertion, *i.e.*, of least exertion for a given result or for a given exertion, a maximum of result.

For brevity's sake we shall refer to these respectively as the principles of *Inertia*, *Facilitation* and *Economy*.

### III.—THE PRINCIPLE OF INERTIA.

The most unambiguous expression of this principle as a psychological principle that I have come across is found in an article contributed, in 1894, by M. Guillaume Ferrero to the February number of the *Revue Philosophique*. It is entitled 'Mental Inertia and the Law of Least Effort'. In a footnote on the first page we read: 'The merit of having introduced the idea of Inertia into Psychology belongs confessedly to M. Lombroso who made use of the idea in order to explain the innate conservatism of the human mind. In the present article I am proposing a fresh application of this same idea which appears to me to be a very fruitful one.'

As regards Mental Inertia, M. Ferrero takes up an extreme position. He maintains that when the brain is not stimulated by sensations, it exists in a state of absolute inertia. The law of mental inertia is for him merely the statement of the fact that man receives from without the impulse to feel, or think, or strive. It is the *tabula rasa* in another form. The impulsion towards psychical activity once received, man's main tendency, in accordance with the principle of Least Effort, is to make the least mental effort possible. Man, naturally, has a supreme horror of work in any form, and the law of Least Effort expresses this tendency of a man to employ such processes, muscular or mental, as require the least exertion.

M. Ferrero then appeals to the facts of Evolutional Sociology as proving in a most marvellous manner that this law of Least Effort controls the psychical activities of man. Man's tendency throughout, he argues, has been, when confronted by the necessity of change, to aim at such provisional adaptations as involved the least outlay of effort, even at the cost of obtaining only the most insignificant and fleeting results.

Such a clearly defined attitude as that of M. Ferrero lends itself easily to criticism. Let us first take M. Ferrero's conception of mental inertia as a fundamental psychological

fact. It is the attempt, as I take it, to pass straight from Physics to Psychology with a blind leap over the facts of Biology. This may be a consistent illustration of the principle of Least Effort but it is unjust to Psychology. Before any mental process whatsoever can take place the organism must have taken in its necessary nourishment, digested and assimilated it. This assimilation brings with it internal changes of one kind and another which issue frequently in spontaneous movements. Thus the movements of an amoeba, to take the humblest of organisms, take place usually without any external cause, being determined from within by the ceaseless fluctuations of its unstable jelly-like substance. These fluctuations are themselves no doubt excited by the stimulating effect of the food it has taken in, but this is not an argument in favour of M. Ferrero. The apparent dilemma that food becomes nourishment only through the digestive activities of the organism and that these activities are made possible only through the stimulating effects of food, is not for the psychologist to solve. The biological fact is that the spontaneity of the organism and the dependence of this spontaneity on food supply are always found together. M. Ferrero seems, by implication, to ignore the fact that so long as the stimulus is 'external,' it cannot affect the organism in any way, and that the irritability of the organism is needed in order to make the stimulus effective. But this irritability is precisely the sign of the non-inertness of the organism.

What is true of the amoeba is true of all organisms. Spontaneous movement and assimilation of food are found everywhere, *e.g.* in the human foetus, to be inseparable concomitants. In so far then as life is prior to consciousness, does spontaneous movement precede sensuous perception. The impulse to action, as Höfding says,<sup>1</sup> 'is given before the consciousness of the actual world and cannot be derived from it'. In a word the purely biological fact of spontaneous movement precedes the psychical fact of sensation.

We start our psychical life with inherited tendencies to movement, and these of two kinds; (1) the definitely co-ordinated congenital activities, usually called instincts, and (2) those random, undifferentiated impulses to movement which, in virtue of a certain inherited organic plasticity, are perhaps the most effective factors in the acquisition of individual experience and skill. Consciousness comes obscurely into being amid the play of inherited instincts and inherited impulses.

<sup>1</sup> *Outlines of Psychology*, p. 810.

A state of vital activity, then, precedes the advent of Consciousness. How Consciousness associates itself with these inherited activities I am not prepared to say. It is a metaphysical problem, though the most simple statement of the facts themselves seems to me to be that when certain vital conditions are realised these activities become consciously active. Consciousness would then be aggressive from the outset, a conscious striving. But the main question for us is whether mental process as Psychology has to consider it, is a process that seeks to further these original activities, to satisfy inner needs and cravings, or a process that needs the constant influx of fresh stimuli to keep it going at all. All the facts seem to point to the conclusion that there is a spontaneous call for the stimulus on the part of the conscious organism, not a mere grudging response to the merciless pricking of the outer world. Were the latter the case it is hard to see how natural selection should not by this time have devised insulating sheaths for the sense-organs so as to preserve intact the sanctity of such a fundamental tendency.

I can only concede this much of truth to M. Ferrero's position, namely, that apart from stimuli we should have no sensations. But this points not to inertia but to an indissoluble co-operation between organism and environment, for it is equally true that apart from a certain appropriate activity of mind, the stimulus would be a mere blank sequelless physical change. Attentive mental process does not then mean a compulsion to feel interested in despite of natural propensity, but an interest that is at least spontaneous, often voluntary. Mental Exuberance, if you will, but not mental inertia.

The collapse of mental inertia as a principle of Psychology brings with it the confusion of the daughter-principle, that of Least Effort. Once we admit with Lloyd Morgan 'the restlessness, the exuberant activity, the varied playfulness, the prying curiosity, the inquisitiveness, the meddlesome mischievousness, the vigorous and healthy experimentalism of the young,' it is a far cry to the lotus land of Least Effort. Moreover, I consider there is a fundamental confusion in M. Ferrero's treatment of the subject. I would fully admit the inherent antipathy to constraint, even to control, as a mark of all activity that is restless and exuberant. The apologist of a principle of least discipline might bid for a good hearing. But it is just the natures which revel in superfluous efforts that are the most averse to constraint and discipline. The Vandal may have a horror for work,

*i.e.*, for such forms of activity as cramp his restless energies into orderly grooves, but he has no objection to making the most strenuous efforts at hunting and killing, looting and drinking. We must surely distinguish between the least effort of inertia and the least disciplined effort of exuberance.

The illustration which M. Ferrero draws from social evolution involves similar confusions. Leviathan moves slowly as we all know but it does not follow that he has been sparing his efforts. A climb up a slippery height takes time, not because one's exertions are less, but because one is apt to lose almost as much ground as one gains. There is nothing to show that the small advance made at any time doesn't represent the difference between the results of a great effort in a forward direction and an equally great effort to avoid being pushed back by circumstances beyond the point one started from. Moreover even if the fact of slow but continuous progress in one direction is accepted, the slowness of the advance may well be a sign not of least effort but only of least hurry. We should distinguish between a spurious and a genuine conservatism. The body politic like Wordsworth's cloud tends to move together, if it move at all. This is the true conservative tendency to avoid plunging too far forward in any single direction at the expense of the other connected interests of a complex organisation; but the conservative is not necessarily a lazybones. It is surely not in the service of Least Effort that the wheels of God grind slowly.

M. Ferrero's illustration turns out as we see to be a negative instance confirming the fundamental psychological principle of the tendency to cumulative activity. If a system of politics or of science proves faulty, it is modified, but no further than the defect requires. It is supplanted by another system involving another principle of unity only when its cumbrousness is more burdensome than the consequences of its removal. Thus the Ptolemaic system of the heavenly movements went on accumulating its epicycles and excentrics for a century or two after the outraged astronomer-king made that costly declaration—for it is stated to have cost him his throne—that had the Almighty only seen fit to consult him at the Creation, things would have been managed more simply. Even the Copernican change when it came was not a complete wrench from the old order of things. It only did away with the first main presupposition of the Ptolemaic system, to wit that the earth was certainly at rest and the celestial movements observed, the real movements; it left unchallenged the second main pre-

supposition that the heavenly bodies were divine and incorruptible and must therefore move in circles. Even Kepler himself, to whom the refutation of this hoary prejudice is due, only refuted it by following up into its consequences a presupposition that was strangely similar to it, namely that the Creator must have been a geometer, and that the orbits of the heavenly bodies must have been arranged if not on a circular, then on some other geometrical pattern.

This tendency not to renew where it is possible to modify and not to supersede where it is possible to renew is an indispensable condition of all continuous growth. The abrupt supersession of one system or one interest by another would mean discontinuity of growth and involve a violation of the fundamental principles of mental development. But to take full advantage of accumulated experience in any direction is not a matter of Least Effort, but a matter of continuous interest. Interest implies a concentration of conative activity either for the breaking down of obstructions or the furtherance of success. Where we are genuinely interested we lavish our energy, the interest is in fact a sign that powers of ours have found suitable material, that some hungry expectant activity sees a chance of getting food.

Are we to conclude then that the principle of Least Effort has no place among the fundamentals of Psychology? As a positive principle of mental development I should unhesitatingly condemn it as a fiction that totally misrepresented the facts. Attentive mental process means striving to know and do, not striving to know and do as little as possible. But it cannot mean a striving to know and to do everything. This would involve a dissipation of interest that could only succeed in disintegrating, instead of building up, the Unity of Consciousness. The greater the persistency, indeed, and the intensity, with which any single interest is followed up, the greater the indifference to what we may call alien or outlying interests. Hence I should be fully prepared to admit as a negative principle of Psychology the law of Relative Inertia or Relative Least Effort, if by this is simply meant the fact that attentive mental process involves a complete lack of interest in whatever is unrelated to the process, and that when once interested in anything we give no attention except under compulsion to whatever distracts us from that interest, and that if compelled to give a certain attention, we give the minimum and that grudgingly.

Relative mental inertia in any direction means then complete lack of interest in that direction, and it is clear that where such complete lack of interest exists there will be a

tendency to make as little effort as possible. And this is a *genuine* case of least effort, for by least effort here is not meant the effort to set aside the uninteresting intrusion as speedily as possible by doing just what is most essential and leaving the rest, but the lazy inclination to get rid of the duty anyhow, to spare effort of brain as well as of hands. If the discomfort produced by the feeling that a certain work is being left undone is more disturbing than the actual doing of it would be, the work is done. If the effort to resist the pressure of some external compulsion is greater than the effort entailed in acquiescence, the work is again done. Otherwise endless postponement, and the relapse of least effort into complete relative inertia.

It is important to emphasise this word 'relative'. The absolute inertia of M. Ferrero is a physiological disease. It implies an inability on the part of the organism to give expression to the natural functioning of psychological activities. If this organic defect is credited to the mind as its characteristic feature and the principle of least deflection from absolute inertia transformed into the formative principle of psychological activity, Psychology becomes nothing more nor less than a department of the more general science of Pathology. Relative Inertia is a fact that in no way requires this obverted relationship of the two sciences, for it exists only in virtue of concentration of interest along normal channels of mental activity. We are relatively inert not because we object to the making of an effort, but because we object to the abrupt transference of effort from one direction to another. We may thus willingly admit that we are, as psychological agents, relatively inert and make the least effort possible in every direction except that in which we happen to be exercising our normal activities, but we must hasten to add that it is only in the one excepted direction that any mental development takes place.<sup>1</sup>

<sup>1</sup> It is a significant fact that the principle devised by M. Ferrero for the elucidation of the psychological life corresponds very closely to the fundamental principle of mechanics as enunciated by Hertz. Hertz's primary law of mechanics is to the effect that were the connexions of a mechanical system momentarily severed, the various masses would each and all pursue some rectilinear path with uniform velocity, but that as such a severance is not possible, the masses in their actual movements all tend to deviate as little as possible from this their free and natural form of motion. 'Every free system persists in its condition of rest or of uniform motion in a straightest possible path.' This is a combination of the laws of Inertia and of least deflection from Inertia when under constraint. And this is also M. Ferrero's psychic principle.



In our treatment of the conception of mental inertia we have reached the following main result. The positive aspect of that continuity of Interest which gives primary unity to Consciousness is essentially an effort at self-realisation, but this positive conation implies a corresponding negative, namely, relative inertia. Relative Inertia is as essential to the unity of mental growth as is continuity of Interest each is in fact implied in and limited by the other. The principle of Least Effort may be taken as the abstract negative expression of the unifying principle of mental development in this sense, that mental progress depends on the elimination of all interests that are alien to the interests that give unity to the mental life, and a renunciation of all free effort on behalf of these eliminated interests.

#### IV.—THE PRINCIPLE OF FACILITATION (IN THE LIGHT OF CERTAIN LOGICAL DISTINCTIONS).

In dealing with Facilitation as a so-called principle of mental process, I propose to deal with two conceptions of lessening effort, the one abstract and negative, the other concrete and positive. In dealing with the former I shall lay special stress on the limitations imposed upon the principle by its abstract character, and I hope to show, in dealing with the latter, that the positive, concretely conceived conception of lessening effort may when rightly interpreted, be identified with a most fundamental and fruitful conception in Psychology.

If we consider the two essential processes that go on simultaneously in every complete process of mental development: (1) the *elimination* of the random, unserviceable and irrelevant in experience, and (2) the *elaboration*, through mutual adjustment and co-operation of what is relevant; and if further our way of considering these processes is to fasten on some abstract common element and raise it in virtue of its extreme generality to the rank of a unifying principle of mental process, we fall in my opinion into the most grievous error. For we identify the result of one or more successive processes of abstraction, the so-called abstract universal with the result of a comprehensive synthesis based on a previous thorough-going analysis, the so-called concrete universal. And the principle, whether abstract or concrete, bears its birth-mark stamped upon it. A product formed by mere abstraction is a product possess-

ing the same abstract kind of generality which is proper to the concepts from which it is abstracted: it is only fit to stand above them in a classificatory system. If adopted as an explanatory principle of the concepts from which it is abstracted it is set to achieve the impossible, for how can  $a$  explain  $a + b$  and  $a + c$ ? How can the abstract element common to elimination and elaboration, the element of lessening effort explain either the processes of elimination or of elaboration? It is true that effort is lessened as in the formation of a habit both by the elimination of irrelevant movements and the elaboration of the relevant, but it is equally lessened whatever be the irrelevancies eliminated or the nature of the elaboration provided the net result is the same. And this is the inevitable outcome of introducing into Psychology principles of an abstract, quantitative cast: these abstractedly derived principles of number and magnitude cannot explain qualitative distinctions and purposive elaborations. It is emphatically true here that what is gained in generality is lost in explanatory power.

The concrete universal, on the other hand, is the pure fact itself as reconstructed in the mind. It is a coherent mental structure. In forming it we start, not from abstract concepts, but from the fact itself, analyse the fact, eliminate what is unessential for our purpose and reconstruct the remaining elements into a complex coherent whole which is what we call the concrete universal—a purified, purposive reconstruction of some aspect of real fact. The concrete universal, further, is that reconstructed conception of an actual fact which supplies a coherent context in the light of which the various elements of the analysed fact receive a certain fulness of meaning of which they are incapable when considered apart from that context. It is not necessarily the articulated thought-structure representing a realised ideal. At any stage of its growth the fact of mental process have been represented, after the proper analyses and syntheses can be been gone through, as a concrete universal.<sup>1</sup>

<sup>1</sup> If we ask ourselves what is the animating principle that gives to the concrete universal such coherency as it may be capable of possessing, and having discovered it or one aspect of it abstract it in idea from the processes which it systematises, so that it stands apart abstracted from that which it unifies, we obtain what I should like to call, in opposition to the *mere* abstract universal already alluded to, the *true* abstract universal. The true abstract universal differs from its maimed and artificial counterpart (1) in its *genesis*, for it is only abstracted after the necessary analyses and syntheses have been made, whereas the *mere* abstract universal is derived not from a process of conceptual analysis

The abstractly conceived principle of lessening Effort is thus in no sense a *formative* principle of mental development in the sense that a psychologist can deduce from it the way in which psychical processes are elaborated, even as the mathematician deduces from the physical principle of Least Action the actual paths that moving bodies must take. It may be a guiding thread or clue,<sup>1</sup> but a clue is not a formative or synthetic principle. The thread of Ariadne cannot explain the killing of the Minotaur, it cannot explain the sword of Theseus. It cannot even explain how Theseus found the Minotaur; it can only explain how he found a safe way back through the labyrinth. So it is with the abstract principle of Facilitation. It cannot explain either the actual discovery of psychological principles and laws, nor does it supply from its armoury any weapons for attacking them. It can only guide the psychologist over ground that he has already covered, and at best serve him as an analytical principle of rearrangement. Thus we might conceivably systematise the subject-matter of Psychology by answering in detail the following question: 'What are the conditions that facilitate the exercise of the various psychical activities, retention, reproduction, discrimination, association, etc.?'

The illusive explanatory power of an abstract product of this sort is due to its undeniable generality, to the fact that the common element it expresses is a general characteristic of the whole process in all its parts. Be it elimination or elaboration, lessening of effort does take place. The inference is then made that it must therefore be an essential factor in mental process. This may possibly happen to be the case, thus it might have happened to be true that the direct impulse or aim of mental process was at all costs to lessen

and synthesis, but from a process of mere comparison; (2) in its *function*. The true abstract universal, as could be inferred in advance from its mode of genesis, is explanatory, and is the genuine universal of all abstract Science after it has reached the explanatory stage. The mere abstract universal is at best descriptive and is the universal proper to a system of classification. In illustration of these distinctions we might take the two processes of elimination and elaboration as they take place in mental development. A mere abstract universal stating an element these two processes share in common we have found in the abstract conception of lessening effort. A true abstract universal giving unity to the two processes as they actually take place might be found in Control, when by control I mean the reinforcing or inhibiting of motor tendencies, ideal or corporeal, in view of satisfying some desire or carrying out some design. In elimination, Control exercises its inhibitory function, in Elaboration, its reinforcing function.

<sup>1</sup> Cf. Helmholtz, *Über die physikalische Bedeutung des Princips der kleinsten Wirkung*, pp. 142, 143.

effort, but there is nothing in the process by which the idea is suggested to prove that it is so. We can in fact only say that it covers the facts—not that it explains them. It is just as likely to be a mere common element in the results of the various processes concerned, and not a vital formative factor at all.

The true concrete conception of Facilitation derives its meaning and value from the correspondingly concrete conception of effort. An analysis of the fundamental fact of conation shows that it is essentially an effort to satisfy a felt need, and that when the need is felt no longer the effort ceases to exist. In finding ease it finds its own natural ending. 'Hunger disappears after a full meal; intellectual curiosity disappears when a problem is solved, and so on.'<sup>1</sup> The tendency of all striving is to pass out of effort into ease, and this can only be done through processes marked by a progressive lessening of effort. But the lessening of effort is here no longer abstractly and negatively conceived. It is to be understood only in the light of the coherent context of reconstructed fact, the product of conceptual analysis and synthesis. It is no longer a mere lessening of Effort and nothing more, a facilitation that derives all its meaning and worth from the abstract conception of facilitation, it is a process whose specific meaning and worth is entirely determined by its psychical context. It shares the full meaning of the mind's effort at self-expression, the vital factor which gives primary unity to Consciousness; it is the expression of the fact that we are ever endeavouring to express ourselves smoothly and efficiently, with the ease that means in the long run not only the appeasing of a passing impulse, but the complete satisfaction of a whole system of related interests. The principle of lessening effort is not a principle of lessening activity, but a principle expressive of the fact that the striving which issues in mental development is continually passing, through the subdual of resistance, into the frictionless, effortless activities that are effective in proportion to their ease.

This concrete interpretation of the principle of lessening effort puts a principle into our hands which makes intelligible the evolution of the spoken forms of language. In all languages that have shown any growth there has been a constant process of elimination and elaboration going on, word-endings and other dispensable parts of words being gradually dropped and the remainder being worked over in

<sup>1</sup> Stout's *Manual of Psychology*, p. 66.

the direction of greater manageableness and agreeableness. Like every form of effort or motor activity, the motor service of speech shows a constant tendency to easy and effective utterance. It is probable that the pleasure felt at the harmonious co-operation of the muscles concerned, and the discomfort produced by difficult muscular combinations act as the guide of effort in the direction of motor ease. The process of the facilitation of pronunciation is sometimes spoken of as though it were a merely physiological process. I think this view ignores the psychical influences of comfort and discomfort. A certain muscular combination bringing a certain relief from effort is unconsciously stamped for repetition by the pleasure that it brings, just as the tendency to eliminate other combinations seems prompted by the corresponding discomfort which accompanies them.

This process of facilitation shows itself in many ways in the evolution of spoken language.<sup>1</sup> Let us take the case of the evolution of Latin into French.

(1) There is a general weakening of the Latin letters when they pass into French; thus the *c* and *g* pronounced hard by the Romans before *e* and *i*, as in *cedere, civitatem*, soften into *ç* and *j* sounds as in *ceder, cité*. Similarly the Latin *p* is softened into *v*.

(2) Letters in contact that do not represent easy vocal transitions are assimilated. Thus *dr* becomes *rr*; e.g., *ulhipare, arriver*. But the inverse substitution of *d* for *rr* never takes place.

(3) Recurring letters that produce through recurrence a hard effect have their hardness frequently softened through the replacing of one of them by a kindred but softer letter. Thus if a Latin word has two *r*'s, in French the pronunciation will be softened by the change of the one *r* into *l*, as *peregrinus, pèlerin*; *luscinola, rossignol*. This is known as dissimilation.

(4) We have that displacement of a consonant which is known as metathesis. Thus *paupertatem* which in the Old French texts is met with as *pauverté*, becomes *pauvrete* by metathesis of the *r*.<sup>2</sup>

All these changes follow what M. Brachet calls the Law of Transition. 'Permutation,' he writes,<sup>3</sup> 'moves on step by step, and never more than one step at a time. A letter

<sup>1</sup> Cf. Baudry, *Grammaire Comparée*, pp. 85, 86.

<sup>2</sup> Cf. Brachet, *Étymological French Dictionary*, Introduction, pp. xxvii-xxix.

<sup>3</sup> Quoting from M. Baudry's work, *Grammaire comparée du Sanskrit, du Grec et du Latin*, p. 83.

does not at a bound change its order, degree, or family ; it can only make one of these changes at once'. And he adds : 'The classical *putrere* did not turn at once into the French *pourrir* ; it passed in the Merovingian Latin into the forms *putrire*, *puđrire*, and in Old French through the successive forms *podrir* and *porrir*, whence finally *pourrir* : the *tr* had to become the intermediate *dr* before it reached *rr*.'<sup>1</sup> Whitney gives the physiological reason for this when he points out that 'one sound passes into another that is physically akin to it, *i.e.*, that is produced by the same organs, or otherwise in a somewhat similar manner.'<sup>2</sup> All this mutual adaptation and adjustment of sounds may with great truth be referred to a guiding principle of *ease* or *facilitation*, provided (1) the principle is understood *concretely*, in which case the impulse to ease is no other than the impulse to the harmonious play of effort, the impulse towards pleasurable forms of activity ; and provided (2) the principle is not confused with the principle of economy. The impulse to ease with organic pleasure as its guide is doubtless the primary impulse whence the ideal of economy springs, but it is no more economy than unity of interest is unity of acquired meaning or skill. It is shortsighted and therefore often wasteful in its results. Thus, as Whitney points out, in such words as *ougunnon*, begun ; *pluccian*, to pluck ; *etan*, to eat ; the lost final syllables are those which showed the grammatical form of the words, being plural ending and infinitive ending.

The impulse to ease in the performance of work finds expression through that same cumulative process which we have seen to be so eminently characteristic of the continuity of mental growth. The reward of a difficulty overcome is, as we know, a greater ease in overcoming the next. There could be no facilitation, no easing of effort, were not the products of past achievements instrumental in, so to speak, pointing out to effort the way of ease.

#### V.—THE PRINCIPLE OF ECONOMY.

We proceed now to a brief examination of that form of the Principle of Least Action usually known as the Principle of Economy, the principle of obtaining the maximum of result with the minimum of effort.

<sup>1</sup> Cf. Brachet, *id.*, p. xcix.

<sup>2</sup> Whitney's *Origin of Language*, p. 58.

The principle has to-day at least two accredited champions, Mach and Avenarius.<sup>1</sup> Mach has set it up as the fundamental principle of scientific thinking; Avenarius has claimed for it the leading place among the principles of Philosophy. I shall content myself with a brief attempt at estimating the real significance of these claims from our present point of view.

We should note, in the first place, that we are no longer dealing with a tendency, but with a deliberately entertained scientific ideal, and we have to consider whether the principle of Economy can be considered as a principle of the Unity of the Scientific Consciousness, and if so, in what precise sense.

Now we may say that the activity of the Scientific Consciousness takes place mainly in one of two ways: either in the work of discovery or in the work of systematising what has been discovered. The work of discovery, according to Mach, must be in conformity with the principle of cumulative activity. The object must be given time to unfold itself before the observer, *i.e.*, the observer must be continuously utilising his previous impressions of an object in order to penetrate more deeply into the meaning of the object: only in this way can the phenomenon exercise its full effect on the mind. This cumulative aspect of the principle of Continuity does not, however, impress Mach so much as the principle of Continuity itself. What he means by continuity may be gathered from the following extract: 'Once we have reached a theory that applies to a particular case we proceed gradually to modify in thought the conditions of that case, as far as it is at all possible, and endeavour in so doing to adhere throughout as closely as we can to the conception originally reached'. 'There is no method of procedure,' he adds, 'more surely calculated to lead to that comprehension of all natural phenomena which is the simplest, and also attainable with the least expenditure of mentality and feeling.'<sup>2</sup> So elsewhere he writes: 'The principle of Continuity, the use of which everywhere pervades modern inquiry simply prescribes a mode of conception which conduces in the highest degree to the economy of thought'.<sup>3</sup>

Cf. H. Cornelius, *Psychologie als Erfahrungswissenschaft*, p. 84; cf. also William James, *Principles of Psychology*, vol. ii., pp. 188, 289, 240; cf. *Text-Book of Psychology*, pp. 844-845.

<sup>1</sup> Mach, *Science of Mechanics*, p. 140 (translated by J. McCormack). See also on this question Mach, 'The Economical Nature of Physics' in a volume entitled *Scientific Lectures*.

<sup>2</sup> Mach, *id.*, p. 490.

The subordination of the fundamental principle of Continuity to the principle of Economy, implied in these last words, and indeed in Mach's statements generally, seems to me to be psychologically incorrect. That Knowledge should proceed gradually from the known to the unknown is surely a more primary consideration of the man of science than that the mentality and feeling of himself or others should be spared. Economy seems here to be rather the happy effect of Continuity—not its final Cause.

The systematic activity of the Scientific Consciousness is again, to my mind, dominated by the principle of Continuity. This is shown in the deductive form all such systematisation takes. The mechanics of Lagrange which Mach refers to as a stupendous contribution to the Economy of thought<sup>1</sup> is the classical instance of the deduction of a science through the continuous application of a single principle. Mach himself points out the fundamental importance of this principle in the deductive development of the system of mechanics, but here again he subordinates the principle to that of Economy. 'Mathematics,' he says, 'may be defined as the Economy of counting,'<sup>2</sup> and adds: 'It is the method of replacing in the most comprehensive and economical manner possible new numerical operations by old ones done already with known results.'<sup>3</sup> Perhaps his most explicit recognition of Continuity and the cumulative factor it involves is given when he tells us that 'the object of all arithmetical operations is to save direct numeration by utilising the results of our old operations of counting'.<sup>4</sup>

But though the deductive instinct seems to me to be a more fundamental instinct of the scientific consciousness than the instinct for economy, it is undoubtedly true that the scientific consciousness does deliberately set itself to economise labour by such devices as that of abbreviation. This is due to obvious considerations of time and memory. Mach, indeed, lays the very greatest stress on this fact. 'Within the short span of a human life, and with man's limited powers of memory, any stock of Knowledge worthy of the name is unattainable except by the greatest mental economy; science itself, therefore, may be regarded as a minimal problem, consisting of the completest possible presentment of facts with the least possible expenditure of thought';<sup>5</sup> and Mach further points out how, in mathematical science, the whole system of symbols, semimechanical

<sup>1</sup> Mach, *id.*, p. 107.

<sup>2</sup> *Ibid.*, *id.*, p. 486.

<sup>3</sup> *Ibid.*, *id.*, p. 195.

<sup>4</sup> *Ibid.*, *id.*, p. 486.

<sup>5</sup> *Ibid.*, *id.*, p. 490.



devices, e.g., determinants, and finally calculating machines are all devised in the spirit of this conviction.<sup>1</sup>

As regards Mach's general position in this matter, while we must allow the truth of the fact that 'Physics is experience arranged in economical order,'<sup>2</sup> we do not consider that he has touched the heart of the matter when he says that 'the goal which physical science has set itself is the simplest and most economical abstract expression of facts'.<sup>3</sup> We should be much more inclined to agree with Descartes in making an essential point of the deductive method of inquiry and laying only subordinate though still very strong emphasis on the necessity of economy.<sup>4</sup> And this is to give continuity the primacy over economy.

With Avenarius<sup>5</sup> the principle of economy, or, as he puts it, the principle of least expenditure of force, is the guiding principle of Philosophy. It is (1) the principle from which philosophy springs, i.e., in pursuing that principle into its consequences we are led to Philosophy, and further, led to Philosophy of a certain kind, for (2) it is the principle which determines the central problem of Philosophy, the attempt to unify the world under one general concept; (3) it is the principle which, rigidly carried out, determines the structure of Philosophy; and (4) it is the principle which inspires the methods of Philosophy.<sup>6</sup>

The characteristic of philosophic thought that brings it thus under the law of economy is its essentially conceptual nature. By this is meant specifically the subsuming of presentations under general notions, and, more generally, the apprehension of the unknown in terms of the known. The latter process is characteristic of all apperception whatsoever, the former, in its fulness, of Philosophy only, for it is only Philosophy that carries the process of subsumption to its natural issue, it is only Philosophy that seeks to bring

<sup>1</sup> Mach, *id.*, pp. 487, 488.

<sup>2</sup> *Ibid.*, *id.*, p. 197.

<sup>3</sup> *Ibid.*, *id.*, p. 207.

<sup>4</sup> Cf. *Regulæ*, xiii., xiv., xvi., xviii., xx.

<sup>5</sup> *Philosophie als Denken der Welt nach dem Princip des kleinsten Kraftmaasses*, Leipzig, 1876. A brief but excellent summary of this treatise can be found in *MIND* O.S., vol. i., p. 298; it is also summarised and discussed at greater length in a leading article of the *Literarisches Centralblatt*, 15 (1876); cf. also *Revue Philosophique*, 8<sup>e</sup> Année, p. 216.

<sup>6</sup> Cf. Avenarius's own introduction to his *Critique of Pure Experience*. It is a significant fact that the principle of economy which so dominates the *Prolegomena* should be completely ignored in the *Critique* itself. The inference is that in serving the abstract office of a 'Leitfaden' or guiding clue to the most economical conception of experience its real value had been exhausted, but this, of course, is not the meaning of Avenarius in dealing with the principle.

the object of its thought—the world—under one general concept.

Avenarius points out at some length that these two characteristics of apperception are both essentially economical processes,<sup>1</sup> and as they are, taken together, peculiarly the processes proper to philosophic thinking, Philosophy has its roots in the principle of economy.

The main objection I would make to the reasoning of Avenarius is that it does not appear to me to be founded on a true psychological analysis. Avenarius proves fully, and often most ingeniously, that an element of economy is to be found in all the various specific processes he treats of, but he does not prove what he avowedly aims at proving—that conation in its theoretical aspect as apperception is a striving to think economically. The element of economy that he invariably discovers is not shown to be the element that dominates the striving, and this, in my opinion, renders the whole argument artificial and misleading.

As a typical instance of the method of Avenarius, let me take the following. After pointing out with true psychological insight that in systematic thought we have (1) the domination and continuous application of a central idea; (2) a perpetual strengthening of the meaning of the interconnected ideas through their connexions with one another and the central idea; (3) a facilitation in applying the idea brought about through constantly applying it, he adds: 'These are, collectively, effort-saving considerations'.<sup>2</sup> They undoubtedly are, but they are also the considerations for effective, *i.e.*, successful work, and success to the striver is of much more importance than economy. Avenarius cheats us throughout by presenting us with an abstract universal, a uniformly present common element, in the place of the concretely determined universal. Economy *per se* is a mere formal principle, and as such incapable of determining its own limits. Let us take the case of Descartes' treatise on Geometry. Its conciseness is such that even Newton found it hard to master. Descartes confided to a friend that he had purposely abbreviated the solutions in order that critics might not say to him, 'Well, any one could have discovered that'. Here we have the most rigid economy, from the quantitative point of view, the maximum number of solutions with the minimum outlay of means. But this is not

<sup>1</sup> Avenarius, *id.*, p. 10: 'The impulse to apperceive is nothing else than the endeavour of the mind to economise its force'.

<sup>2</sup> Avenarius, *id.*, p. 6.

the economy that gives value to science. What is wanted is not economy in the abstract but a wise economy, *i.e.*, an economy limited and defined by the more fundamental consideration of effectiveness. A wise economy implies the keeping the end in view so steadfastly that only such means are employed as the end requires for its attainment; it means putting essentials before accessories according to a principle of Order; it means the keeping of Economy within the limits of clearness, so that there shall be no obscurity due to economising what is essential or relevant, and no obscurity due to the intrusion of the accidental and irrelevant. The true nature of economy is given only in the light of the more fundamental requirements of clearness, continuity, method, and it is only in subordination to these that it finds its true meaning.<sup>1</sup> Descartes saw all this with masterly clearness nigh three centuries ago. He realised, with true mathematical instinct, the value of the economy of thought, but he gave it its true subordinate place. Above all clearness as to one's starting-point, then method, lastly economy in the application of the method.

I do not pretend to have done full justice to the admirable, though unsatisfactory treatise of Avenarius. But I am convinced that its merits whatever they may be cannot save the principle of Economy from being relegated to a second rank among the principles that express the unity of the Scientific Consciousness. What is fundamental in the Scientific Consciousness is not a striving after economy, but a striving after Clearness, Method and Fidelity to Fact. It is the effort to think clearly and deeply that yields the economical virtues of simplicity, relevancy and precision.

<sup>1</sup>On the directionless character of all these abstractly conceived principles, with special allusion to the principle of least action or least resistance, see James Ward, *Naturalism and Agnosticism*, vol. I, pp. 206, 275; vol. II, pp. 26, 88, 290.