100. [The following proofs of the "Equal Bisectors Theorem" ...]

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It must be therefore possible to throw the given equation into the form

$$\Sigma(a \sin A) . \Sigma(ua \operatorname{cosec} A) + \lambda . \Sigma(P\beta\gamma) = 0.$$

Hence $v \sin C \operatorname{cosec} B + w \sin B \operatorname{cosec} C + \lambda P = 2u'$

$$\dots\dots\dots$$

and $u + v + w - 2u' \cos A - 2v' \cos B - 2w' \cos C = 0.$

R. F. DAVIS.

99. [P. 2. a.] Extension of Problem 394.

Problem 394 will be found true for a curve S of any order, the theorem on which it depends being the following :

Let κ be the curvature of a curve S at a point P , the radius vector from the origin O to which is r . Corresponding to the tangent at P is a point P' on the reciprocal curve Σ (taken with respect to centre O , radius k). Let κ', r' be the curvature and radius vector at P' . Then $\kappa r^3 = \frac{k^4}{\kappa' r'^3}$ so that the quantity κr^3 is inverted by reciprocation.

The following corollaries are obvious :

If S has a cusp at P , Σ has an inflexion at P' .

If S has a tacnode at P , the curvatures of the branches being κ_1, κ_2 , Σ has a tacnode at P' , the curvatures being κ'_1, κ'_2 , where

$$\kappa_1 : \kappa_2 = \frac{1}{\kappa'_1} : \frac{1}{\kappa'_2}.$$

We may also obtain from this theorem an expression for the curvature of a conic S at any point P . Take the origin O at a focus of the conic S . Let P be a point on the conic, $OP=r$. Reciprocate S with respect to origin O , radius r . The reciprocal conic Σ is a circle of radius $\frac{r^2}{l}$, where l is semi latus-rectum of S . Since the circle whose centre is O , radius r , reciprocates into itself, the tangent at P to this circle will touch the circle Σ at a point P' . Let $OP'=r'$, then the theorem $\kappa r^3 = \frac{k^4}{\kappa' r'^3}$ gives $\kappa = \frac{1}{l} \left(\frac{r}{r'} \right)^3$ or radius of curvature at $P = l \sec^3 \theta$ where $\theta = \angle POP'$.

This expression is easily reducible to the normal form $\kappa = \frac{ab}{CD^3}$.

A. P. THOMPSON.

100. [K. 1. b.] The following proofs of the "Equal Bisectors Theorem" are perhaps worthy of record, if they have not already been published. Mr. Davis's Proof in the December Number, 89 [K. 1. b.], employs unnecessarily "heavy metal" (Ptolemy's Theorem, etc.).

The theorem is an obvious consequence of this :

If in the triangle ABC the side AB is greater than AC , the bisector BE of the angle B terminated in AC is greater than the bisector CF of the angle C terminated in AB .

First Proof. Make the angles ACH, BCK each equal to $\frac{1}{2}B$,
then since $C > B$, CH, CK will lie on opposite sides of CF .

The triangles ACH, ABE are equiangular and $AB > AC$,
therefore $BE > CH$.

Also in the triangles BCK, BCE ,
 BC is common, $BCK = CBE$ and $KBC > BCE$,
therefore $BE > CK$.

But CF , lying between CK , CH , must be less than the greater, so that, being less than (or equal to) one of them which is less than BE ,
 CF is less than BE . Q.E.D.

Second Proof. Describe the circular arc BFC , cutting the bisector of B in G .

Then $BFC = A + \frac{C}{2}$, $BEC = A + \frac{B}{2}$, so that $BFC > BEC$, and $\therefore BGC > BEC$, hence G lies between B and E .

Also since BCF or $\frac{1}{2}C > EBC$ or $\frac{1}{2}B$, arc $BF > \text{arc } CG$,
 hence arc $BG > \text{arc } CF$ and chd. $BG > \text{chd. } CF$.
 $\therefore$ à fortiori, $BE > CF$. Q.E.D.

This proof is due to the Rev. H. W. WATSON, D.Sc., F.R.S., somewhat improved by the late H. R. DROOP. R. B. HAYWARD.

PROBLEMS.

[Much time and trouble will be saved the Editor if (even tentative) solutions are sent with problems by their Proposers.]

430. [L. 10. d.] Given two straight lines of fixed lengths and placed perpendicular to each other, what must be their relative positions in order that a real parabola may pass through their extremities? E. N. BARISIEN.

431. [K. 1. b. a.] If I be the incentre of a triangle ABC and BE , CF the external bisectors terminated by the opposite sides, then if IB , IA , IC are in geometrical progression, $BE = CF$. R. B. HAYWARD.

432. [A. 3. a.] The discriminant Δ of a polynomial $f(x)$ being expressed in the form $\phi(x)f(x) + \chi(x)f'(x)$, prove that when $\Delta = 0$ the common factor of $f(x)$ and $f'(x)$ is also a factor of $\chi(x)$ and $\phi(x) + 2\chi'(x)$. R. W. H. T. HUDSON.

* This follows only if BF , BG are on the same side of the diameter through B of the circle BFC .
 $\left[BF = \frac{BC}{BC+CA} \cdot BA < BC; \therefore \text{diameter does not lie between } BF \text{ and } BG \right].$