13. That of twenty-two lighthouse observers between Cape Wrath and the Mull of Galloway, who were situated on the older formations (Laurentian, Cambrian, and Metamorphosed Lower Silurian), eleven felt the shock, whilst of thirteen observers situated on newer rocks, it made itself known only to two of them ; and that the earthquake was therefore more generally felt on the older rocks of Scotland.

14. That stations situated near one another, and on the same formation, did not necessarily both receive the shock, and that faults or trap dykes did not seem to affect the passage or intensity of the wave in any way.

15. That the observations of time at Armagh, Belfast, and Omagh, show that the shocks at these places were most probably propagated direct from Phladda in Scotland, and that the severity of the shock, and the "rumbling" noises heard in and around Leterkenny, were probably due to a second and local source of disturbance, generated by the arrival of the shock from Phladda.

2. On the Classification of Statistics. Part I. By Mr P. Geddes.

3. On Professor Cayley's Theorem regarding a Bordered Skew Determinant. By Mr Thomas Muir, M.A.

(This paper is to be found in the *Quarterly Journal of Mathematics*, vol. xviii.).

4. The Law of Extensible Minors in Determinants. By Mr Thomas Muir, M.A.

5. Additional Note on a Problem of Arrangement. By Mr Thomas Muir, M.A.

1. The present note is a continuation of a short paper which appeared in the Proceedings of the Royal Society of Edinburgh for session 1876–77. The problem in question is that referred to in Professor Tait's "Memoir on Knots," viz. :—*To find the number of possible arrangements of a set of* n *things, subject to the conditions that the first be not in the last or first place, the second not in the*

first or second place, the third not in the second or third place, and so on.

It being understood that U_n denotes the said number of arrangements, the concluding lines of the paper referred to will show to what extent the problem was solved.

"Hence for the determination of U_n when U_{n-1}, U_{n-2}, ... are known we have

$$\left.\begin{aligned} U_n = (n-2)U_{n-1} + (2n-4)U_{n-2} &+ (3n-6)U_{n-3} \\ &+ (4n-10)U_{n-4} + (5n-14)U_{n-5} \\ &+ (6n-20)U_{n-6} + (7n-26)U_{n-7} \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &+ \frac{1-(-1)^n}{2}, \end{aligned}\right\} \quad (1)$$

where the coefficients proceed for two terms with the common difference $n-2$, for the next two terms with the common difference $n-4$, for the next two terms with the common difference $n-6$, and so on.

"And as it is self-evident that $U_2 = 0$, we obtain

$$\begin{aligned} U_3 &= 1U_2 + 1 &&= 1 \\ U_4 &= 2U_3 &&= 2 \\ U_5 &= 3U_4 + 6U_3 + 1 &&= 13 \\ U_6 &= 4U_5 + 8U_4 + 12U_3 &&= 80 \\ U_7 &= 5U_6 + 10U_5 + 15U_4 + 18U_3 + 1 &&= 579 \\ U_8 &= 6U_7 + 12U_6 + 18U_5 + 22U_4 + 26U_3 &&= 4738, \end{aligned}$$

and so forth."

What is aimed at now is to reduce the above equation of differences and thereafter to obtain the generating function of U.

2. From (1) we have

$$\left.\begin{aligned} U_{n-2} = (n-4)U_{n-3} + (2n-8)U_{n-4} &+ (3n-12)U_{n-5} \\ &+ (4n-18)U_{n-6} + (5n-24)U_{n-7} \\ &+ \cdots\cdots\cdots\cdots\cdots\cdots \\ &+ \frac{1-(-1)^n}{2}, \end{aligned}\right\}$$

and therefore by subtraction

$$\begin{aligned} U_n - U_{n-2} = (n-2)U_{n-1} &+ (2n-4)U_{n-2} \\ &+ (2n-2)(U_{n-3} + U_{n-4} \ldots\ldots + U_3) \quad . \ . \ (2). \end{aligned}$$

Consequently, of course,

$$U_{n-1} - U_{n-3} = (n-3)U_{n-2} + (2n-6)U_{n-3} + (2n-4)(U_{n-4} + U_{n-5} + \ldots\ldots + U_3)\,,$$

and thus we are enabled to eliminate $U_{n-4} + U_{n-5} + \ldots + U_3$. The result of doing so is

$$\begin{aligned}&(n-2)(U_n - U_{n-2}) - (n-1)(U_{n-1} - U_{n-3})\\ &= (n-2)^2 U_{n-1} + \left.\begin{matrix}(n-2)(2n-4)\\ -(n-1)(n-3)\end{matrix}\right\} U_{n-2} + \left.\begin{matrix}(2n-2)(n-2)\\ -(n-1)(2n-6)\end{matrix}\right\} U_{n-3}\,,\end{aligned}$$

or

$$(n-2)U_n = (n^2 - 3n + 3)U_{n-1} + (n^2 - 3n + 3)U_{n-2} + (n-1)U_{n-3} \quad (3).$$

Putting $(n-1)$ for n we have also

$$(n-3)U_{n-1} = (n^2 - 5n + 7)U_{n-2} + (n^2 - 5n + 7)U_{n-3} + (n-2)U_{n-4}\,,$$

and therefore from this and (3) by subtraction

$$(n-2)U_n = (n^2 - 2n)U_{n-1} + (2n-4)U_{n-2} - (n^2 - 6n + 8)U_{n-3} - (n-2)U_{n-4}\,,$$

or $$U_n = nU_{n-1} + 2U_{n-2} - (n-4)U_{n-3} - U_{n-4} \quad \ldots\ldots \quad (4)$$

Further, partitioning the term $2U_{n-2}$ into $\frac{n}{n-2}U_{n-2} + \frac{n-4}{n-2}U_{n-2}$, this may be written in the form

$$U_n - nU_{n-1} - \frac{n}{n-2}U_{n-2} = \frac{n-4}{n-2}\left(U_{n-2} - \overline{n-2}\,U_{n-3} - \frac{n-2}{n-4}U_{n-4}\right)$$

or, say,

$$V_n = \frac{n-4}{n-2}V_{n-2}\,,$$

whence

$$V_n = \frac{n-4}{n-2} \cdot \frac{n-6}{n-4} \cdot \frac{n-8}{n-6} \cdots\cdots \frac{2}{4}V_4\,,$$

or

$$= \frac{n-4}{n-2}\;\frac{n-6}{n-4}\;\frac{n-8}{n-6} \cdots\cdots \frac{3}{5}V_5\,,$$

according as n is even or odd. But

$$V_4 = U_4 - 4U_3 - 2U_2 = -2\,,$$

and

$$V_5 = U_5 - 5U_4 - \frac{5}{3}U_3 = \frac{4}{3}.$$

Hence

$$V_n = -\frac{4}{n-2}\ (n \text{ even}),$$

$$= +\frac{4}{n-2}\ (n \text{ odd}),$$

that is

$$U_n - nU_{n-1} - \frac{n}{n-2}U_{n-2} = (-1)^{n-1}\frac{4}{n-2},$$

or

$$U_n = nU_{n-1} + \frac{n}{n-2}U_{n-2} + (-1)^{n-1}\frac{4}{n-2} \quad . \quad . \quad . \quad (5).$$

From this the successive values of U_n are got with ease.

3. To obtain the generating function of U we return to (4), and write it in the form

$$U_n = (n-1)U_{n-1} + U_{n-1} + 2U_{n-2} - (n-3)U_{n-3} + U_{n-3} - U_{n-4},$$

so that if u, a function of x, be the generating function, its differential equation is at once seen to be of the form

$$u = x^2\frac{du}{dx} + xu + 2x^2u - x^4\frac{du}{dx} + x^3u - x^4u + \phi(x).$$

By trial, however, $\phi(x)$ is readily found to be $x^5 - 2x^4 + x^3$, consequently the equation is

$$(x^4 - x^2)\frac{du}{dx} + (x^4 - x^3 - 2x^2 - x + 1)u = x^5 - 2x^4 + x^3.$$

Integrating in the usual way we first find

$$\int \frac{x^4 - x^3 - 2x^2 - x + 1}{x^4 - x^2}\,dx = x + \frac{1}{x} - \log\frac{x^2-1}{x},$$

and $\therefore$
$$\left.\begin{aligned} u = exp\left(-x + \frac{1}{x} + \log\frac{x^2-1}{x}\right) \\ + \int\left\{ exp\left(x + \frac{1}{x} - \log\frac{x^2-1}{x}\right) \times \frac{x^5 - 2x^4 + x^3}{x^4 - x^2}\right\} dx. \end{aligned}\right\}$$

$$= \left(x - \frac{1}{x}\right)e^{-\left(x+\frac{1}{x}\right)}\int\left\{ e^{x+\frac{1}{x}} \times \frac{x}{x^2-1} \times \frac{x^3(x-1)^2}{x^2(x^2-1)}\right\} dx$$

$$= \left(x - \frac{1}{x}\right)e^{-\left(x+\frac{1}{x}\right)}\int e^{x+\frac{1}{x}}\frac{x^2}{(x+1)^2}\,dx.$$

This does not really differ from Professor Cayley's result (Proc. R.S.E. 1876–7). The apparent difference is due to the fact that in the one case u is assumed to be of the form $U_3x + U_4x^2 + \ldots$, and in the other of the form $U_3x^3 + U_4x^4 \ldots$.