PRIME NUMBERS IN ARITHMETICAL PROGRESSION.

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All such numbers are contained in the general series
 $a, a+b, a+2b, a+3b, \dots a+(n-1)b \dots (1),$
 where a must be an odd prime, b an even number prime to a
 except when a and b are unity, and n may be any number.

"Three prime numbers cannot be in arithmetical progression, unless the common difference of them be divisible by $2 \times 3 = 6$; except 3 be the first of the prime numbers, in which case there may be three prime numbers in arithmetical progression, whose common difference is not divisible by 6, but there can be only three."—Barlow's Theory of Numbers, page 64.

Barlow evidently overlooked the cases 1, 2, 3; 1, 3, 5; and 1, 3, 5, 7. 1 and 2 are prime numbers.

I. To find three prime numbers in arithmetical progression.
 The general series for three numbers is

$$a, a+b, a+2b,$$

when $n = 3$ in (1).

1. Take $a = 1, b = 1$, then the numbers are

$$1, 2, 3,$$

which are the least three prime numbers in arithmetical progression, and the *only* three consecutive numbers that are primes.

2. Take $a = 1, b = 2$, then we have

$$1, 3, 5.$$

3. Take $a = 1, b = 6$, then we get

$$1, 7, 13.$$

4. Take $a = 1, b = 18$, and we find

$$1, 19, 37.$$

5. Take $a = 1, b = 96$, and we get

$$1, 97, 193.$$

And so on, and we can find as many sets of three prime numbers in arithmetical progression as we please for $a = 1$.

6. Taking $a = 3$, and $b = 2, 4, 8, 10, 14, 20, 28, 34$, etc., we get the following sets of three primes in arithmetical progression, viz:

$$3, 5, 7; \quad 3, 7, 11;$$

$$3, 11, 19; \quad 3, 13, 23;$$

$$3, 17, 31; \quad 3, 23, 43;$$

$$3, 31, 59; \quad 3, 37, 71;$$

and we can find as many sets of three primes in arithmetical progression as we please for $a = 3$.

7. Take $a = 5$, and $b = 6, 12, 24, 42, 48, 54, 66, 84$, etc., and we have

$$\begin{array}{ll} 5, 11, 17; & 5, 17, 29; \\ 5, 29, 53; & 5, 47, 89; \\ 5, 53, 101; & 5, 59, 113; \\ 5, 71, 137; & 5, 89, 173; \end{array}$$

and so on, and we can find as many sets of three primes in arithmetical progression, first term $= 5$, as we please.

8. Take $a = 7$, and $b = 6, 12, 30, 36, 60, 66$, etc., and we find

$$\begin{array}{ll} 7, 13, 19; & 7, 19, 31; \\ 7, 37, 67; & 7, 43, 79; \\ 7, 67, 127; & 7, 73, 139; \end{array}$$

and so on. Many other sets may be found beginning with 7.

9. Take $a = 11$, and $b = 6, 18, 30, 36, 48, 60$, etc., and we have

$$\begin{array}{ll} 11, 17, 23; & 11, 29, 47; \\ 11, 41, 71; & 11, 47, 83; \\ 11, 59, 107; & 11, 71, 131; \end{array}$$

and so on. Many other sets for $a = 11$ can be found.

10. Take $a = 13$, and $b = 24, 30, 48, 84, 90, 114$, etc., and we get

$$\begin{array}{ll} 13, 37, 61; & 13, 43, 73; \\ 13, 61, 109; & 13, 97, 181; \\ 13, 103, 193; & 13, 127, 241; \end{array}$$

and so on. Other sets for $a = 13$ can be found.

11. Take $a = 17$, and $b = 6, 12, 42, 66, 90, 120$, etc., and we get

$$\begin{array}{ll} 17, 23, 29; & 17, 29, 41; \\ 17, 59, 101; & 17, 83, 149; \\ 17, 107, 197; & 17, 137, 257; \end{array}$$

and so on. Other sets for $a = 17$ can be found.

12. Take $a = 127$, and $b = 36$; then we have

$$127, 163, 199.$$

It seems that there can be three primes in arithmetical progression beginning with any odd prime.

II. To find four prime numbers in arithmetical progression.

The general series for four numbers is

$$a, a+b, a+2b, a+3b,$$

taking $n = 4$ in (1).

For four numbers, a can not be 3; for, in that case, $a+3b$ would be divisible by 3 and not a prime; b must not be a multiple of a when a is greater than 1.

13. Take $a = 1$, and $b = 2, 6, 36, 210$, etc., and we find
 $1, 3, 5, 7$; $1, 7, 13, 19$;
 $1, 37, 73, 109$; $1, 211, 421, 631$;
 and so on. Other sets beginning with 1 can be found.
14. Take $a = 5$, and $b = 6, 12, 18, 42$, etc., and we have
 $5, 11, 17, 23$; $5, 17, 29, 41$;
 $5, 23, 41, 59$; $5, 47, 89, 131$;
 and so on. Many other sets for $a = 5$ may be found.
15. Take $a = 7$, and $b = 12, 30, 102, 150$, etc., and we get
 $7, 19, 31, 43$; $7, 37, 67, 97$;
 $7, 109, 211, 313$; $7, 157, 307, 457$;
 and so on. Other sets beginning with 7 can be found.
16. Take $a = 11$, and $b = 6, 30, 60, 126$, etc., and we find
 $11, 17, 23, 29$; $11, 41, 71, 101$;
 $11, 71, 131, 191$; $11, 137, 263, 389$;
 and so on. Other sets may be found beginning with 11.
17. Take $a = 13$, and $b = 30, 48, 90, 150$, etc., and we have
 $13, 43, 73, 103$;
 $13, 61, 109, 157$;
 $13, 103, 193, 283$;
 $13, 163, 313, 463$;
 and so on. Other sets may be found beginning with 13.
18. Take $a = 17$, and $b = 12, 150, 252$, etc., and we get
 $17, 29, 41, 53$;
 $17, 167, 317, 467$;
 $17, 269, 521, 773$;
 and so on. Other sets beginning with 17 may be found.
19. Take $a = 19$, $b = 174$, and 264, and we have
 $19, 193, 367, 541$;
 $19, 283, 547, 811$.

Other sets beginning with 19 may be found.

III. To find five prime numbers in arithmetical progression.

The general series for five numbers is

$$a, a+b, a+2b, a+3b, a+4b;$$

taking $n = 5$ in (1).

Here a must be an odd prime other than 3, and b must not be a multiple of a when a is greater than unity.

"There cannot be five prime numbers in arithmetical progression, unless their common difference be divisible by $2 \times 3 \times 5 = 30$; except when the first term of the progression is 5, in which case there may be five prime numbers in arithmetical progression, whose common difference is not divisible by 30, but there

can be no more than 5."—Barlow's Theory of Numbers, page 65.

20. Take $a = 1$, $b = 330$, and we find

1, 331, 661, 991, 1321.

21. Take $a = 5$, and $b = 6, 12, 42, 48$, and we get

5, 11, 17, 23, 29;

5, 17, 29, 41, 53;

5, 47, 89, 131, 173;

5, 53, 101, 149, 197.

Other sets of five numbers beginning with 5 may be found.

22. Take $a = 7$, and $b = 30$, and 150, and we have

7, 37, 67, 97, 127;

7, 157, 307, 457, 607.

Other sets of five numbers beginning with 7 may be found.

23. Take $a = 11$, and $b = 30$, and 60, and we find

11, 41, 71, 101, 131;

11, 71, 131, 191, 251.

Other sets of five numbers beginning with 11 may be found.

24. Take $a = 13$, and $b = 90$, and 150, and we get

13, 103, 193, 283, 373;

13, 163, 313, 463, 613.

25. Take $a = 17$, and $b = 150$; then we have

17, 167, 317, 467, 617.

IV. To find six prime numbers in arithmetical progression.

The general series for six numbers is

$$a, a+b, a+2b, a+3b, a+4b, a+5b$$

when $n = 6$ in (1).

Here a can not be 3, because then $a+3b$ would be divisible by 3 and not a prime; nor can a be 5, for in that case $a+5b$ would be divisible by 5 and not a prime.

26. Take $a = 7$, and $b = 30$, and 150, and we have

7, 37, 67, 97, 127, 157;

7, 157, 307, 457, 607, 757.

27. Take $a = 11$, and $b = 60$, and we get

11, 71, 131, 191, 251, 311.

28. Take $a = 13$, and $b = 30$, and 90, and we find

13, 43, 73, 103, 143, 173;

13, 103, 193, 283, 373, 463.

V. To find seven prime numbers in arithmetical progression.

The general series for seven numbers is

$$a, a+b, a+2b, a+3b, a+4b, a+5b, a+6b,$$

when $n = 7$ in (1).

Here a must not be 3 or 5; for, if $a = 3$, $a+3b$ and $a+6b$

would be divisible by 3 and not primes; and if $a = 5$, $a + 5b$ would be divisible by 5, and not a prime number.

"Seven prime numbers cannot be in arithmetical progression, unless their common difference be divisible by $2 \times 3 \times 5 \times 7$; except the first of those prime numbers be 7, in which case there may be seven prime numbers in arithmetical progression, of which the common difference is not divisible by 210, but there cannot be more than seven. And, generally, there cannot be n prime numbers in arithmetical progression, unless their common difference be divisible by $2 \times 3 \times 5 \times 7 \times 11$, etc., n ; except the case in which n is the first term of the progression, in which case there may be n such numbers, but not more."—Barlow's Theory of Numbers, pp. 67-8.

29. Take $a = 7$, and $b = 150$; then we have

7, 157, 307, 457, 607, 757, 907,

seven prime numbers in arithmetical progression.

As the number of terms in the progression increases, the difficulty and labor of finding such series becomes greater and greater since they must be found by trial from a table of prime numbers; but enough has already been done to indicate the methods to be pursued in seeking them, and to show that the subject is practically inexhaustible.

MOVING PICTURES IN GERMAN EDUCATION.

The use of moving pictures in education has had a real impetus in German official circles, according to information recently received at the United States Bureau of Education. The Prussian Ministry of Education is now considering the feasibility of employing cinematograph films in certain courses in higher educational institutions, and a number of film manufacturers are being given an opportunity to show the authorities what films they have that are adapted to educational purposes.

A well-known philanthropist has recently donated two fully equipped moving picture machines to the schools of Berlin. One is to be used in the Continuation Institute for Higher Teachers and the other in the high schools of greater Berlin.

Moving-picture films are now available in Germany for anatomical, biological, and bacteriological courses, and the manufacturers are confident that an enormous field for their products will be opened up when educators fully realize the value of moving pictures in education.