TOTAL REFLECTION.

BY ELMER E. HALL.

IN a former article¹ the writer has discussed the theory of penetration during total reflection, when reflection takes place from a thin film contained between two denser media. The discussion was based upon the electromagnetic theory of light and particular application was made to the case of a film between the hypotenuse surfaces of two right-angled glass prisms placed almost in contact. If the electric forces be regarded as the light vectors, the ratio of the intensities in the two glass prisms can be obtained from the ratio of the electric forces within the prisms. Neglecting losses due to absorption, and representing the component of the electric force perpendicular to the plane of incidence in the first and second prisms by E_s and D_s respectively, it has been shown that²

$$\frac{D_s^2}{E_s^2} = \frac{4 \cos^2 \varphi \left(\frac{\sin^2 \varphi}{n_1^2} - 1 \right)}{(1 - n_1^2)(n_2^2 - 1) \sinh^2 u + (\cos \varphi + n_1 n_2 \cos \theta)^2 \left(\frac{\sin^2 \varphi}{n_1^2} - 1 \right)} \quad (1)$$

Here φ is the angle of incidence, within the first prism, on the boundary of the film; θ is the angle of refraction within the second prism; n_1, n_2 are the indices of refraction from the first prism into the film and from the film into the second prism respectively;

$$u = \frac{2\pi d}{n_1 \lambda} \sqrt{\sin^2 \varphi - n_1^2},$$

d being the thickness of the film and λ the wave-length of light within the film. The above equation expressing the ratio of the squares of the electric forces gives also the ratio of the squares of the amplitudes of the incident and emergent light. This will be the ratio of the intensities only when the two prisms have the same

¹ PHYSICAL REVIEW, Vol. XV., p. 91.

² Loc. cit., p. 97.

index of refraction. If n_1 is not numerically equal to $1/n_2$ the relative intensities will not be measured by the squares of the amplitudes merely, but the rates at which the energy is propagated by the corresponding waves in the two media must be taken into account. In this case, if the two media enclosing the film be different, the ratio of the intensities will be

$$\frac{D_s^2}{E_s^2} \tan \varphi \cot \theta = \frac{4 \cos^2 \varphi \left(\frac{\sin^2 \varphi}{n_1^2} - 1 \right) \tan \varphi \cot \theta}{(1 - n_1^2)(n_2^2 - 1) \sinh^2 u + (\cos \varphi + n_1 n_2 \cos \theta)^2 \left(\frac{\sin^2 \varphi}{n_1^2} - 1 \right)}. \quad (2)$$

If now the beam of light be reversed, φ and θ will interchange and $1/n_1$ replace n_2 and *vice versa*. Inserting these changes in (2) the ratio of the intensities within the two prisms becomes

$$\frac{D_s'^2}{E_s'^2} \tan \theta \cot \varphi = \frac{4 n_1^2 n_2^2 \cos \theta (n_2^2 \sin^2 \theta - 1) \tan \theta \cot \varphi}{(1 - n_1^2)(n_2^2 - 1) \sinh^2 u' + (n_1 n_2 \cos \theta + \cos \varphi)^2 (n_2^2 \sin^2 \theta - 1)}. \quad (3)$$

Since

$$n_2^2 \sin^2 \theta = \frac{\sin^2 \varphi}{n_1^2}$$

the second members of (2) and (3) may be reduced to an identity. It therefore follows that the penetration for any such system is the same whether the light passes from the denser to the rarer of the two media enclosing the film or in the reverse direction. Similar treatment yields the same conclusion for light polarized perpendicular to the plane of incidence. In the former discussion of the theory the factor $\tan \varphi \cot \theta$ was omitted and hence erroneous conclusions drawn.¹

To test the conclusion under better experimental conditions than in the former measurements, a flint-glass prism, whose index of refraction for sodium light was 1.7597, was used with a crown-glass

¹ I am indebted to Professor Voigt for calling my attention to the error, by letter and also in Nach. d. K. Gesellschaft d. Wissenschaften zu Gött. 1903 Heft 3 S. 1 where a different treatment yields the same conclusion.

prism of refractive index 1.5164. The hypotenuse surface of the crown-glass prism was slightly convex. The prism angle for the flint-glass prism, $27^{\circ} 54'$, was such that when the two prisms were used in combination no deviation was obtained with sodium light. The source was a narrow yellow band from the spectrum of a Nernst lamp filament, the current being maintained practically constant by means of an electro-dynamometer and an adjustable resistance. The following table gives the average of a number of closely agreeing readings. The light was polarized perpendicular to the plane of incidence; d is the depth of penetration in microns; i_1 and i_3 , for the same line, are so related that i_3 is the angle of emergence from the flint-glass when i_1 is the angle of incidence on the crown-glass prism.

Crown-glass, Air, Flint-glass.				Flint-glass, Air, Crown-glass.			
i_1	θ	d	d/λ	i_3	θ	d	d/λ
5°	$40^{\circ} 42' 20''$	1.98	3.36	$12^{\circ} 30' 00''$	$34^{\circ} 58' 45''$	1.85	3.15
3	43 1 30	1.32	2.24	14 22 30	36 0 45	1.28	2.18
0	45 0 0	1.06	1.80	17 8 00	40 2 20	0.99	1.69

The differences in the values for the two cases are greater than the experimental errors. Repeated sets of measurements, taken with different intensities, all gave the same order of differences for the two cases. Light polarized parallel to the plane of incidence also gave the same result, the penetration is apparently greater when the light is incident on the crown-glass. The intensity observed, however, is not the intensity in the second prism but the intensity in air after the light has left the second prism, and furthermore it is the intensity in air before incidence on the first prism and not the intensity of the light incident on the film that is constant. The Fresnel coefficients for transmission from air to the first prism and from the second prism to air must be taken into account. The intensity of the light transmitted by the system for any given angle of incidence is not the same when the light is made to travel the same path, but in the reverse direction, unless the amount of light getting through the film varies directly as the intensity of the light incident on the film. This, however, is not the case, the penetration is a complicated function of the intensity.¹ For the condition under

¹ PHYSICAL REVIEW, Vol. XV, p. 100.

which the measurements were taken the results for the two directions therefore will not be identical. The penetration will be the greater for that direction in which the light incident on the film has the greater intensity. Using Fresnel's coefficient for light polarized perpendicular to the plane of incidence, the intensity of the light incident on the film when crown-glass is the first medium, for $i_1 = 5^\circ$, is to the intensity of the light incident on the film when the flint-glass is the first medium, for the corresponding angle of incidence $i_3 = 12^\circ 30'$, in the ratio of 959 to 929. Absorption has not been taken into account. Since the flint-glass has a distinct yellow tinge the losses due to absorption would increase rather than diminish the ratio of intensities as given. The greater value for the penetration obtained in the above table, when light was incident on the crown-glass, is thus accounted for, and it may be concluded that the penetration is independent of the order in which the light passes through the prisms.

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