CONCERNING THERMAL CONDUCTIVITY IN IRON.

BY EDWIN H. HALL.

HAVING had occasion of late to examine with some care the information given by Landoldt and Börnstein and by Winkelmann concerning the thermal conductivity, k , of wrought iron and the temperature coefficient, α , of k , I have discovered so many inaccuracies, especially in Winkelmann, and have been so much impressed by the inconclusiveness of the showing, that I have felt moved to write a somewhat critical résumé of the methods and results in this limited but much worked field of investigation. I am the more induced to undertake this labor by the fact that even the most recent books published in English follow the custom of quoting the results obtained by Forbes as if they were the only ones worthy of especial mention, in spite of the fact that these results, for the best known of his bars, were long since disproved and superseded by work done on the same bar in the same laboratory.

It is true that Holborn and Wien published¹ some years ago the results of a discussion somewhat similar to that which I propose, but they did not publish their discussion; and, moreover, I find that in a number of particulars, especially in regard to the value of α , my conclusions are different from theirs. Where so much experimental work has been done, there is room for more than one attempt to estimate the value and results of that work. It is my hope to reach conclusions which may help to direct and make profitable future investigations. The recent interesting paper² of Liebenow, *Zur Thermodynamik der Thermo-ketten*, may seem to bring near the possibility of determining the thermal conductivity of metals from a study of their electrical conductivity and thermoelectric properties. But, whatever may be the final outcome, one of the first effects of Liebenow's paper must be to stimulate renewed

¹ Zeitschr. Ver. Deutsch. Ingen., Vol. 40, 1896.

² Wied. Annalen, June, 1899.

interest in the results obtained from the methods heretofore used for the measurement of thermal conductivity.

Landoldt and Börnstein give ¹

	Temp.	k .	Observer.
Eisen,		.1638	Neumann.
“	über 0°	.1587	Berget.
“	0°	.1988	Angström.
“	0°	.1665	Lorenz.
Schmiedeeisen,	0°	.2070	Forbes.
“	39°	.1485	H. Weber.

and the following temperature coefficients, that is, values of the constant α in the formula $k = k_0(1 + \alpha t)$:

Eisen	— 0.0002282 Lorenz.
“ gewöhnl, 0 bis 300°	— 0.000611 Mitchell.
“ gekühlt, 0 bis 300°	+ 0.000706 “

Winkelmann ² gives in his table, without the dates,

Metal.	Temp.	k .	Observer.	Date.
Eisen,	0°	0.1665	Lorenz,	1881
“	0°	0.1988	Angström,	1861-4
“	über 0°	0.1587	Berget,	1890
“	ca. 15°	0.1648	Neumann,	1862
“	ca. 15°	0.1133	Wiedemann u. Franz,	1853
“	0°	0.1509	Mitchell,	1887
“	0°	0.172	Stewart,	1893
Schmiedeeisen,	0°	0.2070	Forbes,	1862-5
“	39°	0.1485	H. F. Weber,	1880

The temperature coefficients given by Winkelmann are

— 0.000038	from the work of Lorenz,
— 0.000517	“ “ “ “ Angström,
— 0.0006838	“ “ “ “ Forbes.

I shall follow in general a chronological order in my discussion, departing from it whenever there seems to be good reason for so doing.

¹ Physikalische Chemische Tabellen.

² Handbuch der Physik., part 2 of vol. 2.

WIEDEMANN AND FRANZ.

“Es wurden zwei Stangen I. und II. von 5 mm. Dicke untersucht.” This is, I believe, the whole explicit description of the iron, although it can be inferred from the description of the apparatus that the length of the bars was about 50 cm.

They worked with a considerable number of metals, but obtained comparative results only. It therefore seems idle to attempt now to derive absolute values of k from their data.

They appear to have been the first to make a systematic comparison of thermal and electric conductivities, although Forbes had many years before pointed out the apparent connection between these two properties in metals. Wiedemann and Franz did not measure, even comparatively, the electric conductivity of the bars; but trusted to the values found by others for similar materials. The table given by Winkelmann, Vol. 2³, p. 277, to show their results differs in several particulars from that given by Wiedemann and Franz, and it is so badly aligned that in some cases it is impossible to tell to which metal the numbers given apply.

It is to be observed that Landoldt and Börnstein give no absolute values on the authority of these early experimenters.

Forbes, Angström and Neumann, working each by his own peculiar method, obtained results which they published about the same time. The method of Forbes was simpler, in theory, than the others, and will be discussed first.

FORBES (and MITCHELL).

No other experiments upon thermal conductivity are so familiar to English-speaking people as those of Forbes, and the method which he originated and used is so well known that description of its general features is unnecessary here; and yet it is time for us to recognize the fact that the numerical results obtained by Forbes have only an historical interest, and that, if their publication in physical tables of the present day is to perpetuate a belief in their numerical accuracy, it would be far better to exclude them altogether from such tables. This statement I shall undertake to verify by what follows.

Forbes used two wrought-iron bars of somewhat different sizes. The better known of the two, called *D*, he describes a "a beautiful bar, $1\frac{1}{4}$ inch square and fully eight feet long." He gives its specific gravity as 7.79. The other bar was 1 inch square. Reduced to the C. G. S. system, the values which he found for k from the two bars were

$1\frac{1}{4}$ inch bar.	1 inch bar.	
0.207	0.1536	at 0° C.
0.1567	0.1293	" 100° "

The great difference between the values of k given by the two bars at the same temperature has been explained as probably due to a difference in the quality of the iron; but in view of the subsequent history of bar *D*, and the general average of values obtained for iron by other experimenters, this explanation seems inadmissible.

Each bar gave a large negative temperature coefficient for k . Tait, after Forbes, worked with bar *D* and applied corrections, which Forbes had not done, for change of specific heat with change of temperature. The result was to make the temperature coefficient almost or quite disappear. Then Mitchell worked with the same bar. In accordance with the advice of Tait he used sometimes the ordinary method of Forbes, heating one end of the bar while all the rest of its length was freely exposed to the air of the room, and sometimes combined the heating of one end with a cooling of the middle in order to get a steeper gradient of temperature in the cooler parts of the bar. This variation of method produced a marked change in the values found for k .

Forbes had expressed his results in terms of the foot, minute, and thermal capacity of a cubic foot of water. Mitchell departed from this tradition far enough to express his results in terms of the foot, minute, and thermal capacity of a cubic foot of the iron at some one temperature. It was therefore not necessary for him to determine the specific heat of the iron; but he did undertake to introduce into the values obtained for k at different temperatures correction for change of specific heat with temperature. It seems probable that he used a somewhat too large temperature coefficient in making this correction.

Certain thermometer corrections were applied by Mitchell, the

effect of which corrections was to lessen considerably the values of thermal conductivity found for high temperatures as compared with those found for low temperatures.

The numbers which follow and the words accompanying them I have copied from Mitchell's paper :

Temperature C.		0°	100°	200°	300°
Iron (Ordinary).	Not corrected for specific heat change.	.0127	.0115	.0103	.0091
	Corrected for specific heat change.	.0131	.0123	.0115	.0107
Iron (Cooled Bar.)	Not corrected for specific heat change.	.01154	.01162	.01170	.01178
	Corrected for specific heat change.	.0119	.01274	.01358	.01442

"Iron (Ordinary)" refers to the "ordinary" Forbes's method, heat applied to one end of the bar the rest of which is exposed to the air. This is the "Eisen gewöhnl" of Landoldt and Börnstein's table. "Iron (Cooled Bar)" refers to the varied method, in which the middle of the same bar was cooled by a stream of water while the end was heated. This is the "Eisen gekühlt" of the same table. The great difference between the results of these two very similar methods throws much doubt upon the accuracy of the results obtained from either or both, so far, at least, as relates to the temperature coefficient, which has a large positive value in one case and a still larger negative value in the other case.

The values given by Mitchell are to be turned into C. G. S. units by multiplying them by $15.48 C\rho$, where C is the specific heat at some one temperature, probably the temperature of the air, and ρ is the density. Winkelmann gets the value 0.1509 at 0°, which he puts opposite the name of Mitchell, by multiplying 0.0119 ("Cooled Bar," 0°) by $C = 0.105$ and $\rho = 7.8$. This reduction is probably not very far wrong, though a somewhat higher value for C might well have been used, but in reducing the values given by

Mitchell for k at higher temperatures, as Winkelmann does on page 280, he multiplies each by the specific heat of iron for the higher temperature in question, 100° , 200° , etc. The values which he thus gets for these higher temperatures are therefore doubly "corrected" for change of specific heat, once by Mitchell and again by Winkelmann. Fortunately, Winkelmann does not put these values into his final table.

Landolt and Börnstein give no value of k from Mitchell; but, as we have seen, they give two values of α , its temperature-coefficient from his work, not perceiving, we may suppose, that "Eisen gewöhnl," "Eisen gekühlt," and "Schmiedeeisen" (Forbes) are one and the same bar D , one of the national monuments of Scotland. The temperature-coefficients which they give are obtained correctly from the values of thermal conductivity "corrected for specific heat change," quoted above from Mitchell. Taken together these two coefficients almost completely extinguish each other; and I can not help thinking that they should be allowed to do so entirely, and disappear from tables of physical constants.

Whatever may be thought of the accuracy of the value k at 0° ; as finally obtained by Mitchell using his "cooled bar" method, it seems plain that the values 0.207 at 0° (Forbes) and 0.1509 at 0° (Mitchell) ought not to appear in the same table without an express statement that they were found from the same bar. Indeed the history of bar D goes far to show that the Forbes method is not well suited for the nicer problems of thermal conductivity, the uncertainty in estimating surface emissivity being too great.

We have not sufficiently definite data for a profitable estimation of the ratio between the thermal and electric conductivity in bar D .

STEWART.

R. W. Stewart also followed the method of Forbes, with some variations of detail, using a bar which he describes thus: "The iron bar was a $\frac{3}{4}$ -inch square bar, about $4\frac{1}{2}$ feet long, of ordinary commercial wrought iron, having its surface filed up and very lightly polished with black lead to secure uniformity of radiating power." "The density of the iron was found to be 7.556 at 0°

C." The specific heat was found by Bunsen's ice-calorimeter to be represented by the formula

$$C_t = 0.1095 (1 + .00073t),$$

t referring to the centigrade scale.

The thermometers of Forbes were replaced by a single thermo-electric couple, made of "a good soft iron wire and German silver," each wire about 0.5 mm. in diameter. The depth of the holes bored in the bar to receive the thermo-electric junction was about 1.4 cm., the diameter about 0.1 cm.

"One end of the experimental bar was immersed in a vessel of melted lead maintained at a constant temperature, and the rest of its length was exposed to the air, but protected from draughts and external radiation by a wide trough of sheet zinc." The dimensions of this trough are not given, but from the figure shown it appears to have been about 27 cm. deep, the bar being suspended so as to hang about half way between the bottom and the top, which was not covered.

"In taking a series of observations of the distribution of temperature along the bar, the heating was allowed to go on for five or six hours, and then the deflections corresponding to each hole along the bar were observed, up and down the bar, time after time, until a set of perfectly constant and concordant readings were obtained."

"The distribution of temperature along the bars [copper as well as iron was studied] in the stationary state was determined by inserting one junction successively into the small holes drilled in the bar, while the other junction was maintained at a constant temperature¹ by inserting it in a hole in a short iron bar resting on the bottom of the zinc trough. The deflections obtained were, in this way, proportional to the difference of temperature between the several points on the bar and the temperature of the surrounding air. It was found necessary to carefully insulate the ends of the wires of the couple from one another just above the junctions, and from the bars in which they were inserted; this was done by filing down the

¹ "This temperature was indicated by a thermometer placed with its bulb in a hole drilled in the [short] bar close to the small hole in which the junction was placed. In no case was the variation of temperature more than a fifth of a degree. The bar was a short massive iron bar, not the one used for cooling experiments."

wires, inserting a thin slip of silk between them, and then whipping them around with a fine silk thread. The outer surface of the thread charred slightly at the higher temperatures, but not sufficiently to destroy its efficiency." "The rates of cooling of the bars at different temperatures were determined by heating a piece of each bar, about a foot long, in a sand-bath to a temperature higher than required, and observing its rate of cooling under the same conditions as those to which the experimental bar was exposed. For this purpose it was suspended in the position originally occupied by the long bar, the thermo-electric junction inserted in a hole drilled at its middle point, and the deflection of the galvanometer observed at regular intervals during the cooling."

Winkelmann quotes from the abstract of Stewart's work given in the *Proceedings* of the Royal Society. But in the *Transactions*, a mistake having been discovered, Stewart gives

$$k_t = 0.175 (1 - .0015t).$$

I am of the opinion that little importance should be attached to the value of k obtained by Stewart and still less to his value of the temperature coefficient. A thermo-electric junction such as he describes, placed in a hole in the hot bar, but not in metallic contact with it, the greater part of the length of the wires being exposed to the air and the other junction being at a temperature near that of the air, would surely not have the temperature of the iron adjoining it. Moreover, we are told nothing about the location of the short bar carrying the colder junction except this, that it rested upon the bottom of the zinc trough—whether near the hot end or the cold end or the middle of the long bar, we do not know; but this one short bar was assumed to have the effective average temperature of the surroundings of the long bar and to hold the cooler junction at this same temperature.

Finally, the method of noting fall of temperature in the cooling bar, one foot long, by means of a single thermo-electric junction inserted at its middle point, is apparently inaccurate. The ends of the bar would certainly cool faster than the middle.

Near the end of his paper Stewart remarks, "Angström's method should be adopted in preference to Forbes'." Having now done

with researches by the method of Forbes, I return to the consideration of work done by his contemporaries.

ANGSTRÖM.

Angström used two bars of iron; the second, from which he obtained his best known results, was 117.8 cm. long and 3.5 cm. thick. I have found no description of its quality. Angström takes 0.8862 as the product of its specific heat by its density; but he uses precisely the same quantity for his other iron bar. It does not appear that he determined either the specific heat or the density.

Angström's general method of sending waves of temperature along the bar by alternate heating and cooling of one end, the remainder of the bar being exposed to the air, and noting the fluctuations of temperature at certain chosen points, is familiar to physicists and perhaps even more familiar to mathematicians. The mathematical discussion of the method, as given by Angström, treats specific heat and thermal conductivity as constants, it being assumed that the variation of these quantities, within the range of temperature fluctuations in the part of the bar studied, is too small to affect seriously the result arrived at. As to the validity of this assumption there has been discussion, which I do not propose to renew at present.

Whatever may be the merits of Angström's method, it is easy to show that he made near the end of his second paper¹ a large numerical error, the easy correction of which will bring his value of k for iron much nearer to the values obtained by most other investigators who have attempted to determine it.

The paper just referred to in Poggendorff contains the following record of data and experimental results:

EISENSTAB.

No.	Period.	$\frac{k}{c\delta}$	Temp.	
1	32'	10.487	43°.4	Einfache Reihe: Therm. No. 2 u 3
2	32	10.429	43 .7	Doppelte Reihe: Therm. A und B
3	32	10.279	18 .9	

¹ Poggendorff, Vol. 118, p. 423, has a translation of the original paper.

This is followed, after a page or more devoted to the discussion of a similar record for copper, by a passage, quoted below, in which Angström professes to deduce a formula for k , as dependent on temperature, from the data which I have just given. A brief examination is enough to show that his formula, which has a negative temperature coefficient for k , cannot be derived from the numbers he refers to, which evidently give a positive temperature coefficient for k . Suspecting a mistake in the data as given in Poggendorff, I went to the original paper¹ and found the following table, which differs from that given above in one place of one number. We must therefore discard the data as given in Poggendorff, and consider whether the value given for k by Angström is consistent with the original data as here reproduced:

JERNSTÅNGEN.				
No.	Period.	$\frac{k}{c\delta}$	Temp.	Anmärkingar.
1	32m	10.487	43°.4	Enkel serie; therm. no. II. o. III.
2	32	10.429	43 .7	Dubbel serie; therm. A o. B.
3	32	11.279	18 .9	“ “ “

The following discussion, by means of which Angström deduces his value of k , I quote from the article in Poggendorff, having compared the formulas in this discussion with those in the original passage and found them to be the same.

“Berechnet man den Werth von $\frac{k}{c\delta}$ aus den in der Tafel aufgestellten zwei letzten Bestimmungen für das *Eisen*, welche vollkommen vergleichbar mit einander sind, so erhält man

$$\frac{k}{c\delta} = 13.458 (1 - 0.002874 t)$$

und, wenn man setzt

$$c\delta = 0.8862$$

wird

$$k = 11.927(1 - 0.002874 t)."$$

Now if we take the last two lines of the original data and calculate for them the value of $\frac{k}{c\delta}$ for 0°, we get 11.927. But this is

¹Öfversigt af Kongl. Vetenskaps-Akademiens Förhandlingar, 1862, p. 24, Stockholm.

precisely the value which Angström gets for k at 0° . According to the data referred to by Angström, he should get

$$k_0 = 11.927 \times 0.8862 = 10.57;$$

and in general

$$k = 10.57 (1 - .002874t),$$

for this revision requires no change in the temperature-coefficient which will be found perfectly consistent with the data.

The fact appears to be that Angström, having before him the quantity 0.8862 which he was presently to use as a multiplier, accidentally let it get in as a divisor, so that its final use as a multiplier merely gave back the value of $\frac{k}{c\delta}$, instead of giving the value of k .

In at least one other particular the calculations of Angström can profitably be revised. He treats the product $c\delta$ as a constant, using the same value for one bar at 0° as for another at 50° . But I shall now take the values of $\frac{k}{c\delta}$ which he gives as the results of experiments at $43^\circ.7$ and at $18^\circ.9$, apply to these the values of $c\delta$ which appear suitable to the temperatures, and work out thus values for k at $43^\circ.7$ and at $18^\circ.9$, and finally at 0° .

There can be no great error in taking 7.8 as the value of δ at all the temperatures considered, if Angström used good wrought iron, as in all probability he did. For the values of c I shall use numbers which are obtained by a combination of those given by Lorenz with those found by Naccari.

Temp. Sp. ht.	Lorenz.				Naccari.		
	0°	50°	75°	100°	0°	15°	100°
	0.1050	0.1107	0.1136	[0.1165]	[0.1080]	0.1091	0.1151

The numbers in brackets are my extrapolations. Then

$$c = \left\{ \begin{array}{c} 0^\circ \\ 0.1050 \\ 0.1080 \end{array} \right\} 0.1065 \quad c = \left\{ \begin{array}{c} 100^\circ \\ 0.1165 \\ 0.1151 \end{array} \right\} 0.1158.$$

By interpolation

$$\begin{array}{cc} 18^{\circ}.9 & 43^{\circ}.7 \\ c = 0.1083 & c = 0.1106. \end{array}$$

Using these values of c , we get

$$k \text{ at } 43^{\circ}.7 = 10.429 \times 7.8 \times 0.1106 = 8.999,$$

$$“ “ 18^{\circ}.9 = 11.279 \times 7.8 \times 0.1083 = 9.528,$$

and by extrapolation k at $0^{\circ} = 9.93$. The general formula now becomes

$$k = 9.93(1 - .0021t),$$

in the units employed by Angström—the centimeter, gram and minute. Reducing to the ordinary system by dividing by 60, we have

$$k = 0.1655(1 - .0021t).$$

Holborn and Wien, in the paper to which reference has been made, give as the result of Angström's work

$$k = 0.1628(1 - 0.002874t),$$

but do not tell in what way they have revised his calculations.

The value 0.1988, k at 0° , which both Winkelmann and Landolt and Börnstein give in their tables on the authority of Angström, is obtained from his 11.927 by dividing by 60. The temperature coefficient of k from Angström's work is given by Winkelmann as—0.000517. I do not understand how this value was found. A little earlier, on p. 282, Winkelmann gives Angström's result, revised for the change of $c\delta$ with temperature and reduced to C. G. S. units, as

$$k = 0.1842(1 - 0.001562t),$$

but here again there is nothing to show just how the numbers are derived. One value of the temperature coefficient is a little more than three times as great as the other.

Angström recognized the fact that for a satisfactory comparison of thermal and electric conductivities both quantities should be found for the same piece of metal in the same state. Accordingly, he attempted to measure the electrical resistance, x , of his bars, but

encountered difficulties which baffled him, although he expressed the hope of overcoming them. Whether he finally succeeded I have not discovered. His results therefore tell us nothing directly in regard to the ratio $k \div \alpha$.

Angström's method has not proved attractive to investigators in general. Tait tried it and gave it up. I am not aware that anyone except Angström has published results obtained by it for iron. Probably the very large value of k standing in the tables against the name of Angström has heretofore discredited his method. This consideration is now in great measure removed by the examination which has just been detailed. This objection, however, holds against all methods using temperature waves, that it is difficult for the ordinary reader, if not for every one, to see what effect will be produced in the result by small inaccuracies of theory or data. Such methods may give sufficiently accurate results, but they do not carry conviction.

NEUMANN.¹

Neumann studied several metals. His whole published account of his work on these is contained in three pages; and the description is vague as well as brief. He used a method of variable temperature, heating one end of a bar or one section of a ring for a certain time and noting the changes of temperature after heating was stopped. Some features of his method can be seen in the work of H. Weber, which is presently to be described; for Weber avowedly followed Neumann as a guide, though perhaps with considerable deviations.

Neumann writes, "Les barres employées avaient 3 à 4 lignes de côté." The length is not given. "Dans quelques expériences j'ai aussi remplacé les barres par des anneaux." It seems not quite clear whether the results published were obtained from the bars or from the rings; but it is likely they came from the bars.

The specific gravity of the iron used was 7.74. Neumann gives 193 as the thermal conductivity of iron in the units employed, and says that this value can be reduced to the units used by Angström by multiplying by 0.0509. To reduce to C. G. S. units the

¹ Ann. de Chemie et Physique, Vol. 66, 1862.

193 should be multiplied by $0.0509 \div 60$, which gives $k = 0.1637$, practically the same value which Landolt and Börnstein put opposite the name of Neumann. I find nothing in Neumann to indicate the temperature for which his value of k is supposed to hold.

Neumann was especially interested in the ratio between thermal and electric conductivity, and he deduced comparative values of this ratio for the several metals which he used. He did not, however, determine the electric conductivity in absolute measure, or give any data from which we can now reduce it to that measure; so that the real ratio between the thermal and electric conductivities of the iron which he used cannot be found.

H. WEBER.

H. Weber "of Brunswick," not H. F. Weber as in Winkelmann, used the method of Neumann. The particular volume, No. 146, of Poggendorff, in which the original account of his work appears, is missing from the set which is at my command, and I have therefore had recourse to the translation of his paper which appeared in the *Philosophical Magazine*, vol. 44, 1872.

1. "Iron rod (annealed)," round; density 7.761; specific heat 0.1125 (temperature not given); length of bar 23.035 cm., diameter 0.75168 cm. "The density and the specific heat were specially determined, the latter according to the method proposed by Neumann."

After describing briefly the method of Angström, Weber says, "Neumann has now shown that the same problem can be treated in another way, and one more favorable for observation—and that it is more suitable for the determination of the two conducting-powers ["internal" and "external"] to subject not merely *one* end of a bar regarded as unlimited, but *both* ends of a bar of infinite length to the same periodic change of temperature in such a manner that when the two ends in the *oth* period [really the *first* period of observations after the bar has been brought into a state of regular alternation] have the temperatures u_0 and u_1 they in the next following take the temperatures u_1 and u_0 , in the 2d period (on the contrary) they are in the same state as in the *oth*, and in the 3d as

in the 1st, etc." The temperature u_1 was that of steam, u_0 about $4^\circ.6$. The length of a period was 5 minutes.

Assuming for the purposes of mathematical discussion, that the specific heat, the density, the internal and external conductivities, are constant quantities, an assumption which Weber defends only as an unfortunate necessity, he proceeds to show that determinations of k , the internal conductivity, and h , the external conductivity, can be found from observations of the temperature of the air, the temperature of the middle point of the bar (which temperature should prove to be a constant), and the difference of temperature between two other properly chosen fixed points on the bar at various intervals during each period.

Thermoelectric effects, by means of wires soldered to the bar at the points mentioned, were used to measure the temperatures required. "A thin iron wire and a thin German-silver wire were soldered in the middle of it [the iron rod] * * * opposite to each other. The ends of these two wires were soldered to two copper wires," the points of junction being near together "in the surrounding air" at a point the temperature of which was read from a sensitive thermometer. At two other points, one of which was $\frac{1}{6}$ of the length of the bar from its end and the other $\frac{4}{6}$ of the length from the same end, "two German-silver wires were soldered to the iron bar * * * and their ends, at about 6 inches from the bar, were soldered to the two copper wires which led to the same galvanometer as the wires before mentioned." Readings of the deflections produced by this latter thermo-electric couple were taken at intervals of 15 seconds, and, as this deflection was changing rapidly during much of the time, a careful study of the characteristics of the galvanometer was necessary. The mean temperature of the air was about $5^\circ.7$; that of the middle point of the bar, $33^\circ.1$. The estimated mean temperature of the whole bar during a "period," of 5 minutes, was $39^\circ.23$, which, according to Weber, "can only be regarded as a rough approximation."

There is a tone of intelligence, frankness and thoroughness about the paper of Weber which inspires confidence. I cannot help feeling, however, that the description and discussion of the thermo-electric

devices is too meagre. How thick and how long were the "thin" wires soldered to the middle of the bar? How much solder was used in making the junctions, and how great was the thickness of solder between the wires and the bar? These seem to me important questions in a case where, as here, some of the points of attachment were rapidly varying in temperature and all were much above the temperature of the room. It is unfortunate that Weber did not make such variations in his thermo-electric apparatus as to remove reasonable doubts concerning its accuracy.

On the other hand we feel somewhat reassured by the fact that he did vary considerably the surface condition of the bar, without finding any corresponding important change in the value derived for the thermal conductivity.

After giving a series of values obtained from "even periods," which agree well among themselves, and a corresponding self-accordant series from "odd periods," he says, "If we take the mean of the values of K [the internal conductivity] and H [the external conductivity] given by the even periods, and likewise of those derived from the odd periods, we obtain :

$$\begin{aligned} K &= 15.14, & H &= 0.00266, \\ K &= 14.56, & H &= 0.00266." \end{aligned}$$

The difference between "odd" and "even" is attributed to "want of absolute coincidence of the soldering-places with the points $x = \frac{1}{6}l$ and $x = \frac{4}{6}l$." For the mean we have

$$K = 14.85 \qquad H = 0.00266.$$

"The surface of the iron bar had not the highest polish. In order to try whether the quality of the surface exerted a perceptible influence on the value of K , the previous experiments were repeated with the same iron rod after its surface had been evenly coated with soot from a gas flame. The thickness of the coating was such that the metallic surface was just perceptible through it. Here the duration of the periods amounted to 10 minutes, in order to make the

observations in even and odd periods alternately, not consecutively as before." The mean results from the iron thus coated were

$$K = 14.79 \quad H = 0.00328,$$

an extremely satisfactory agreement for K , with the previous result.

It is unfortunate that the temperature at which the specific heat was found is not given. The value used, 0.1125, seems rather high for the mean temperature of the bar. An error of two or three per cent. may occur here, making the value obtained for K too large by that amount.

Weber's values of K and H are expressed in terms of millimeters, milligrammes and seconds. To reduce to C. G. S. units we must divide K by 100 and H by 10. We are concerned with K only, from which we get

$$k = 0.1485,$$

as given by Landoldt and Börnstein and by Winkelmann. This is referred to the mean temperature of the rod, about 39°.

Weber found the ratio of the thermal conductivity to the electric conductivity, which henceforth I shall call the ratio $k \div x$, to be 2458×10^4 at 44°.3C, in the units which he used. Reduced to the C. G. S. system this ratio becomes 2458.

L. LORENZ.¹

The iron used by Lorenz appears to have been exceptionally pure. It had a high density, 7.828 at 0°, and an electric conductivity greater than Landoldt and Börnstein quote for any other specimen of iron.

Lorenz gives from his own observations on this iron

Sp. Ht.	Elect. Cond.
$C_0 = 0.1050$	$x_0 = 10.374 \times 10^{-5}$
$C_{50} = 0.1107$	$x_{100} = 6.628 \times 10^{-5}$
$C_{75} = 0.1136$	

His bar was 30 cm. long and 1.5 cm. in diameter.

¹Wied., N. F., 13, pp. 422 and 582, 1881.

In his main experiments Lorenz used two methods. One was that of Forbes with various improvements. The other, which he considers the more exact, consisted in measuring the time rate of increase of mean temperature throughout a certain part of a bar, with a known difference of temperature gradient at the ends of the portion considered, and with a surrounding jacket kept at a known fixed temperature by flow of cold water or steam. Moderate differences of temperature between different parts of the bar and between the bar and jacket were used. The temperature measurements were made by means of thermo-electric junctions, usually of copper and German-silver, but sometimes of copper and iron. Heat was brought into the bar by end contact with a bar of copper kept hot by means of a lamp. Of course it was necessary to know the density of the bar and its specific heat at various temperatures. Careful observations to determine these quantities were made upon the bar itself or upon a portion cut from it.

It is impossible to read the papers of Lorenz on thermal conductivity without being impressed by the thoroughness of his work and the probable accuracy of his results. But his method as a whole does not strike one as simple or easy to carry out, and there is one detail of the apparatus which suggests to me the possibility of considerable error. The thermo-electric junction commonly used consisted of a copper wire 0.01 cm. thick and a German-silver wire 0.03 cm. in diameter soldered together end to end in a straight line. This junction rested within a hole 0.04 cm. diameter bored straight through the bar from side to side, and both wires were insulated from contact with the metal of the bar, the German silver within the hole being covered with a thin wrapping of silk for this purpose. For the purpose of finding the temperature gradient leading heat into or out of the parts considered, "thermo-electric double elements were used which consisted of short German-silver wires with both ends soldered to copper wires." Now it seems probable that the ends of any one of these German-silver wires would, under the circumstances described, differ decidedly less in temperature than the parts of the bar at the two holes connected by the wire. This objection must have occurred to Lorenz, though he does not, so far as I have observed, mention it. It is easy to see that error from this defect of

apparatus would in some measure eliminate itself, since it is the difference of two temperature gradients which enters into the calculations. Moreover, Lorenz states that he sometimes used copper-iron thermo-electric junctions, and sometimes junctions made by copper wires joined directly to the bar, without finding any material difference in the results obtained. Finally, it is to be said that the Forbes method, which Lorenz used for checking the results obtained by his first method, gave nearly the same values for thermal conductivity in most cases. The agreement in the case of iron was especially good, the Forbes method giving a result about $1\frac{1}{2}$ per cent. smaller than the other. It is true that similar errors may affect both methods as applied by Lorenz.

The original method used by Lorenz was particularly well designed for giving the temperature coefficient of k ; and the value which he found for this coefficient would be entitled to much consideration, even if it should appear that the values which he found for k were somewhat erroneous.

Lorenz found, by his preferred method, for iron

$$k_0 = 0.1665, \quad k_{100} = 0.1627,$$

in C. G. S. units.

The temperature coefficient from these values is 0.0002282.

Landolt and Börnstein report k and its temperature correctly from Lorenz; but Winkelmann in deducing the temperature coefficient forgets to divide by k_0 .

The ratio $k \div x$, as found by Lorenz for his iron bar at 0°C. , is 1605.

BERGET.¹

Berget made first a determination of the thermal conductivity of mercury in absolute measure, and afterward found the thermal conductivity of various solid metals, including iron, by comparison with the mercury. He found the gradient of temperature in a vertical column of mercury, of which the upper end was exposed to steam and the lower end was kept cool by melting ice. A "guard ring" of mercury, exposed to the same terminal temperatures as this

¹ Compt. Rend., 1887-'90. Journal de Physique, Vol. 7, p. 2, 1888, and Vol. 9, p. 135, 1890.

column, surrounded the column and prevented lateral escape of heat. The amount of heat transmitted by the shielded column was calculated from the amount of ice melted at its lower end, this latter quantity being found sometimes by use of a Bunsen ice-calorimeter, sometimes by weight. The temperature gradient was measured by means of thermo-electric couples, formed by iron wires leading through the guard cylinder into the interior column, with which they were in metallic contact.

The self-consistency of the results which Berget obtained for mercury is marvelous, but not greater than the admirable simplicity and directness of his method seems capable of giving. There was no need of determining specific heat or "surface conductivity."

In working with solid metals Berget superposed a column of mercury upon a column of the solid, each surrounded by its "guard ring," found the gradient of temperature in each by thermo-electric means after a stationary condition was reached, and then deduced the conductivity of the solid from that of mercury by the simplest of calculations.

The whole account of his work given by Berget is brief, and some details of apparatus or procedure are described less fully than the reader could desire; but the general method is so excellent that one can have little doubt concerning the accuracy of all parts of the investigation. It is therefore the more to be regretted that Berget has not told us the degree of purity, or even the specific gravity, of the metals which he studied. Perhaps it is not yet too late for this omission to be made good.

He determined, for solid metals, the mean thermal conductivity between 0°C. and 30°C. and declared its value to be, for iron, 0.1587.

He found the mean electrical conductivity of the same bar of iron to be, between 0°C. and 30°C. , 9.41×10^{-5} .

The ratio of the mean values of the two conductivities between these limits of temperature is therefore, according to Berget,

$$k \div x = 1687.$$

This may be taken as equivalent to

$$k_{15} \div x_{15} = 1687.$$

B EGLINGER.¹

Probably the broadest piece of work ever done by one person upon the thermal conductivity of iron is that of W. Beglinger, who determined the density, specific heat and thermal conductivity of 52 pieces of iron, including steel and cast iron, of many varieties. The following passages freely translated from his own descriptions will give an account of the general method and arrangement of apparatus.

"A broad cylinder, which in all points had the same initial temperature N , was from a determined moment, which we take as the zero point of the time, subjected to a continuous cooling by keeping its base continually at the temperature 0° , while the remainder of its surface was allowed to give off heat into a space kept likewise at the constant temperature 0° . The time-rate of fall of temperature at any point of the cylinder can be arrived at by calculation; and after certain simplifications of the expression thus obtained for the temperature, there results the method for the determination of the internal heat conductivity." The method thus indicated is credited to H. F. Weber,² who had applied a somewhat similar method to the study of heat conduction in several metals³ about 1880. "The temperature was measured thermo-electrically. Since it would not do to break the continuity of the material by means of small canals, one junction of the thermo-element was placed at the middle of the upper end surface [of the cylinder], the other junction kept at the constant temperature 0° [so-called] of the cooling water. The thermo-electric current was led through a galvanometer of such a character as to permit the variable electromotive force to be followed exactly after the lapse of a few seconds." * * * "From a given moment, after the cylinders had been heated, its lower end surface was played on by a stream of water 3 cm. thick from the public water supply (having a pressure about 7 atmospheres), while care was taken that the upper end as well as the curved surface of the cylinders should not be touched by the water."

"In the center of the upper end surface [of the cylinder] a drill hole 0.2–0.25 cm. broad and 0.15 cm. deep was filled with a piece

¹ *Verhandlungen des Vereins zur Beförderung des Gewerbflusses*, vol. 45, 1896, pp. 33–61.

² *Berliner Monatsberichte*, 1880, and *Wied. Ann.*, Bd. 13.

³ *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zurich*, vol. 25, 1880.

of soft copper. In this was bored a fine hole in which the junction [of the thermo-element, of thin iron and German-silver wire] could be easily and securely placed."

The narrowest cylinders used by Beglinger had a diameter of nearly 6 cm.; the greater number had a diameter of 15.8 cm. or more. The least length was 4.44 cm.; the length of the greater number ranged from 6.5 cm. to 8 cm.

Each cylinder when in use was mounted on, or rather in, a broad thick supporting plate of iron, having at its center a hole widening conically toward the upper face and just wide enough at the bottom face to admit the cylinder. The lower end surface of the cylinder was made flush with the lower surface of the plate, being supported in that position by three small lugs attached to the under face of the plate. There could be side contact between cylinder and plate only along a strip $\frac{1}{2}$ mm. high. The narrow space between the two was filled with a cement consisting of yellow wax and "Kolophonium." Above the cylinder was placed on the plate a double-walled cap of sheet copper containing water of the same temperature as the other cooling water. The water jet, about 3 cm. wide, was directed against the middle of the base of the cylinder from a distance of 3 or 4 cm. with very great force, and "erst am Rande des Gestelles [support] in seiner Bewegung in horizontaler Richtung gehemmt, nahm es seinen Weg nach unten." At the maximum this stream delivered about 3.7 liters of water per second with a velocity of 5.3 meters per second. A considerable reduction of the flow was accompanied by a perceptible loss of effectiveness of the stream.

The warming of the cylinder was effected by means of a Bunsen burner; and it lasted from 3 to 5 minutes, at times as much as 10 minutes. After the operation the under surface of the cylinder was always carefully cleaned and the water jet was put into position. Two or three minutes were thus allowed to elapse, in order that for the subsequent cooling it might be assumed that the initial temperature was everywhere the same. The warming was observed by means of the galvanometer, and "im Allgemeinen wurde nicht über 40° Temperaturdifferenz hinausgegangen." I think 40° is here a mistake for 4°; for I find later, as a typical case, "Kühlwassertemper-

atur $3^{\circ}.3$ C." * * * " $\bar{n} = 2^{\circ}.0$; also gilt das Resultat für die Temperatur $5^{\circ}.3$ C."

As the "external" conductivity plays a certain part, though a subordinate one, in the cooling, certain direct observations upon rate of cooling by external conductivity alone were made.

In some cases the effect of removing the thermoelectric junction from the middle of the cylinder end to an excentric position was tried.

Each cylinder was tested in both of its upright positions; that is, the face which had been the base in one set of observations was made the top in another set; this gave a rough test of the homogeneity of each cylinder, which test all of the wrought-iron cylinders except one and all of the steel cylinders except one bore well. But with 14 out of 31 cast-iron cylinders the values of k obtained from the two positions differed from each other by 2% or more.

For the determination of its specific heat each cylinder was heated in a water bath to " $30, 40$ oder 50° ," from which it was after a time quickly transferred to the calorimeter. The giving out of its heat in this calorimeter occupied from 4 to 8 minutes.

Beglinger uses the minute as his unit of time. His values of k are, therefore, to be divided by 60 in order to reduce them to the C. G. S. system, and the reduction is made in the following table, which gives the results of his work with malleable iron (Schmiede-eisen):

Schweisseisen.					
No.	l	$2R$	ρ	c	k
	cm.	cm.			
1. Schweisseisen, Gerlafingen	7.387	15.91	7.835	0.1110	0.1502
2. Sandwyk-Eisen	6.992	19.96	7.884	0.1104	0.1492
3. Siegener Schweisseisen	6.987	19.96	7.803	0.1107	0.1329
4. Schweisseisen, Gerlafingen (E.)	6.982	19.96	7.812	0.1115	0.1279
5. Frischfeureisen, Gerlafingen	7.447	20.05	7.693	0.1112	0.1263
6. Packeteisen "	7.285	20.05	7.806	0.1110	0.1262
7. Hayange, No. 2, gewalzt	6.970	15.92	7.521	0.1134	0.1078
8. Hayange, No. 2, geschmiedet	6.987	15.91	7.550	0.1122	0.1112
Flusseisen.					
9. Flusseisen, Gerlafingen	7.275	20.05	7.849	0.1108	0.1346
10. Flusseisen	6.045	15.87	7.657	0.1125	0.1241

l = length.

ρ = density.

$2R$ = diameter.

c = specific heat.

k = thermal conductivity.

Beglinger does not tell, as a rule, for what temperature his values of c or of k were determined; but from various remarks it seems probable that the temperatures for k lie between 5°C. and 20°C. The mean temperatures for which c was experimentally determined probably lay ten or fifteen degrees higher. Whether the values of c , as given above, are reduced to the lower temperatures for which k holds, we are not, I believe, directly told; but it appears as if these values of c were used in finding k . The mean of the values given for c is about what Naccari and also Lorenz would give for the specific heat of iron at 50°C.

Without questioning the great value and merit of Beglinger's work, one may suspect his results for k to be affected by considerable errors. For example, the description of the method of warming the cylinder leaves it very doubtful whether its temperature was everywhere the same at the beginning of the cooling. Moreover, it is even possible to doubt whether the lower face of the cylinder, in spite of the great size and velocity of the water stream directed against it, had everywhere the temperature of the water in the stream. The parts directly attacked by the full force of the stream would, perhaps, soon attain and keep a temperature very nearly the same as that of the stream itself; but other parts would probably be less affected. We might, therefore, expect narrow cylinders to be cooled somewhat more rapidly, other things being equal, than broad cylinders. As to the possible action along the curved surface of the cylinder Beglinger remarks: "Es ist zu bemerken, dass aus der Erwärmung des Gestelles [during the warming of the cylinder by means of a Bunsen burner] kein Fehler entstehen konnte; ebensowenig ist ein Einfluss der Kittschicht, trotz vielfacher Modifikationen konstatirt worden." It is to be remarked, however, that if there is any flow out through the cement layer during the cooling, this flow would hasten the cooling more, other things being equal, in narrow cylinders than in broad cylinders. We find here, then, two possible influences tending to make k appear greater in narrow cylinders than in broad cylinders. Reasoning in this way, I was curious to see whether the results, as given by Beglinger, would give any indication of such a result. It happens that he used 11 pairs of cast-iron cylinders, the individuals of each pair

being of the same material and differing, so far as was known beforehand, only in the diameter, which was about 6 cm. for one and 16 cm. for the other. Taking mean values I find

	ρ	c	k
For broad cylinders,	7.102	0.1152	0.1192
For narrow cylinders,	7.093	0.1159	0.1225

The mean k found for the narrow cylinders is, therefore, about 2.5% greater than the mean k found for the broad cylinders; and as the mean density and mean specific heat are nearly the same for one set as for the other, it seems probable that the apparent difference in the mean values of k is due to some fault in the method of experimentation.

This is not a very grave criticism of the work of Beglinger, which from its scope was necessarily somewhat rough in details; but it should be borne in mind hereafter, if there should be a disposition on the part of any one to discredit the values obtained by others on the ground that these values do not agree with any of those given by Beglinger. The maximum value of k which he found for wrought iron is 0.1502, which is a very little smaller than the value which I have found for a less dense piece of iron, about 5% less than the value found by Berget, and 10% less than the value found by Lorenz for iron of very nearly the same density, as Beglinger's iron.

Beglinger's results seem to show that, in the case of malleable iron, density is an important, though by no means the sole, factor in determining the value of k . The greatest value of k was not found in the most dense piece, No. 2, nor the least value of k in the least dense piece, No. 7; and yet, when the values of k for the ten pieces of malleable iron are plotted as ordinates with the densities as abscissas, it appears that an increase of 1% in the density is accompanied by an increase of several per cent. in the value of k , and there is an indication that the rate of increase of k rises with the increase of ρ . The other variables upon which k in soft iron depends are, according to Beglinger, very obscure. He calls attention to the fact that k in cylinder No. 1, which came directly from the iron-works, was about 15% greater than k in No. 4, a cylinder of the same material which came through a commercial

firm. The density and specific heat were about the same in one of these cylinders as in the other. It seems a pity that serious effort was not made, by dissection and minute examination of these two cylinders, to discover the cause of their great difference in k .

It may be well to add a few words concerning the results of Beglinger for steel and cast-iron.

The 11 cylinders of the steel group were all "Flussstahlsorten." In these

ρ	varied from	7.84	to	7.92,
c	"	"		0.1110 to 0.1135,
k	"	"		0.1327 to 0.0940.

There was among these no apparent connection between ρ and k .

Among the 11 cylinders four pieces are numbered twice each, once in the soft condition and once in the hard condition. "In 3 Fallen hatte der gehärtete Cylinder ein kleineres k als der weiche, aber nur um 11, 5 und 6% ; in eniem Falle hatte sogar der gehärtete Cylinder ein um 2% grösseres Leitungsvermögen." "Die Ergebnisse für das Härten bestätigen die Resultate des Herrn Kohlrausch nicht." In fact, Beglinger comes to the conclusion that hardness and softness in steel, whether due to its mode of manufacture or to subsequent treatment, has comparatively little influence on k . It is to be observed, however, that Beglinger used pieces of steel about 4.5 cm. thick, and, as he does not describe his method of hardening them, it may well be doubted whether he succeeded in making them very hard throughout.

Within the cast-iron group of 31 cylinders,

ρ	varied from	6.849	to	7.217,
c	"	"		0.1120 to 0.1210,
k	"	"		0.0932 to 0.1533.

Here, as in steel, there is no apparent connection between ρ and k . It seems to be a matter of mere chance whether a very dense piece of cast iron has a high, low or medium thermal conductivity. In the eleven pairs of cast-iron cylinders already mentioned, the individuals of each pair being ostensibly of the same material, Beglinger found

always a difference of k between these individuals, the smallest difference being less than $\frac{1}{4}\%$, the largest 21%.

Beglinger gives the chemical analysis of none of his cylinders, and expresses a doubt as to whether such analysis would clear up the differences observed, in view of the comparatively small effect produced by the presence of a large amount of carbon. "Der gebundene Kohlenstoff drückt beim Gusseisen das k etwas hinunter im Beispiel des Cylinders No. 40 gegenüber No. 33, wo der Kohlenstoff zum grössten Theil in Form von Graphitblättchen ausgeschieden ist." "Ueber den Einfluss von Mangan, Silicium und Schwefel etc., auf das Wärmeleitungsvermögen von Stahl und Eisen fehlt jeder Anhaltspunkt."

I find from the values given by Beglinger :

for 10 pieces of soft iron	the mean k	= 0.1300,
" 11 " " steel	" " "	= 0.1100,
" 31 " " cast iron	" " "	= 0.1175.

HALL.

The general method which I have followed in measuring the thermal conductivity of a plate of wrought iron is the same as that used by Mr. Ayres and myself in our work on the thermal conductivity of cast iron.¹ Certain changes of apparatus or procedure, the most important being the substitution of platinum thermometers for thermoelectric junctions in measuring the change of temperature of the water passing over the disk, will be described in the Proceedings of the American Academy ; but the character of the iron used and the results obtained will be given here.

The metal was Yorkshire wrought iron, recommended to me by an engineer friend of much experience as the softest wrought iron to be found in the Boston market. At the place of purchase the metal was described to me as containing 99.7% iron and no carbon. Chemical analysis made at my instance showed

iron	99.93%,
carbon	0.059%.

¹ Proceedings of American Academy of Arts and Sciences, Vol. 34, p. 283, 1899.

The values which I find for the thermal conductivity, k , of this iron are, at

28°.2C.	0.1528,
58°.3C.	0.1514,

if the specific heat of water at each of these temperatures is called 1. This would give for the temperature coefficient 0.0003, very nearly.

If the values of k are revised in accordance with the values proposed by Winkelmann¹ for the specific heat of water at the given temperatures, they become at

28°.2C.	0.1513,
58°.3C.	0.1511,

and the temperature coefficient deduced therefrom will be too small to be worth writing down, 0.0000?, let us say.

But the recent work of Callendar and Barnes² gives for the specific heat of water

0.9992	at 25°C.
0.9987	at 30°C.
0.9992	at 55°C.
1.0000	at 60°C.

If these values given by Callendar and Barnes are adopted as correct, my first values of k will stand almost without change; and I shall therefore leave them for the present without correction for variation in the specific heat of water.

My value for the temperature coefficient, 0.0003 or a little less, is, so far as it goes, a corroboration of the substantial accuracy of the temperature coefficient found by Lorenz, 0.0002282, although it may be doubted whether the last three figures of this number are of much significance. This agreement is eminently satisfactory to myself; for a comparison of the work of Lorenz with that of other investigators has convinced me that his value of the temperature coefficient is entitled to an especial degree of confidence.

¹Part 2 of Vol. 2, p. 340.

²This REVIEW, April, 1900.

Measurements of the electrical resistance of the iron were made on 9 cylinders, each 2 cm. long and about 0.23 cm. in diameter, cut from the same great bar as the disk on which the measurements of k were made. The length of the cylinders, like the thickness of the disk, was taken parallel to the length of the bar; and the cylinders were cut from a part of the bar adjacent to that from which the disk was cut. The extreme difference in the specific resistances of these cylinders was apparently about 5 per cent. The mean specific resistance was found to be 12240 at 18° C. The mean specific conductivity, κ , at the same temperature would, therefore, be 817×10^{-7} C. G. S.

SUMMARY AND DISCUSSION.

In accordance with the preceding exposition I have made the following table of results for wrought iron, in which ρ = the density, κ = the electric conductivity, k = the thermal conductivity :

Observer.	ρ	$\kappa \times 10^7$	k
<i>Method of steady flow with "external conduction."</i>			
Forbes,	7.79	?	0.207 (?) at 0°C. (see Mitchell).
Mitchell,	7.79	?	0.1509 (?) at 0°C. (sp. ht. uncertain).
Stewart,	7.556	?	0.175 (?) at 0°C.
<i>Methods using waves or oscillations of temperature.</i>			
Angström,	?	?	0.1655 (?) at 0°C. (sp. ht. uncertain).
Neumann,	7.74	?	0.1637(?) at ? (account too brief).
H. Weber,	7.761	604 at 44°C.	0.1485 at 39°C.
<i>Methods of rise or fall of temperature with internal and external conduction.</i>			
Lorenz,	7.828	1037.4 at 0°C.	0.1665 at 0°C.
Beglinger,	7.74	?	0.130 (?) at ? (method rough).
<i>Method of steady flow with guard ring or jacket.</i>			
Berget,	?	941 at 15°C.	0.1587 at 15°C.
Hall,	7.785	817 at 18°C.	0.1528 at 28°C.

Temperature coefficient of k.

Lorenz,	$a = -0.0002282.$
Hall,	$a = -0.0003$ or slightly less.

The only two values here given for a were obtained by very different methods, which, however, have this very important feature in common, that each of them used precisely the same piece of

metal in finding the low temperature value of k as in finding its high temperature value, the whole apparatus being used at two different levels of temperature, with little or no relative change within it in passing from one level to the other. In the Forbes method of finding α , followed by Mitchell and Stewart, the high temperature value of k is found at one part of a bar and the low temperature value at a different part, where the quality of the metal may not be quite the same. For this reason, as well as for others which will be apparent to any one reading the preceding discussion of work by the method of Forbes, I have retained none of the values of α obtained by that method. The value to be derived from the data given by Angström seems also too uncertain for a place in this summary.

If we take -0.00025^1 as the value of α in wrought iron of high grade, I believe that we may have much confidence in the ciphers and some confidence in the first digit.

In four cases we have both k and x in C. G. S. units. Reducing to 0°C ., using -0.00025 as the value of α and $x_0 = x \div (1 - 0.0051t + 0.000013t^2)$ as the temperature formula for electrical conductivity, we get in these four cases

Observer.	ρ	x	k	$k+x$
Lorenz,	7.828	10374×10^{-6}	0.1665	1605
Berget,	?	10160 “	0.1593	1568
Hall,	7.785	8958 “	0.1539	1716
H. Weber,	7.772	7540	0.1500	1989

The mean of the four values here given for k is 0.1574. The mean value deduced by Holborn and Wien was 0.156 at $(?)^\circ\text{C}$.

It is plain that diminution of density is here accompanied by fall of both x and k . It will be remembered that Beglinger's results also, for soft iron, indicated a fall of several per cent. in k with a fall of one per cent. in ρ . It would be unwise, however, to draw definite numerical conclusions from the numbers before us; for all the values of x and k are in some degree uncertain, and in the case of Weber's iron even the ρ is a little dubious; for he does not tell the temperature for which ρ is 7.761, and I have assumed that it

¹ Holborn and Wien, *loc. cit.*, retaining the values of α given by Mitchell and Stewart, get, as the most probable value, $\alpha = -0.001011$.

was 39°C., which he gives as the mean temperature for which he determined k . We should be in better position for speculations if we knew the value of ρ in the iron used by Berget. The x and k for this iron seem to require a high density.

Although k appears to increase rapidly with increase of ρ , x increases still more rapidly, so that, with one exception, we find $k \div x$ to decrease with increase of k . This exception may be due to entirely obscure causes lying in the difference of quality of the iron used by Lorenz and Berget respectively, or it may be due to errors of measurement by one or both of these investigators. To determine this ratio within 1% for any piece of iron would be a considerable achievement. It is worth while to inquire what values of this ratio were obtained by Lorenz and by Berget for the other metals which were examined by both of them. Brass being omitted as a too uncertain quantity, these metals were copper, tin, lead and antimony. Berget's results being given for 15°C., we must reduce them to 0°C.; and for this purpose can not do better than to take the temperature coefficients found by Lorenz. Using these we get for the ratio $k \div x$ at 0°C.

	Lorenz.	Berget.
Copper,	1574	1516
Tin,	1635	1727
Lead,	1627	1532
Antimony,	2011	1629

Thus we have, including iron, four cases out of five in which the value of $k \div x$ found by Berget was less than that found by Lorenz. I am, therefore, inclined to attribute the exception above noted, in the column $k \div x$ for iron, to errors of measurement, and to hold it as a rule that in iron the ratio $k \div x$ decreases with increase of k .

In conclusion I have to make the following suggestions :

1. That every one who undertakes to determine the thermal conductivity of any metal should determine also its electrical conductivity and thermoelectric quality, and should give in general the most definite attainable account of its physical and chemical properties.

2. That every one should have regard not merely to the probable accuracy of his method but also to its convincingness, lest his re-

sults, however accurate, may be received with doubt and so prove ineffective.

3. That the general methods used by Berget are peculiarly simple, direct and convincing; and that his probably very accurate determination of k for mercury puts at the disposal of all other investigators a standard substance by comparison with which they may, using his guard-ring method, measure the conductivity of other metals with far less labor than if they undertook independent absolute determinations.

A striking example of the uncritical use of old data relating to thermal conductivity is furnished by the paper of Liebenow referred to at the beginning of this article. Liebenow derives from his theory the following formula for the thermo-electromotive force in any metal:

$$\frac{dE}{dT} = \pm 2.04 \sqrt{\frac{RL}{T}},$$

where dE is the electromotive force in volts,

dT " difference of temperature in degrees C.,
 R " specific resistance in ohms,
 L " thermal conductivity,
 T " absolute temperature.

He then says, "For copper one finds accordingly at 15° C., if one takes 0.8 as the specific thermal conductivity of copper,

$$\frac{dE}{dT} = \pm 2.04 \sqrt{0.8 \times 0.00001635 \div 288} = \pm 0.0001375 \text{ volts.}"$$

Why Liebenow took the value 0.8 he does not say. The values given by Winkelmann on the authority of various observers range from 0.6296 to 1.11. But having taken this value for copper he follows the curious course of calculating the thermal conductivity for iron by multiplying 0.8 by the value which Wiedemann and Franz found for the thermal conductivity of iron as compared with copper, as if there had been no improvement on the work of these observers during the past forty-five years. The electric conductivities

are taken from Matthiessen's values. Accordingly Liebenow gets for iron

$$\frac{dE}{dT} = \pm 128.2 \text{ microvolts,}$$

"and one gets for the iron-copper element -9.3 microvolts [that is, $128.2 - 137.5$]; which agrees very well with observations."

To show how much importance Liebenow attaches to the case of iron and copper, I will quote one more passage. "Since the electric stream in the copper-iron element, under 276° , flows from copper to iron at the warm junction, though the electromotive force of the copper is in this case larger than that of the iron, it follows that the direction of these forces is opposite to that of the heat flow; that is, the positive electricity is in these metals, with unequal temperature, driven toward the warmer places. Moreover, since all metals investigated by Matthiessen give, in combination with copper, smaller electromotive forces than 137.5 microvolts, one must conclude that all metals when unequally heated become electrically *positive* at the warm end."

But let us see how the argument would run if we were to use the results obtained by Lorenz and by Berget, both excellent and, as compared with Wiedemann and Franz, recent experimenters.

From the data given by Lorenz we get for copper at 0°C .

$$\frac{dE}{dT} = 2.04 \sqrt{\frac{RL}{T}} = 2.04 \sqrt{\frac{k}{xT}} 10^{-9} = 0.0001549 \text{ volt,}$$

and for iron at the same temperature,

$$\frac{dE}{dT} = 2.04 \sqrt{\frac{k}{xT}} 10^{-9} = 0.0001564 \text{ volt.}$$

Thus we find $\frac{dE}{dT}$ greater for iron, the opposite of what Liebenow finds. At 15°C ., the temperature used by Liebenow, the excess of $\frac{dE}{dT}$ for iron would be a little greater than at 0°C .

From Berget's data we get at 15°C . for copper

$$\frac{dE}{dT} = 2.04 \sqrt{\frac{1.045}{651300 \times 288}} = 0.0001519 \text{ volt,}$$

and for iron at the same temperature,

$$\frac{dE}{dT} = 2.04 \sqrt{\frac{0.1587}{94100 \times 288}} = 0.0001561 \text{ volt.}$$

Here again, $\frac{dE}{dT}$ is greater for iron than for copper.

However sound Liebenow's general theory may be, it seems plain that his computations should be revised.

CAMBRIDGE, May 12, 1900.