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X. *On Sir Thomas Wrightson's Theory of Hearing.* By
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IN his recently published book entitled "The analytical Mechanism of the Internal Ear," Sir Thos. Wrightson has put forward a new theory regarding the much debated question of the means by which the ear is enabled to separate out the constituent pitches in a compound aerial vibration. His theory has something in common with that which was proposed by Voigt with regard to the origin of combination tones,* inasmuch as in both theories attention is directed to the geometrical features of the curve which represents the motion or pressure of the air. In Voigt's theory the points of maximum and minimum displacement were shown to lie on certain sine curves having the frequencies of the combination tones, and this geometrical fact was supposed to give rise to sensations of the tones in question. Wrightson takes into consideration these stationary points along with the points where the curve crosses the axis. He groups all these two kinds of points together and calls them "Impulse Points," on the ground that there is "an impulse in the stapes and the liquid moved by it, not only where pressure and velocity become nil, which is synchronous with the crossing points of the pressure air-wave, but also at the crest of the wave where the acceleration of velocity and increase of pressure cease" (*loc. cit.*, p. 93). Having drawn a large number of curves got by compounding simple harmonic vibrations of different frequencies and having examined the spacing of their impulse-points, the author finds that there are in each complete cycle some points whose distance corresponds approximately to the frequencies of the component vibrations, others to the octaves below these pitches, others again to the summation and difference tones. These are all called "effective" points, and are held sufficient to account for the sensations of the corresponding tones. In all the special cases enumerated nearly all the impulse-points are stated to be "effective" in one way or another. The degree of approximation of the spacings to the correct interval is not stated.

* Cf., W. B. Morton and Mary Darragh, "On the Theories of Voigt and Everett Regarding the Origin of Combination Tones," in these "Proceedings," Vol. XXVII., p. 339, 1915.

I have not been able to follow clearly the argument by which the author seeks to establish the effectiveness of the impulse-points in relation to the mechanism of the ear. The purpose of the present note is, first, to indicate a method by which the distribution of the "impulse-points" can be examined without going through the labour of drawing each separate case, and second, to point out some difficulties in the way of accepting as valid the fundamental assumptions of the theory. Let the compound vibration be represented by

$$y = a \sin mx + b \sin n(x + \delta).$$

We shall suppose $m < n$, and shall call m and n the frequencies of the component vibrations. The points where the curve crosses the axis are given by

$$\sin mx / \sin n(x + \delta) = -b/a.$$

Accordingly if we plot the curve $y = \sin mx / \sin n(x + \delta)$ its intersections with the line $y = -b/a$ will give the positions of the points in question. We can thus, by altering the level of a horizontal line crossing the curve, follow the displacements of the zeroes of our compound vibration-curve, as the relative amplitudes of the components vary while a definite phase-relation, determined by δ , is maintained. It is easy to verify that, if the horizontal line is taken at levels both above and below the axis, we can exhaust all possible cases

by letting δ run from 0 to $\frac{\pi}{2mn}$, in separate graphs.

The curve $y = \sin mx / \sin n(x + \delta)$ goes to infinity at the zeroes of the denominator. We can get a more compact graph by working only with amplitude-ratios lying between 0 and 1, so for cases in which the amplitude of the higher note n is the greater we plot the reciprocal $y = \sin n(x + \delta) / \sin mx$. The two curves can be combined in one diagram on which $y =$ whichever of the two expressions is not greater than unity, the two parts of the graph being drawn with a full and a broken line respectively.

The stationary points on the vibration-curve are given by

$$ma \cos mx + nb \cos n(x + \delta) = 0, \text{ or}$$

$$m \cos mx / n \cos n(x + \delta) = -\frac{b}{a},$$

and so can be discussed in a similar manner by plotting the graph $y = m \cos mn / n \cos n(x + \delta)$ or its reciprocal.

The accompanying figures show these graphs for the interval of a major sixth $m=3$, $n=5$, with $\delta=0$, 3° and 6° , the curves for the two classes of impulse-points being combined on one diagram. The positions of the zeroes are given by the curves marked with a nought at the top and bottom, the curves giving the stationary points are marked with a cross.

Using these graphs we can trace as follows the displacements of the impulse-points as we pass from one note alone to the other alone, through all mixtures of the two, while the phase-relation remains constant.

The middle line of the diagram is crossed by the full-line curves at equi-distant points corresponding to the zeroes and crests of the lower note existing alone. If a horizontal line, starting from the central position is pushed upwards its points of intersection with the curves shift, showing the effect on the positions of the impulse-points of the addition of more and more of the higher note, the distance of the horizontal line from the axis being the ratio of the amplitude of the higher note to that of the lower. Fresh intersections appear when the moving line meets the loops of the graph and before the top of the diagram is reached (equal amplitudes) the number of points corresponds to the higher frequency. If now the line is brought down again its intersections with the *dotted* lines give the impulse-points when the higher note has the greater amplitude, the height above the axis now being the ratio of amplitude of lower to that of higher. Finally on reaching the axis again we have the spacing corresponding to the higher note alone. By carrying out the same movement on the lower half of the diagram we get the case in which one of the two component vibrations is reversed in phase.

It is possible to make certain general statements regarding the number of points of each class. The fresh points appear when the line is at a stationary value of y .

Considering first the zeroes, $y = \sin mx / \sin n(x + \delta)$ is stationary when $\tan mx / \tan n(x + \delta) = m/n$. It is easy to show that the corresponding value of y lies between m/n and unity, ($m < n$). For

$$\frac{\sin mn}{\sin nx} = \sqrt{\frac{1 + \cot^2 n(x + \delta)}{1 + \cot^2 mx}} < 1 \text{ since } \cot mx > \cot n(x + \delta)$$

and

$$\frac{\sin mx}{\sin nx} = \frac{\tan mx}{\tan n(x + \delta)} \sqrt{\frac{1 + \tan^2 n(x + \delta)}{1 + \tan^2 mx}} > \frac{\tan mx}{\tan n(x + \delta)} = \frac{m}{n}.$$

FIG. 3 — For $y = a \sin 3x + b \sin 5(x + 6^\circ)$.

It follows that when b/a , *i.e.*, amplitude of the higher note divided by that of the lower, is less than m/n , the number of zeroes is dictated by the lower note; if this ratio is greater than unity, by the upper note. For values of b/a lying between m/n and unity the number of zeroes on the compound vibration-curve depends on the phase relation-between the component vibrations.

The energies of the two components are as m^2a^2 to n^2b^2 , and this may be taken as the ratio of the *physical* loudnesses of the notes. The rule just stated concerning the number of zeroes may then be stated as follows. The lower note dominates when it is louder than the upper, but the upper note does not certainly dominate unless its loudness exceeds that of the lower in a greater ratio than n^2 to m^2 .

When the same method is applied to the maxima and minima of the curve, the critical values of b/a are found to be m^2/n^2 and m/n instead of m/n and unity. In terms of loudness this leads to a statement just the reverse of the former one; it is now the upper note which settles the number of points when it is the louder. The lower note does so, for all phase-relations, only when its loudness exceeds n^2/m^2 times that of the higher note.

Sir Thomas Wrightson's theory rests equally on two foundations, *viz.* : (1) That an impulse is communicated to the ear alike at the maxima and minima of the vibration-curve and at the points where the curve crosses the axis. (2) That the distribution of these impulse-points on the curve is such as to convey the sensations of the pitches which are heard. These two matters can be discussed independently of each other. I confine myself in this note to the second point, assuming for the moment that the supposed impulses actually occur. The following difficulties present themselves.

1. A point is classed as "effective" if it is preceded or followed (but not immediately) by another point at an interval corresponding to the frequency either of one of the two component pitches, or of the octave below either or of a combination-tone. The spacings of the combination-tones may be supposed to give rise to the sensations of those tones, but cannot help the ear to discern the two primaries. Further, there seems no sufficient reason for calling in the assistance of the lower octaves of the primaries. It is stated that these "assist the harmony" (p. 27), but no explanation is offered of the fact that they are not heard as pitches equally with the

primaries. If points reckoned effective by virtue of spacings of these two categories be ruled out, it makes a substantial reduction in the proportion of effective points.

But, independently of this, the fundamental difficulty is to see why the ear should pick out these particular spacings and ignore the others. With so many distances to choose from it is not surprising that a number can be found to approximate to any desired spacing. In order to make out a case for the special effectiveness of a spacing in impressing itself on the ear, it seems necessary to show that it occurs sufficiently often to raise it out of the mass.

I have gone into detail in one special case, the first one used by the author in explaining his theory (p. 26). This corresponds to the top line on my diagram (1), the interval of a major sixth when the notes have equal amplitudes and the two vibrations pass through zero at the same instant in opposite directions, *i.e.*, the case $y=a(\sin 3x-\sin 5x)$. We may call the notes *C* with period 10, and *A* with period 6. The complete cycle of the compound curve has period 30.

In taking a census of the spacings presented by this case I confine myself to distances under the half-cycle 15. Taking the 10 points marked with noughts and crosses (zeroes and stationary points respectively) on the left-hand half of the diagram, each of these is the left-hand limit of nine intervals, measured forward, which lie under the prescribed distance, giving $10 \times 9 = 90$ spacings in all. These are made up of 39 different lengths, which are repeated, through the symmetry of the diagram; one occurs six times, six occur four times, 28 twice, and four once. Among those which occur twice are lengths 9.97 and 5.89. The former is a close approximation to *C*; the latter a worse approximation to *A*. These spacings are indicated by the letters above the diagram. There are two other lengths, each occurring twice, which lie at about two-thirds of a semitone on opposite sides of *C*, *viz.*, 9.64 and 10.39. There is no other distance lying within a semitone of *A*. Thus, among the 90 intervals there are only six which could by any possibility suggest the pitch *C* and only two to suggest *A*. On the other hand, there are six exact intervals of 7.5, corresponding to the note *F* between *C* and *A*, which is not heard at all. There are 10 spacings lying within a semitone of the period 11.

It does not help much to bring in the lower octaves. There are two lengths of 12.01 and two of 12.10. To get approxima-

tions to 20 we should have to expend the range of spacings considered, and this would bring in a largely increased number of useless intervals.

2. Another consideration is the alteration brought about in the spacings of the impulse-points when the relative amplitudes are changed. Suppose, for example, that the ear has seized on the spacing marked *CC* in the case just considered, of equal amplitudes. The physical loudnesses of *C* and *A* are then as 9 to 25. Now suppose the loudness of *C* to be gradually increased to equality with *A*. The displacements of the impulse-points can be followed by bringing the horizontal line down to the level marked 0.6 and watching its intersections with the full curves. It will be seen that the two points in question separate to a distance 11.3, corresponding to a pitch below *B* flat, so this particular spacing would cease to be available long before equality of loudness is reached. The change is still more marked in the case of the points marked *AA*, where there is an approach equivalent to a rise of a fifth.

It may, of course, be urged that as certain pairs of points become gradually unavailable other pairs will become available. But it is hard to reconcile such an inexact and capricious means of determination with the certainty and constancy of the pitch-sensations to be explained.

A similar objection might be based on the alterations in the spacings which are brought about by changing the phase-relation with constant loudness-ratio. This indeed would be a more cogent argument because such progressive changes in phase-relation must always occur except when the tuning is perfect. In any case the actual relation between the phases is a matter of chance and has no influence on the discrimination of the pitches by the ear.

3. The simple harmonic air-motion which gives the sensation of a single pure tone has four impulse-points in each period, viz., two zeroes and two stationary points. In other words if the frequency of the note is n the impulses, according to the theory, fall on the ear with frequency $4n$. One would expect then that also in the case of a compound disturbance the sensation of pitch n would depend on the recurrence of impulses with frequency $4n$, whereas it is sought to be explained by spacings of frequency n .

4. The author adduces as an analogy the case of Seebeck's siren, in which a current of air is directed against a row of

holes spaced round the circumference of a rotating plate. I would suggest that a rough test of his theory might be made if the holes were given the relative positions of the consecutive impulse-points on a compound vibration-curve. This would be analogous to the plan adopted by Koenig when he sought to establish, by his "wave-siren," his views as to the effect of phase-relation upon quality of tone. It would be open to criticisms similar to those urged by Helmholtz against Koenig, but if the arrangement succeeded in giving clearly the two primary pitches this would be recognised as giving support to Sir Thomas Wrightson's views.

The theory has already attracted a considerable amount of attention, specially from those who are dissatisfied with the Helmholtz theory of the mechanism of hearing. Every attempt to discover a fresh point of view in so difficult a matter is to be welcomed.

The book contains a valuable and clear account of the internal anatomy of the ear, in which Prof. Keith has collaborated. Such discussions as I have seen have been directed to the internal processes and have not touched on those preliminary assumptions to which the criticism of the present note is directed.

ABSTRACT.

The theory seeks to explain the power possessed by the ear of analysing into its component tones a compound aerial disturbance. It assumes that (1) impulses act on the mechanism of the ear corresponding to the maxima and minima of the compound vibration-curve, and also to the points where the curve crosses the axis; (2) that among the spacings of these impulse-points there is a preponderance of intervals which approximate to the periods of the component tones, their lower octaves and their combination tones, and that these spacings determine the sensations of the component tones. The present note is concerned with the second of these assumptions. Graphs are drawn which exhibit the way in which the distribution of impulse-points varies when relative intensities and phase-relation of the component notes are changed. Difficulties are found in (1) the large number of other spacings presented to the ear, (2) the variations of the spacings with loudness-ratio and phase relation, (3) the fact that in a single pure tone the spacing is a quarter of the period of the vibration.

DISCUSSION.

Prof. BAYLISS said he would like to make a mild protest against Prof. Morton's statement that physiologists welcomed Wrightson's Theory. It is really the anatomist who says that there is no structure in the ear capable of acting as a resonator, ignoring Helmholtz's demonstration that a membrane stretched unequally in different directions will resonate.

Personally he saw no objection to the resonance theory. On Sir Thos. Wrightson's theory there were four impulse points per vibration. If you reduce the frequency of vibration far enough you cease to hear a sound and hear the separate vibrations. Why do you not notice four impulses per vibration in this case. Another important point is one that Lord Rayleigh raised. We can hear sounds up to 30,000 per second. Now from what we know of nerve fibres, they are completely inactive for at least a thousandth of a second after receiving an impulse so they cannot possibly record more than a thousand separate impulses per second. Thus, on Wrightson's theory, all sounds requiring more than 1,000 impulse points per second should sound alike.

Principal SKINNER was interested to hear Mr. Bryliss's opinion that the material of which the body is made is not acoustically the best. He had always had a difficulty in understanding how the cavity of the mouth could act as a resonator at all; the vibrations should be damped down so quickly.

Prof. BRAGG asked if it was not the case that a sound was recognisable if two or three consecutive vibrations were present. This would explain Mr. Skinner's difficulty, for even if the vibrations of speech were very rapidly damped out, there might still be sufficient of them to give the note its characteristics.