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it might be found in many other places; certainly it must have been formed in the old Franklin furnace, in Sussex county New-Jersey, where so many fruitless attempts were made to work the Franklinite.

Before I conclude these remarks, I must observe, that it does not appear that the presence of zinc affects the properties of iron. In Belgium the iron is of good quality; and it is an interesting fact, that the bar-iron of Ancram is in great demand at £ 120 per ton, a higher price than is at present paid for any imported iron. The castings from the Ancram furnace are in such a repute, that no other pigs are used at the West Point Foundry for the heavy guns (32 and 42 pounders) now casting for the United States' navy.

The Ancram furnace equals, in beauty of workmanship, and economy of means, any that we have seen; and we entertain no doubt, that all works carried on with such admirable perfection, must and will always prove equally honourable and profitable to their owners and directors.

XXIV. *On a Planetary Analogy; or a Law of Motion pervading and connecting all the Planetary Orbits. By Mr. J. UTTING.*

To the Editors of the Philosophical Magazine and Journal.

Lynn Regis, June 21, 1823.

THE following beautiful analogy which obtains in the motions of the planetary orbs has, I believe, never been described by any astronomical writer, or is not generally known, viz. If the mean orbicular motion of each planet in its orbit, be multiplied by the square root of its mean distance from the sun, a product will be obtained common to all the planets: for instance, if the orbicular motion in miles of each planet in one sidereal day, be multiplied by the square root of its mean distance from the sun, the product will be 15.634.588.176 miles, a *constant* quantity for all the planets, as the mean velocity of the planets, multiplied by the square root of their respective mean distance, is always a *constant* quantity.

The same analogy obtains in each respective system of satellites; for, if the velocity of a satellite be multiplied by the square root of its mean distance from its primary, a *constant* product will also be produced in each respective system of satellites; and if this *constant* product be multiplied by the square root of the reciprocal of the sun's attractive power, and that of their respective primaries, the same result will be produced as that which obtains in the planetary motions, as above. Thus a *constant* product, or *quantity*, obtains in the motions of

of the planets, and their respective systems of satellites, extending to the whole planetary system, resulting from the periodic times and mean distances of the planets, with the periodic times and mean distances of their satellites, compounded with the attractive power of the sun, as compared with that of the primary planets, around which each respective system of satellites circulates; viz.

Let $V, V', V'',$ &c. represent the velocities of the planets in their orbits; and $\sqrt{D}, \sqrt{D'}, \sqrt{D''},$ &c. the square roots of their mean distances from the sun. Let also $v, v', v'',$ &c. represent the velocities of their respective satellites; and the $\sqrt{d}, \sqrt{d'}, \sqrt{d''},$ &c. the square roots of their mean distances from their primaries. Let the square root of the sun's attractive power, that of each planet being unity, be denoted by $\sqrt{m}, \sqrt{m'}, \sqrt{m''},$ &c. respectively.

Whence we have $V \times \sqrt{D} = V' \times \sqrt{D'} = V'' \times \sqrt{D''}$ &c. a constant quantity for the primary planets. And $v \times \sqrt{d} = v' \times \sqrt{d'} = v'' \times \sqrt{d''}$, &c. a constant quantity for each respective system of satellites. Also, $v \times \sqrt{d} \times \sqrt{m} = v' \times \sqrt{d'} \times \sqrt{m'} = v'' \times \sqrt{d''} \times \sqrt{m''}$, &c. a constant quantity equal to that in the first analogy. Whence $\frac{V \times \sqrt{D}}{v \times \sqrt{d} \times \sqrt{m}} = 0. = \frac{V' \times \sqrt{D'}}{v' \times \sqrt{d'} \times \sqrt{m'}} =$

$$0. = \frac{V'' \times \sqrt{D''}}{v'' \times \sqrt{d''} \times \sqrt{m''}} = 0. \&c.$$

The following general analogy also obtains, viz. As $V : V' :: \sqrt{D'} : \sqrt{D}$; also as $v : v' :: \sqrt{d'} : \sqrt{d}$ &c. where the product of the two extreme terms will always be equal to the product of the two mean ones, whatever may be the planets or satellites fixed on.

The following table exhibits the result of my calculations in elucidation of this analogy.

Tabular View of the Analogy which obtains in the Planetary System.

Planets.	Sidereal periods in sid. days.	Mean dist. in miles.	Mass of the ☉ each planet = 1.	Square root of the ☉'s mass.	Square root of mean dist. in miles.	Velocity in one sidereal day in miles.	Constant product in miles.
♂	88.2101005	36.387308			6032.19	$2591860 = 15634588170$	
♀	225.3159734	67.993235			8245.80	$\times 1896067 = 15634588170$	
♁	366.2563835	94.000000	337102.	580.605	9695.36	$\times 1612585 = 15634588170$	
♂	688.8604607	143.227108			11967.75	$\times 1306393 = 15634588170$	
♃	4344.4468810	488.908265	1066.09	32.6510	22111.27	$\times 707087 = 15634588170$	
♄	10787.7763273	896.517987	3512.08	59.2628	29941.91	$\times 522164 = 15634588170$	
♅	30772.7323350	1803.218792	19504.	139.6567	42464.32	$\times 368182 = 15634588170$	

Satel-

Satellite of the Earth.

Satellites.	Sid. period in sid. days.	Mean dist. in miles.	Square root of mean dist. in miles.	Velo- city in one sid. day in miles.	Constant product for each system of satellites in miles.	Square root of the ☉'s mass.	Constant product in miles.
☾	27.3964621	239780	$489.674 \times$	$54992 =$	$26928100 \times$	$580.605 =$	15634588170
Satellites of Jupiter.							
1	1.7739813	263410	$513.239 \times$	$932975 =$	$478839200 \times$	$32.6510 =$	15634588170
2	3.5609634	419160	$647.426 \times$	$739605 =$	$478839200 \times$	$32.6510 =$	15634588170
3	7.1741405	668630	$817.699 \times$	$585594 =$	$478839200 \times$	$32.6510 =$	15634588170
4	16.7344602	1176020	$1084.444 \times$	$441553 =$	$478839200 \times$	$32.6510 =$	15634588170
Satellites, and Ring of Saturn.							
1	0.9452910	116360	$341.112 \times$	$773406 =$	$263818000 \times$	$59.2628 =$	15634588170
2	1.3739915	149300	$386.400 \times$	$682760 =$	$263818000 \times$	$59.2628 =$	15634588170
3	1.8929684	184860	$429.965 \times$	$613595 =$	$263818000 \times$	$59.2628 =$	15634588170
4	2.7469802	236950	$486.775 \times$	$541973 =$	$263818000 \times$	$59.2628 =$	15634588170
5	4.5298580	330730	$575.091 \times$	$458741 =$	$263818000 \times$	$59.2628 =$	15634588170
6	15.9899556	766710	$875.618 \times$	$301293 =$	$263818000 \times$	$59.2628 =$	15634588170
7	79.5467885	2234420	$1494.798 \times$	$176491 =$	$263818000 \times$	$59.2628 =$	15634588170
Ring of ϵ	0.4402692	69914	$264.412 \times$	$997754 =$	$263818000 \times$	$59.2628 =$	15634588170
Satellites of Uranus.							
1	5.9087328	222960	$472.186 \times$	$237089 =$	$111950150 \times$	$139.6567 =$	15634588170
2	8.7306375	289240	$537.812 \times$	$208159 =$	$111950150 \times$	$139.6567 =$	15634588170
3	10.9911093	337230	$580.714 \times$	$192780 =$	$111950150 \times$	$139.6567 =$	15634588170
4	13.4927396	386630	$621.797 \times$	$180043 =$	$111950150 \times$	$139.6567 =$	15634588170
5	38.1792417	773480	$879.476 \times$	$127292 =$	$111950150 \times$	$139.6567 =$	15634588170
6	107.9892458	1546980	$1243.775 \times$	$90008 =$	$111950150 \times$	$139.6567 =$	15634588170

NOTE.—The periodic times of the planets and satellites were taken from the fourth edition of Laplace's *Système du Monde*, the time being converted from solar to sidereal days in the proportion of 1.0027378 to 1. The mass or attractive power of the sun, and planets, was also taken from the same work, from which with the periodic time, and constant product, the distances of all the satellites from their primaries were computed. The distance of the ring of Saturn, is the distance from the centre of the planet to the centre of attraction in the cylinder of the ring, or the centre of gravity of a satellite, supposing all the particles of matter in the ring to be condensed into a globular form, and whose sidereal period is equal to that of the rotation of the ring.

XXV. *State of the Thermometer at Smyrna for every Day in the Year 1820 (being the Year of the great Eclipse, and Leap-year,) taken in the Shade four times every Day; viz. 9 A.M., Noon, 6 P.M., Midnight. Communicated from Smyrna by a Correspondent to Dr. T. FORSTER.*