122. The Fundamental Formulae of Spherical Trigonometry

Author(s): C. S. Jackson

Source: *The Mathematical Gazette*, Vol. 2, No. 38 (Mar., 1903), pp. 261-262

Published by: Mathematical Association

Stable URL: <http://www.jstor.org/stable/3605113>

Accessed: 23-12-2015 22:12 UTC

Your use of the JSTOR archive indicates your acceptance of the Terms & Conditions of Use, available at <http://www.jstor.org/page/info/about/policies/terms.jsp>

JSTOR is a not-for-profit service that helps scholars, researchers, and students discover, use, and build upon a wide range of content in a trusted digital archive. We use information technology and tools to increase productivity and facilitate new forms of scholarship. For more information about JSTOR, please contact support@jstor.org.

Mathematical Association is collaborating with JSTOR to digitize, preserve and extend access to *The Mathematical Gazette*.

<http://www.jstor.org>

(1) Let A, B, C, D, E be the 5 pts. By isogonal transformation with regard to ABC as many conics can be drawn through A, B, C, D, E as str. lines through D', E' the isogonals of D, E , i.e. 1.

(2) Let A, B, C, D be the 4 pts. and q the str. line. Transform with regard to A, B, C . Then as many conics can be drawn to pass through A, B, C, D and touch q as str. lines can be drawn to pass through D' and touch conic q' , the isogonals of D and q , i.e. 2.

(3) Let A, B, C be the 3 given pts. and q, r the two str. lines. Transform with regard to A, B, C . Then as many conics can be drawn through A, B, C to touch q, r as str. lines can be drawn to touch conics q', r' the isogonals of q, r , i.e. 4.

(4), (5) and (6) follow from (3), (2), (1) by reciprocation.

H. L. TRACHTENBERG.

122. [K. 20. f.] *The Fundamental Formulae of Spherical Trigonometry.*

The proof, of which an outline note is subjoined, is a slight modification of one given by Mr. W. W. Lane, R.N. in his *Spherical Trigonometry*.* The figure given below possesses the advantage of shewing all lines in their true lengths and angles of their true magnitude. Paper models obtained by cutting out $OABCAO$ or $OADAMAEAO$ and folding them up, are very easily made. The formulae for a right-angled spherical triangle are readily deduced in a similar manner.

Let ABC be a spherical triangle; O the centre of the sphere.

Draw AD, AE, AM perpendicular to OB, OC and the plane BOC respectively.

Join MD, ME . Then

$$\begin{aligned} MD^2 &= AD^2 - AM^2 \\ &= OA^2 - OD^2 - (OA^2 - MO^2) \\ &= OM^2 - OD^2. \end{aligned}$$

*[These proofs are really due to our constant and most valued contributor, Mr. R. F. Davis, who discovered them in 1874, while an Undergraduate at Queen's College, Cambridge. Mr. Davis published them in the *Messenger of Mathematics*, Vol. IV., p. 102, and in the *Gazette*, Vol. I., p. 40. It is curious that such simple proofs have not found their way into general use. It is perhaps worth noting that they will be found in Sir Robert Ball's forthcoming *Spherical Astronomy*.]

In 1878 Isaac Todhunter characterised the proofs as "interesting and elegant," and intended to introduce them into a new edition of his *Spherical Trigonometry*. J. Casey in 1888 wrote of "these beautiful proofs" that "they are certainly new to me, and I have no doubt they are original." He also expressed his intention to insert them in an Appendix to his *Spherical Trigonometry*. We are glad to have the opportunity of once more drawing attention to them.

W. J. G.]

Therefore MD is perpendicular to OB . Similarly ME is perpendicular to OC . Let the triangles ADO , AMD , AME , AEO revolve about OD , DM , ME and EO respectively, into the plane BOC . We thus obtain the annexed figure, and identify a , b , c , B , C therein.

Draw EF perpendicular to OD and MG perpendicular to EF .

We have $OD = OF + FD$, or, R being the radius OA ,

$$R \cos c = R \cos b \cos a + R \sin b \cos C \sin a \quad (FD = MG),$$

$$\text{i.e.} \quad \cos c = \cos a \cos b + \sin a \sin b \cos C. \dots\dots\dots(1)$$

Again $EF = FG + GE$, that is

$$R \cos b \sin a = R \sin b \sin C \cot B + R \sin b \cos C \cos a;$$

or, dividing by $R \sin b$

$$\cot b \sin a = \cot B \sin C + \cos C \cos a. \dots\dots\dots(2)$$

Again $R \sin b \sin C = AM = R \sin c \sin B$;

$$\therefore \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}. \dots\dots\dots(3)$$

The modification of the above figure when b or c is greater than 90° presents no difficulty. C. S. JACKSON.

123. [L.¹ 1. a.] Note on the Parabola.

I. The general equation to the parabola must be of the form

$$(x - A)^2 + (y - B)^2 = (Lx + My + N)^2 \{L^2 + M^2\},$$

where A , B are the coordinates of the focus and $Lx + My + N = 0$ is the equation of the directrix.

The above equation becomes by transposition $(Mx - Ly)^2 + \dots = 0$, showing that the terms of the second degree form a perfect square and represent a line through the origin perpendicular to the directrix, that is, the diameter through the origin.

II. To reduce the general equation $(ax + \beta y)^2 + 2gx + 2fy + c = 0$ to this form.

Assume $\beta x - \alpha y + \kappa = 0$ as the equation to the directrix, where κ is at present undetermined and has to be found. Then

$$(a^2 + \beta^2)(x^2 + y^2) + 2x(g + \beta\kappa) + 2y(f - \alpha\kappa) + c + \kappa^2 = (\beta x - \alpha y + \kappa)^2,$$

and coordinates of focus are

$$\begin{aligned} &-(g + \beta\kappa) / (a^2 + \beta^2), \\ &-(f - \alpha\kappa) / (a^2 + \beta^2). \end{aligned}$$

Also

$$\begin{aligned} (g + \beta\kappa)^2 + (f - \alpha\kappa)^2 &= (a^2 + \beta^2)(c + \kappa^2), \\ \kappa &= \{f^2 + g^2 - c(a^2 + \beta^2)\} / 2(\alpha f - \beta g). \end{aligned}$$

Thus the focus and directrix are determined.

III. The axis is a line through the focus parallel to the diameter through the origin

$$\begin{aligned} \alpha \left(x + \frac{g + \beta\kappa}{a^2 + \beta^2} \right) + \beta \left(y + \frac{f - \alpha\kappa}{a^2 + \beta^2} \right) &= 0. \\ \alpha x + \beta y &= (\alpha g + \beta f) / (a^2 + \beta^2). \end{aligned}$$

IV. The latus rectum is twice the perpendicular from the focus on the directrix

$$= 2 \frac{-\beta(g + \beta\kappa) + \alpha(f - \alpha\kappa) + \kappa(a^2 + \beta^2)}{(a^2 + \beta^2)^{\frac{3}{2}}} = 2 \frac{\alpha f - \beta g}{(a^2 + \beta^2)^{\frac{3}{2}}}.$$

R. F. DAVIS.