ART. XXII.—*On the Genealogy of Modern Numerals.* By
Sir E. CLIVE BAYLEY, K.C.S.I., C.I.E.

NEARLY fifty years have elapsed since James Prinsep first announced his discovery of ancient Indian Numerals, and attempted their decipherment. Towards this latter object gradual advance has been made from time to time, and now, thanks to the successive labours of Thomas, Cunningham, Bhao Dáji, Bühler, Bhagwán Lál and others, it is possible to give to each symbol used its proper meaning.

Nevertheless, there is still much left to settle, and as regards the history of these signs there is yet great uncertainty. It is known that while in European parlance our modern numerals have been termed "Arabic," yet that they differ in several important points from those used in Arabic writing; moreover, Arab writers emphatically declare these last to be an Indian invention. Still, the chain of descent of either form from an Indian source has never been satisfactorily elucidated. And while no doubt the modern system of using, for all purposes of notation and calculation, nine unit figures and a zero, arranged in decimal order, is apparently indigenous in India, it has not yet been distinctly shown how this simplified form was eliminated from the complicated system of notation which was in ancient times used in India. This last, as we now are aware, employed (with many variations of form) no less than twenty signs, capable, in some cases, of being differentiated by certain methods so as to signify higher powers of the original. This ancient system moreover employed no zero.

Much has indeed been written on these points, so much indeed that any undertaking to collect, examine, and analyze all the literature of the subject, would be far beyond the compass of a paper like the present.

All that will be now attempted is to give the leading facts already established and accepted. It is not proposed to add much which is really new; but it is hoped, by a fresh arrangement and collocation of data, to suggest solutions of each separate problem of at least a plausible character.

It will of course not be possible to attempt this except in the shape of a sketch; but, as far as may be, the authorities used will be indicated, so that readers who desire to consider the matter more fully, will be able to test the information tendered, and the arguments employed, more completely than can well be done in a brief memoir.

The whole case naturally divides itself into three parts, viz.

- (1) The origin of the ancient Indian system of numerals.
- (2) The simplification of this system, by the rejection of all the signs except those for the nine units, and by the invention of the zero.

(3) The propagation of this latter system, and of the varying forms of the numeral signs, both in the East and West.

It will be convenient to discuss these three divisions of the subject in their chronological order, and to divide this paper accordingly into three separate parts.

PART I.—ORIGIN OF THE ANCIENT INDIAN SYSTEM OF NUMERATION.

The earliest¹ inscriptions hitherto discovered in India are those at Náná Ghát,² in the Bombay Presidency, which have been ascribed to the early part of the third century B.C., and which are supposed to be memorials of the early Andhra dynasty of Southern India, and of their contemporaries. In these inscriptions are found systems of alphabetical writing

¹ Since this was written I have become aware that Professor Sayce is about to publish yet earlier examples of ancient Indian writing, but they will not, I believe, include any examples of numerals.

² The results of the examination of the Náná Ghát inscriptions by the Bombay Archaeological Survey have not yet reached England. The numerals of the Náná Ghát period are therefore taken from the facsimiles given by Bhagwán Lál Indrají in the *Indian Antiquary* for 1877, vol. vi. pp. 44-47.

and of notation by non-alphabetic signs, both singularly perfect and complete.

But though the alphabetical and numerical systems of India thus first appear together, and in a complete form, on the historical horizon, it by no means follows that they are of identical, or even of contemporaneous origin; and as the subject of the present paper is the system of notation only, questions concerning the alphabet will only be noticed when it may be necessary to do so incidentally, in order to illustrate the main question.¹

Plate I. attached to this paper gives a table of the early Indian numerals, and of the principal modifications which they underwent from their first appearance till the adoption of the simplified mode of notation with a zero, or down to about 650 A.D., that is to say, over a period of nearly a thousand years.²

It is of course natural that the curious phenomenon of a perfect alphabet, and of a very good system at least, of notation, being thus found together at a very early period, for the first time, without any palpable evidence of previous local development, should give rise to much speculation. It has been

¹ *A priori*, numerals are likely, at least in all ancient systems, to be of later date than written expressions of ideas, for they seem to have been originally little else than "shorthand" modes of *writing* numbers. Of course, however, when an alphabet is borrowed from an external source, as the Indian alphabet in its initial form probably was, the alphabet so borrowed may have already had a system of numeration attached to it, which was imported with it. There is, however, one fact which might be held to indicate that the Indian alphabet originally possessed no numeral system. The *old* Pali writings of Ceylon, that is, the books of the Buddhist religion written in the sacred language of Buddhism, do not employ any numerical *signs*. This religion and its language were introduced from India into Ceylon apparently during the fourth, or late in the fifth century B.C. In these books the numbers are either expressed in words, or by a certain arrangement of written syllables. It does not, however, necessarily follow that the Indians were at that time altogether ignorant of numerical notation by separate signs; only that in all probability such a method had not come into use in sacred writings, or in MSS. of any kind. Some such system indeed very probably existed in India even before 400 B.C., though perhaps in a more or less imperfect state, for, as will be seen presently, there is some ground for believing that it received improvements by successive borrowings down to the middle of the second century B.C.—[M. Rodet and Professor Rask, quoted by Pihan, *Signes de Numeration*, pp. 142-43; also Cantor, *Mathematische Beiträge*, pp. 58-59.]

² The references attached to this Plate indicate the authority on which each figure is adduced; for the most part it will be seen that these are given from original photographs or coins, and only when that is not possible from trustworthy facsimiles.

accordingly suggested that they were introduced together, "ready made," by the Brahmanic-Aryan races who overran Upper and Central India before the historic period. Not only, however, is there a total want of evidence in support of this hypothesis, but it is inconsistent with many uncontested facts; it leaves unexplained, the apparent disuse of the systems for a period of nearly a thousand years at least which this theory involves, and the reason why this alphabet was not used by other branches of the same race, who certainly adopted from external sources various alphabets peculiar to the localities in which they settled; as they would naturally do if they had none of their own, but hardly so, if the case were otherwise. Other writers have suggested that the system was *originated* in India itself either by the Aryan invaders or by the races whom they found settled there; but this hypothesis involves a long, independent process of development, of which no evidence of a trustworthy character exists, and indeed the separate and independent invention of such an elaborate system is against all probability and experience. A third hypothesis supposes that both the alphabet and the numeral signs were brought into India from a foreign source, and many points of resemblance have accordingly from time to time been noted between the Indian, and other alphabets and numeral systems.

Dr. Bühler—now Sanskrit Professor at Vienna, who has during his long residence in Bombay given much attention to the subject—has come to the conclusion that the alphabet, in its original shape, may have been introduced into India from a foreign source, and subsequently adapted by gradual alterations to Indian purposes; but that the numeral signs are, at least in their later forms, nothing else but the alphabetical expressions of certain syllables termed "aksharas," which possess in Sanskrit fixed numerical powers. This view, originated by Pandit Bhagwán Lál, has been elaborated by Professor Bühler in his official lectures at Vienna, and he has been good enough to furnish me with a memorandum giving the substance of the arguments he has employed, and this will now be given *in extenso*.

In doing this, however, it will be observed that the hypothesis thus put forward necessitates to some extent an explanation of Dr. Bühler's views on the origin of the alphabet also.¹

MEMORANDUM BY PROFESSOR G. BÜHLER, Ph.D., C.I.E.

"The Southern Indian Alphabet, the oldest form of which we possess in the Maurya and Andhra (Nânaghât and Nasik Karli and Amarâvatî inscriptions), no doubt comes before us as a fully developed system about 300 B.C., and is accompanied both in certain Maurya and in certain Andhra inscriptions, by an equally developed system of numeral figures, which are clearly syllables.

"As far as I can see, there can be no doubt that this alphabet was an old institution in India about 300 B.C., and that it owed its development to the grammatical schools of the Brahmans.

"The arguments proving its great age are—

"1. The enormous extent of territory over which it occurs, from Kathiâvâd to Orissa and the Eastern Coast, and from the Himâlayas down to the Sahyâdris.

"2. The fact that it must have been generally known among the higher classes (and even the lower classes) of this enormous territory; as is shown—

"(a) By the circumstance that Aśoka could hope to improve the morals of his subjects by official placards,

"(b) By the exquisite execution of the inscriptions, which excels (*e.g.* on the Dehli and Allahabad pillars) all the best work of the Roman and Greek stonemasons,

"(c) By the fact that the stonemasons, a low caste in India, used (as Cunningham has lately discovered) the letters (*e.g.* at Buddha Gaya) to mark the pillars, and that the order in which they gave the letters reveals the existence of a Bârâ Khadî, or table of the alphabet, which closely resembles that still in use in our indigenous schools, and proves that the

¹ Dr. Bühler's memorandum was attached in the first instance to a private letter, and was originally intended only for my personal use, and not for publication.

system of instruction now followed was already elaborated 2000 years ago.

"3. The fact that both the Maurya and the Andhra alphabets are sister-alphabets derived from a common source. It is wrong to say that the Andhra is derived from the Maurya alphabet; a comparison of the two alphabets, for example, in Burgess's tables, shows the contrary.

"Take the *ḍa* and *ḍha*; in the Maurya alphabet we have ḍ *ḍa*, ḍ *ḍha*; in the Andhra Ḍ *ḍa* and Ḍ *ḍha*.

"There cannot be any doubt that the *ḍha* was developed from *ḍa* by the addition of a little hook or curve added at the right of the *ḍa*, just as in Ḍ *cha* and Ḍ *chha*, Ḍ *pa* and Ḍ *pha*. Now it is utterly impossible to derive the Ḍ of the Maurya alphabet from the ḍ, but its connection with the Andhra Ḍ is very clear. Hence, I say, it is probable that the latter sign is the older one, and that the Maurya ḍ is not the parent of the Andhra sign. It may either be itself a development of the Andhra sign (by a change of the curves into angles), or an older alphabet may have had both the angular and curved signs. But, however that may be, the South Vindhyan Pali alphabet is not a daughter of the North Vindhyan alphabet. The bearing of this point on the age of the South Vindhyan alphabet is clear.

"4. The fact that the Brahmanical grammarians have developed the Maurya and Andhra alphabets, and brought them into the shape in which we first find them. This point is proved by the following circumstances:—

"(a) Nobody but a native grammarian (who, indeed, wanted the distinctions for his school lore) would have invented five or six separate signs to indicate various shades of the nasal sounds. We have in the Maurya inscriptions Ḍ, Ḍ, Ḍ, Ḍ, as a numeral Ḍ, and the same signs occur again in the Andhra inscriptions. There is a clear tendency to have separate signs for the nasal of each of the five 'Vargas,' or classes of the consonants as arranged by the grammarians: gutturals, palatals, linguals, dentals; and there is besides the *, which is used both as a conjunct nasal for all classes and the curious

nasal *g* sound at the end of words, which corresponds to the French final *n*.

"Now there is no other alphabet in the world which has developed such a number of signs for nasals; most alphabets have only two; some, like the Greek, three. If the Indian alphabet is derived from a Semitic source, these nasals must be mostly an Indian invention. It is also quite clear from the forms, that three at least are only differentiations of one fundamental form. Nobody has ever doubted that the $\mathbf{I}$ is derived from the $\perp$; it seems to be also highly probable that the $\mathfrak{h}$ goes back to the same type, for there is another rare form of the $\perp$ in the Andhra inscriptions $\lfloor$, looking very much like the Maurya u $\lfloor$. The $\mathfrak{h}$ arose out of this by the introduction below of two bands $\lfloor$, and the addition of the top horizontal stroke, or we might also say that the $\mathfrak{h}$ was derived from the $\mathbf{I}$ in its Andhra form, viz. $\mathbf{Z}$, by prolonging vertically the two ends of the lower horizontal line.

"Now who would have fallen on such a cumbrous system of nasals (which by the way in the Prakrit inscriptions serves no useful purpose because at least $\perp$ and $\mathbf{I}$ are used promiscuously)? Certainly not a merchant, for a merchant would only care for brevity, not for phonetic accuracy, and as a matter of fact the merchants in their books never used all the signs of the alphabet, and certainly no vowel signs, till compelled to do so by the English Courts. Again no Prakrit-speaking official or writer would dream of distinguishing between $\perp$ and $\mathbf{I}$, because to him the two letters were interchangeable and meant the same thing, *na* or *na*, according to the country to which he belonged, or according to the dialect which he spoke. But all these forms would be necessary to a Brahmanical grammarian who had in his fine polished school-language carefully to distinguish between *ng*, $\tilde{n}$, η , *n*, *m*, the anusvâra ° and the anunâsika $\lfloor$, and who according to his belief gained heaven, or went to another place—as he pronounced his sacred texts rightly, or wrongly.

"Hence I say the differentiation of the nasals shows the influence of the Brahmanical grammatical schools.

“(b) The same inference may be drawn from the existence of the three sibilants 𑀓 , 𑀔 , and 𑀕 (Khalsi and Pantaleon’s coin), all three go back to one original form, which consists of two little semi-circles, and differ only in the arrangement of these elements. Now Semitic alphabets have two sibilants: whose interest was it to have three? Of course it was necessary for the Sanskrit grammarians and for nobody else. In Prakrit only two sibilants exist, and they are used very promiscuously, according to dialects. A merchant would not be such a fool as to burden himself with such useless ballast.

“(c) A similar inference may be drawn from the careful system of short and long vowels.

“(d) Likewise from the invention of the *la* 𑀭 , which is peculiar to the Andhra inscription, because the sound occurs only south of the Vindhya range.

“But if it be granted that the Maurya and Andhra alphabets have been developed by Brahmans, does not that show that they must have been long in use before the time when we first find them?

“This inference as to a very early cultivation of the art of writing in India, at a time indeed much anterior to 300 B.C., is strengthened by the consideration of the Northern (Bactro-Arian) alphabet, which was clearly worked up by the same class of people who fashioned the southern system of characters. Take, for example, the system of vowel notation, and the system of compound letters, which follow exactly the same principles as those of the Southern alphabet.

“As regards the Indian numerals, my views are as follows:

“I. The Indian numerals, consisting of separate signs for the units, the tens, the hundreds and the thousands, are all *syllables*, which were pronounced as such, not signs for which the numerals were pronounced.

“The reading of these syllables has in general been given correctly by Bhagvânâl, except for the signs — , = , ≡ , 𑀓 ; the former three must be read *u*, *ú*, and *ú-u*, and the last *nu* (Fleet’s discovery). As regards the reading of 𑀔 , 𑀕 (𑀓), 𑀭 (𑀭), it is doubtful as yet whether the pronunciation was *phu*, *gu*, *hu*, or *phra*, *gra*, *hra*. I now incline to the latter view

(though I cannot find any distinct proof of it), because the *u* certainly appears in the hundreds. The proofs are :

“(1) The most certain evidence for the ancient times is furnished by the Rûpnâth and Sahasrâm Edicts. In the former 200 is clearly ρ , while in the latter $\mathfrak{y}$ is used. It is impossible to see in the first sign anything else but the syllable *sû* (not *su*), as the elongation of the right-hand stroke of the *s* shows that something else than the simple *su* is intended, and the natural explanation is that the second *u* sign, which makes the vowel long, has been attached at the top instead of below, ρ instead of ρ . A similar plan for the expression of long *û* is adopted in Dr. Burgess's new inscriptions of Purushadatta from the Stûpa at Jagayyapettâ. There *tû* is several times written $\mathfrak{t}$ and the stroke indicating the length of the vowel attached to the top of the *t*. The cause of this proceeding, as well as the uncouth appearance of the *s* in $\mathfrak{y}$ (Sahasrâm), is the desire to distinguish by the form of the syllables, the cases where they have numeral values, from those where they have merely an etymological value as parts of numerals.

“(2) The second proof is the fact that several syllables change their shapes according to the change of the letters in the various alphabets (Bhagvânâlâl), always with this proviso, that mostly some slight difference is allowed to remain between the form of the syllables as numerals, and those used as parts of words. The change shows that the people pronounced the syllables as syllables, and the differences which frequently occur are due to the reason above given.

“(3) The third proof is that a few signs show such variation as can be explained by phonetic changes, which in the language, too, are of very frequent occurrence. The clearest case is that of the syllable for 100. In the Asôka edicts we have *su*, and the same occurs in the Andhra, and a great many other inscriptions; but the Western Kshatrapas and others use $\mathfrak{c}$, which is clearly *sû* $\mathfrak{c}$. Now in all Indian languages there occurs a wonderful confusion of the sibilants, and in ancient times *sa* and *śa* are in Prakrit equivalents. The one

occurs constantly for the other. If we therefore find ࣳ and ࣴ in the numerals for 100, the conclusion is that the people were in that case as careless as in others, and pronounced indiscriminately *su* and *śu*, because they were accustomed to do this in common life. The same was probably the case for the numeral syllable for five, where side by side with forms which clearly are *nu*, others occur which must be read *no* (𑀭𑀮, Bhagvānlāl's tables).

"(4) Fourthly, there are other cases where certain sects, or the Pandits of certain countries, have misread the ancient signs, and have substituted wrong syllables for them. The best cases are: (a) that of the numeral syllable for 10, which in the oldest forms is *la* ࣳ in the south, and *da*¹ ࣴ in the north. This has been rendered by ࣳ and ࣴ (*lri* and *lū*).

"(b) That of the numeral syllable for 100. The Nepalese have misread (as has sometimes been done by modern epigraphists also) the ࣳ=*su*, as ࣳ=*a*.

"In these cases the important point, which shows that the people pronounced syllables, and not the numerals when reading the signs, is that they always made a new syllable of the old sign, not a mere unintelligible symbol. Had they pronounced *sata* for ࣳ, they would have left it, and not have written a clear ࣳ for it.

"(5) The fifth argument is that down to the present day the numeral syllables are called *akṣarapālī*, viz. 'letter table,' by the Jainas, and are known to represent syllables. Mallinātha (circa 1150 A.D.) speaks distinctly of such a syllable as a *śabda* 'a word.'

"II. The system of numeral syllables as we find it in the oldest inscriptions was settled by the Brahmanical schools. The proof of this assertion lies in the use of the signs ࣳ (*la*) *gñ*, ࣴ (*p*) *ph*, ࣶ (*a*) *kh*, which occur in Brahmanical books, and speak alone. Nobody but a Brahman could have dreamt of making the Anunāsika ࣵ, the Jihvāmūliya ࣶ, and Upadhmanīya ࣴ serve for numerals. The circumstance that the three strokes —, =, ≡ are intended for *u*, *ū*, *ū-u* (*hrasva*,

¹ *da* is the northern equivalent for *la* in the language also.

dirgha, *pluta*) points to the grammatical schools being the originators of the system. The proof for the assertion that the strokes too have a vowel value lies in the manner in which they are used with the hundreds and thousands—100 being expressed by ण (*śu*), 200 ण, and 300 ण; 1000 by q (*dhu*), 2000 by ण, and 3000 by ण. If the strokes had a mere numerical value, the marking would be wrong and unintelligible. We should then require for 200 ण, and for 300 ण. If we pronounce *śu*, *sû*, *sû-u*, the difficulty disappears. Hence, I conclude that wherever we find the strokes —, =, ≡, these, too, are intended as symbols for a vowel, and for the vowel *u*, because in all grammars the vowel *u* is used to illustrate the three stages, short, long and treble (*pluta*). Pāṇini's sūtra is 'u, ú, ú-ũ—hrasvadîrgha-plutâh,' i.e. 'vowels having the duration of *u*, *ú*, *ú-u* are called long, short and *pluta*.' The origin and meaning of these marks seems to have been forgotten very early, and in the Bauddha and Jaina books, *eka*, *dvi*, *tri*, or *sva*, *sti*, *śrî*, *om*, *na*, *mo* (the latter being the usual initial three syllables of books), are substituted.

"III. Though I claim for the Brahmans the oldest form of the Aksharapalli,¹ I do not claim for them its invention. We constantly find in India that something foreign imported into the country is made to assume native Indian forms, and disguised so cleverly that one would swear it was a native invention. As I believe that the Indian alphabets are foreign inventions introduced into India long before the historical times, I think it probable that the numerical system came too from a foreign country. I believe the Southern Indian alphabet came to India from Arabia or from the Persian Gulf, *vid* Suppârâ (Sopheir) or Broach, and that the Southern Indian numerals came by the same road. But I think that, in spite of the resemblances pointed out by Deecke² between the Himyaritic and Maurya letters, we

¹ Since seeing the tables given in Pl. II. Dr. Bühler informs me he is convinced that the "Aksharapalli" is of an origin extraneous to India, though he still finds it difficult to believe that its signs are borrowed from four or five different sources.—E. C. B.

² "Zeitschrift der Deutschen Morgenländischen gesellschaft" for 1877, vol. xxxi. p. 598.

have not yet found the alphabet from which the Southern Indian characters are derived. I think that there is much less chance of making out anything about the numerals, and of saying from what other system they are immediately descended. But it is not at all clear that originally they may not have come from Egypt, but probably through some Arabian traders either from the Arabian Coast or from the Gulf."

It now remains to set out the theory which it is proposed to maintain and illustrate in the following remarks; and it may be said, once for all, that for this purpose it is not necessary to deal with that part of Dr. Bühler's memorandum which treats purely of the origin and growth of the alphabetical signs.¹

Briefly this theory is, that the Indian numeral system was, if not wholly, at least mainly, of extraneous origin; but that it was not adopted all from the same source, or at the same time.

In short, it will be attempted to show that it was, so to speak, of an eclectic character; that it began by the adoption of a primitive and imperfect system—probably from the early Phœnicians; and that it was subsequently improved by successive adoptions at various times, and from different sources

¹ It is by no means intended to intimate any dissent from Dr. Bühler's views on this part of the subject; on the contrary, they seem at least *a priori* reasonable. It may perhaps be a question how far the examples given by Dr. Bühler go to prove the derivation of the Maurya forms of writing from the Andhra,—they seem at least equally consistent with the supposition that both may have been derived from some earlier common original, to which perhaps, in its square and more archaic forms, the Maurya may preserve a closer resemblance than the Andhra does. The more rounded forms of the latter may perhaps be due to the nature of the substance written upon—which to some extent is even still locally employed for writing, viz. the palm-leaf. On the other hand, Dr. Bühler's arguments may perhaps be held to make it at least probable that the early modifications of the letters which he traces to Brahman influence actually grew up for the most part in the Andhra alphabet, and that they were adopted thence by the Mauryas. Indeed, this is *a priori* probable, inasmuch as the Andhra court seems to have flourished for some time previous to the consolidation of the Maurya power, and would thus have furnished a centre of civilization and learning, where Brahmans would be more likely to find extensive patronage, than elsewhere in Central India at that day. Indeed, even afterwards, the atmosphere of the Maurya Court was possibly not (especially during its later years) altogether favourable to the development of Brahmanical ideas. Otherwise the divergence between the Andhra and the Maurya alphabetical types hardly seems greater than local circumstances would usually produce in India, within a moderate period of time.

(Bactrian, Egyptian, and possibly Cuneiform), of additional signs, until the whole grew up into a convenient, complete, and almost perfect form.

This theory of course implies a very considerable degree of intercourse between India and the nations from whom the signs are supposed to have been borrowed; and it will be endeavoured to show that such an intercourse actually did exist for commercial purposes.

This theory, it will be seen, is in fact little else than an amplification of the third hypothesis explained above, and is also in no respect contradictory to that advanced by Dr. Bühler; for it leaves entire room for the influence of the "aksharas" in the gradual modification of the numeral forms; in fact, as will be seen hereafter, the theory now advanced attaches even greater importance to the existence of the "aksharas" as a factor in the growth of the general science of enumeration, than is assigned to it by Dr. Bühler.¹

On one point, however, Dr. Bühler's memorandum requires a few words of explanation. He appears to have understood me as intending to suggest that the Indians were induced to borrow the numeral signs for *the purposes of commerce*. Such a suggestion, however, forms no part of the theory to be here maintained. In fact, it is almost certain, from their cumbrous character, that these early signs were not employed for purposes of calculation or account. Such an use of them probably did not spring up at least until the simplification of the system, and the employment of the nine units alone for purposes of numeration.

On the other hand, that the early Indians did adopt their numeral signs from foreign nations, and that they acquired their knowledge of these nations, and of all the incidents of foreign civilization—including the use of numeral signs—from an intercourse which was purely commercial in its

¹ It may be observed that the influence of the aksharas and the Brahmanical manipulation of the numerals would hardly begin to take effect till the numerals were employed for manuscript purposes,—or perhaps for use in sacred MSS.; and, as will be seen from what has been said in a previous note (p. 337), of Professor Rask's remarks on the ancient Cingalese numerals, this use did not probably begin till the fourth or fifth century B.C.

initiation and object, is not only part of, but the basis of the hypothesis on which this paper is founded.

It will perhaps make the theory thus put forward more clear, if the circumstances which suggested it be briefly stated.

An examination of the principle of the ancient Indian mode of differentiating the signs for the 'hundred' and 'thousand,' so as to make them express higher powers of those numbers, was suggested by an inquiry which will be mentioned further on.¹

For the purpose of this inquiry, it became necessary to examine the ancient Egyptian numerals, to which attention had been drawn, as affording some points of resemblance to the ancient Indian numerals, by Barth, Burnell, Rodet, and other writers.

That system was found, as will be shown immediately, to present not only an identical system of differentiation, but to afford a complete explanation of the causes which produced it, which causes did not exist in the Indian system. It became palpable therefore that it was borrowed by the Indians from the Egyptians. Further inquiry, however, as to other resemblances between the Indian and the Egyptian numerals, whether these last were hieratic or hieroglyphic, showed that such resemblances could be readily detected in three instances only—viz. in the earlier symbol for a 'thousand,' which appeared to be taken from the Egyptian hieratic, and in the latest symbols for a 'hundred' and a 'thousand,' which were both clearly referable to hieroglyphic origins.

The evidence of borrowing, however, thus obtained, invited inquiries in other directions. General Cunningham long since indicated the Bactrian *alphabet* as the original source of the unit numerals from '4' to '9,' and a comparison of these last in their oldest form, as given on the Nánághát inscription, with the *lapidary* (not numismatic) forms of Bactrian writing of almost contemporaneous date, found in the Shábázgiri edicts of Asóka, fully confirmed this conjec-

¹ See p. 370.

ture.¹ But it was not possible to carry the identification beyond these unit signs. In proceeding, however, to examine the Bactrian *numerals*, as explained by Professor Dowson [J.R.A.S. Vol. XX. o.s. for 1863, p. 228], it became apparent that the second (rather rare) *cruciform* shape of the Indian 'four' was identical with the 'four' of the Bactrian *numeral* system, and, moreover, that one of each of the signs for '20' and for '100' in the second oldest stage of the Indian system were only slightly disguised forms of the Bactrian signs for those numbers.

After this result there remained unidentified only the forms for 'ten,' and one other (the earliest) form of 'twenty,' the (earliest) sign for a 'hundred,' and the signs for the higher powers of 'ten' (except that for 'twenty,' which has been already dealt with above). The signs for 1, 2, and 3, were of course common to every ancient (non-alphabetical) system of numerals. Induced, however, by the analogy of the Bactrian numeral system to that of the Palmyrene Phœnician, an examination of the various Phœnician systems was next begun, and among the remaining groups of Nánághát numerals, it was found possible to identify a complete series of archaic non-alphabetical Phœnician signs for the numerals of higher denomination than units, *viz.* the ten, the twenty, and the hundred; the first no doubt in an altered shape, but the hundred only slightly, and the twenty not at all disguised.

The figures for 30, 40, 50, 60, 70, 80, and 90 thus remained alone without any discoverable archetypes; and this partly because, except for the '60' and '80,' no examples can be found in the Nánághát inscriptions; indeed (except the 50, which occurs in the Asóka inscriptions) no other examples save those of a comparatively late date exist. Some conjectural remarks, however, on their possible derivation and mode of formation will be given later on.

¹ See General Cunningham's paper, J.A.S.B. vol. xxiii. for 1854, p. 703, note. It will be observed further on, that I have not entirely adopted the details of General Cunningham's identifications; indeed, with the fuller knowledge of the Bactrian letters which we now possess, and which is so largely due to General Cunningham's own labours, that writer would not probably himself now maintain them all, or indeed the theory founded on the facts as then understood by him.

In Plate II. Figs. 1, 2, 3, and 4, will be found tables showing the figures attributed to each of the sources thus assigned, arranged according to the supposed order of time in which the originals were severally laid under contribution, and the alterations (if any) to which they were subjected in the process of adoption.

In Fig. 1 are given the numerals attributed to the archaic Phœnician,¹ which, by reason of its simpler form, its greater antiquity, and possible relation to the original source of the Indian alphabet, it is proposed to consider the earliest in date.

It will be seen that the signs for one, two, and three, are in the Indian system drawn horizontally. This probably may be, as Dr. Bühler suggests, the result of Brahmanical influence in order to bring them into correspondence (when used for differentiation) with the written expression (in composition) of the powers of the vowel *u*; still in Egypt these signs were sometimes written horizontally also, and their adoption in this form may have been at first a mere matter of convenience in writing.

The earlier units were probably all expressed, as in the older non-alphabetical Phœnician,² and in the hieroglyphic,³ by groups of simple lines, till this method was, ere long, superseded by methods borrowed from the Bactrians.

The ten of the early Indian system, it will be observed, seems to be altered from one of two archaic Phœnician forms (or a compound of both), turned, however, from left to right, instead of from right to left, to suit the Indian mode of writing.

In the oldest shape (that found at Náná Ghát), the Indian sign for *ten* thus derived seems to express the syllable *thú*, as it would appear, written in Indian characters of that date, which may possibly be an *akshara* (though not that used later on) for *ten*; but this also may be a mere accident, for the ordinary *akshara* for ten does not seem to be *thú*.

¹ Cf. Gesenius, *Monumt. Vet.* pp. 80-88; Pihan, *Signes de Numeration*, p. 165; and De Luynes, *Numismatique des Satrapies et de la Phénicie*, pp. 112-114.

² See Pihan, *Signes de Numeration*, p. 164.

³ Pihan, *Signes de Numeration*, p. 26.

The *twenty* of the Phœnician, a simple circle, remains unaltered in its Náná Ghát shape. The Phœnician figure for a *hundred* is but very slightly changed in the Náná Ghát shape, and even still less so in that employed by Asóka for his inscriptions in Eastern India (where probably, as will be explained presently, more archaic types in general prevailed), though it is there differentiated in order to make it represent two hundred.

It seems probable that the Bactrian *numeral* system was that next laid under contribution; for it is hardly likely that the Bactrian *numeral* four should have been separately taken *after* the Bactrian *alphabet* had supplied, as will be seen, a full series of signs for the units above three. The loans from Bactrian numerals are therefore shown in Pl. II. (both in their original shape, and as reversed to suit Indian writing) Fig. 2. Nothing further need be remarked in regard to the identity of the figure for four;—as has been said, the use of this four in Indian numerals is rare—but it is found under more than one modification on the silver coins of the Guptas. It gave rise certainly to one of the two signs for forty, and, as will be shown in the sequel, eventually prevailed as the model for all the modern Indian and European types of *four*.

The *twenty* of the Bactrian system (a double ten) will, *if written reversed* to suit the Indian writing, and with top and bottom joined by a curved “rounding” stroke, give the exact form of the “Cave” variety of the Indian “twenty,” which, slightly modified, was employed almost exclusively¹ on all later coins and inscriptions.

The *hundred* of the Bactrian system, if also reversed, requires but little alteration to convert it into the likeness of the Indian “S,” in which shape it occurs on Asóka’s Western (dated) Inscription, and to which form it was doubtless adapted under Brahman influence, in order to fit it to the alphabetical expression of the “akshara” “su,” which denotes a hundred.²

¹ The old form, however, occurs on one of Kumára Gupta’s inscriptions at Garhwa (of 129 Gupta), Cunningham, Arch. Survey, vol. x. p. 7, pl. iv.

² Though in this case also the sign is differentiated to express *two* hundred.

To pass to Fig. 3, Pl. II., it may be taken as likely, for the reason already assigned, that the adoption of certain signs from the Bactrian alphabet was of later date than the borrowing from the Bactrian numerals. It was *probably*, on the other hand, anterior to the borrowings from the Egyptian systems, for *some* of these latter were certainly of very late date, later it will be demonstrated than that of the Nánághát, or even of the Maurya inscriptions; and the use made of these Bactrian signs to improve on the Egyptian system of differentiation, seems to imply some familiarity with their use at a comparatively early period, and before the mode of differentiation was obtained from Egypt.

It is necessary, however, to explain Fig. 3 more fully, and it can only be said that the proof of borrowing consists solely in the almost *absolute* identity of the numerals, with the older lapidary Bactrian forms of certain letters. It will be seen that the 4 = the Bactrian letter *chh* (not *ch*), the 5 = *p*, the 6 = *g*, the 7 = *a*, the 8 = *b*, the 9 = *h*.

Now, so far as it has been possible to trace the matter, it may at once be said that the numeral force thus assigned to the several letters cannot be explained by any known system of alphabetical numeral notation, Aryan, Semitic, or Turanian. Nor do the letters appear to give the initials of the names of all or even most of the units they represent, in any known language.¹

This fact, however, is not in itself sufficient to overthrow the evidence derived from the absolute identity of the two sets of forms. One system of notation by letters, well known, and certainly of very considerable antiquity, the Arabic *abjad*, appears to be equally arbitrary in its assignment of numerical powers to letters.

Still there doubtless was some reason for the adoption of this system of notation, and for its adoption as an *integral* system, for it will be observed that it overlaps the older system supposed to be already in vogue in India, and gives a second and alternative form for '4.'

¹ Indeed of hardly of any except the five.

To confess the truth plainly however, it is only possible with the present total want of evidence on the point to conjecture this reason, or chain of reasons. A mediæval writer, Radulphus, Bishop of Laon, who died in 1131 A.D., declared that the Greeks obtained the *peculiar names* for the units, used by the Neo-Pythagorean school, *with the abacus*, from Chaldæa. It will be shown presently that this assertion is in a great measure corroborated by facts; and from analogy there seems no great improbability in guessing that the Indians obtained not the names, but the *signs* for the "abacus," together with that instrument. For it has been already pointed out that the signs for the *units only*, which alone would be used with the "abacus," appear to be taken from the Bactrian alphabet, and that there seems to have been no further borrowing from *that* source. If this conjecture be correct, it would show, what is not antecedently impossible, that the knowledge of the "abacus" was obtained by the Indians *through Bactria*.¹

But if even this hypothesis be granted, the question is only advanced one step towards solution; for why, it may be asked, came it to pass that the signs of the "abacus" were indicated by these particular letters? even in Bactria before their transfer to India?

Two guesses may perhaps be permissible; the letters may perhaps represent the initials of some lost system of unit "aksharas," and perhaps, if this be so, this explanation might cover also the origin of the "abjad" system. For, as will be explained a little further on, there seems reason to suspect that a system of "aksharas" or phonetic notation was very early and very widely employed.²

But it may be noted that two Arabic traditions assign to the "Abjad" system, a mnemonic formula as its origin, one declaring it to be expressed by a sentence which gave the

¹ It need hardly be said that the use of the "abacus" is still common in every village bazar in India, and has been universal apparently from time immemorial.

² The term "akshara" (from the negative "a" and "kshar"), signifying "indestructible," "incorruptible," seems to be a term invented after the introduction of writing, or at least of numeral signs, as indicating the superiority in respect of durability and accuracy of the phonetic signs.

names of the first six kings of Midian, and the other by the names of the inventor of the Cufic alphabet and of his sons.¹

Such an origin would, too, not have been at all incompatible with Indian usage, for, as will be shown, the 'aksharas' themselves were practically strung together in a metrical form, so as to give a sort of 'memoria technica.'

The subjoined conjectural solution of the point is therefore offered, though with diffidence, for it must be admitted that some of the suggestions made are not in exact accordance with the strict rules of orthography, at least of the *Sanskrit* branch of the Aryan tongue.

It will be observed that the fourth letter in the group is the initial letter *a*; manifestly if they be the initials of syllables, this letter must mark the beginning of a fresh word. Supplying the inherent vowels, this word would become *abaha* or *abah*. The accepted etymology of *abacus*, or at least that generally favoured, refers it to a Semitic word equivalent to the Hebrew (Ex. ix. 9, Is. v. 24, Ez. xxvi. 10) אַבָּק *abak*, or أَبَق (which means 'very fine dust'). This word might well be written in dialects with the final letter ה, corresponding to both the Arabic ح and خ, and might therefore thus pass to أَبَح and أَح *abakh* and *abah*, by a very slight change, or *abah* might represent the name of the instrument, in a form derived from the word for 'dust.'

Admitting this, the preceding syllables must refer to some objects connected with the *abacus*. If they refer to the signs themselves, the expression would be in the plural number, and this plural would probably be denoted by a suffix; the last letter, *g*, would then represent this suffix, and it naturally recalls the modern Persian plural form of *gán*, used where a word in the singular ends in a silent *h*; as *bundah*, *bundagán*, *istádah*, *istádagán*, *zindah*, *zindagán*, etc., etc.

¹ M. Woepeke (Sur l'Introduction de l'Arithmétique Indienne en Occident, p. 68) quotes from Sibth ul Máridíní, who died in 1527-28 A.D., another mnemonic arrangement of letters according to the *Abjad* system, in groups according to the powers of each unit, thus 1, 10, 100, 1000; and 2, 20, 200, 2000, etc.

If this be so, the remaining syllables would represent *chhapah*, or *chhápah*, a word which comes very near, indeed, to the Hindi word *chhápá*, which signifies a 'sign,' 'stamp,' 'seal,' or 'mark.' It has also a technical meaning in connection with the 'Vaishnava'¹ creed, denoting the sectarian marks, such as the 'trident,' 'lotus,' etc., which the devotees of Vishnu delineate on their bodies. Under the form of چاپ, the same root seems to occur, and with a similar general force, in the modern Persian; that is to say, in the sense of 'stamp,' or 'mark.' It is very improbable, either that a word of foreign origin should be employed in the technical religious sense which it possesses among the Indian Vaishnavas, or that the Persians should have borrowed the word from the modern Hindi. The alternative is to accept it as derived in both instances from some common Aryan source. It is true that the word *chhápá* has not been admitted by etymologists as referable to any known Sanskrit root. The form in *chh* seems to indicate an original in *ksh*, but the nearest root in Sanskrit is *kshi*, from which *chhápá* does not come by any regular rule. It is possible, however, that in this latter form it may be only a corruption of some other, such as *kshépa*; and though that word does not occur in the same sense, one meaning of the root appears to be 'to smear,' or even 'to depress,' from which the modern meaning may have come in a secondary form; or the word may be even derived from some lost Aryan root. If it be in any way referable to an Aryan origin, it would not be out of place in a Bactrian sentence. Nor is it perhaps too much to assume, that the modern Persian plural in *gán* is, perhaps, a relic of some (perhaps popular) form of an ancient plural suffix of the same character. If these conjectures be admissible, the sentence would of course stand as *chhápágán-(i)-abah* 'signs of the abacus.'

After all, however, this interpretation can only be offered as purely conjectural; the argument as to the Bactrian origin of the signs cannot rest upon it, but must be based solely on

¹ See Thompson's Hindi Dictionary; also Fallon's Hindustani Dictionary, *in voce* "Chhápá." In the latter, a quotation of a Hindi verse will be found, in which the word occurs in this sense.

the almost exact identity of their forms with those of the Bactrian letters.

Passing, however, from Fig. 3, Plate II., the borrowings from Egyptian sources (Fig. 4) remain for consideration.

Of these, by far the most important in every way is the system of differentiation. It might be imagined, if the ancient Indian system of numerals came from Phœnician sources, that the Indians would in the first place have adopted also the Phœnician mode of differentiation. This was what actually happened in the Bactrian system, which appears to have been taken from the quasi-Phœnician Palmyrene. The Phœnicians using a certain symbol for a hundred, differentiated it with the unit representing its powers; thus for one hundred, they wrote the hundred symbol with a single stroke on the right hand (i.e. *before* it); in writing two hundred, they added a second stroke: exactly as in England we write £1 for one pound sterling, £2 for two pounds, or \$4 for four dollars, etc., etc.

But the actual Indian mode of differentiation as first found existing, is wholly and markedly distinct, and, as it occurs in its Indian form, appears arbitrary and based on no principle. Thus, the simple symbol for a hundred (or a thousand) stands for one hundred (or one thousand); but when it is given with a *single* differentiating mark (to the *right*, that is, *after* the symbol), it stands for *two* hundred; and with *two* such marks, for *three* hundred; that is, there is always one mark *less* than the power to be indicated. When, however, four hundred is reached, then the differentiating mark takes the form of the *unit* 'four,' and similarly in the case of the remaining higher powers of a hundred below one thousand.

If we turn to the column showing the Egyptian hieratic forms in Fig. 4, Plate II., it will be seen that this is also the principle of that system, and that *there* its adoption is easily explained. In the original *hieroglyphic* signs, two hundred was denoted by a group of two of the symbols for a hundred, four hundred by a group of four, eight hundred by a group of eight symbols, etc., etc. The hieratic, however, which was

a species of 'shorthand' writing, after once indicating the symbol for one hundred, did not repeat it at length in the case of numbers above one hundred, but merely placed *after* it a square dot (or a line representing a group of dots) for each symbol for a hundred which it omitted to write at full length. Thus, one dot was added for two hundred, two dots for three hundred, and so on. The number of dots being therefore, as in the Indian system, always exactly *one less* than the power of the hundred which it was desired to express. In the case of the seven hundred and nine hundred, however, as the hieratic Egyptian possessed special symbols for the units 'seven' and 'nine,' these were used to differentiate the hundred symbol (they had also a hieratic symbol for five in occasional use, but did not employ it to differentiate) instead of groups of dots, or strokes; exactly on the same principle as that which governs the use in the Indian system of the unit signs above three for purposes of differentiation. The Indian system was therefore in principle altogether identical with the hieratic Egyptian; but so far improved on its model, that having apparently already adopted from the Bactrian alphabet special symbols for each unit above three, they employed them to express not merely the seven hundred, and nine hundred, but all powers above three hundred, instead of the short spur-like side strokes with which they replaced the Egyptian dots.

Now no two other known systems of numeration in the world possess this peculiar mode of differentiation, which, it must be admitted, is hardly one which in *all* its details is likely to have been twice invented. The history of its growth is not to be traced in the Indian system—it is easily seen in the Egyptian, where its development can be explained by natural causes. Moreover, it was employed in Egypt at least in 1200 B.C.,¹ at a period far more ancient than any probable Indian use of numerals.

There need be therefore little hesitation in affirming that

¹ The presumed date of the "Rhind" Papyrus, but the first use of these symbols was probably older still.

the Indians adopted this system exactly in all its principles, as they found it already developed in the Egyptian hieratic, improving it, however, to the slight extent already mentioned, in a few of its details.

They effected another improvement, moreover, for they used this mode of differentiation both for the hundred and the thousand symbol. The Egyptians employed for the thousand another and clumsier mode, allied to the ancient Phœnician. This last adaptation may perhaps indicate that the Indians adopted their mode of differentiation when as yet they had no separate symbol for the thousand; a supposition which is not indeed unlikely, for they had been hitherto borrowing only from systems belonging to the Phœnician family, and, so far as is known, no *Phœnician* system had any separate symbol for a thousand, which number they appear to have represented by a *group* of figures.¹

The Indians, it may be observed, seem likewise to have borrowed *all* their signs for the thousand from the Egyptians; the earliest form being apparently merely the Egyptian hieratic symbol deprived of its side stroke; the later one a cursive form of the hieroglyphic 'lotus' symbol. The latest Indian symbol for the hundred also (which does not occur till *after* 225 B.C.) seems to be, in its original form, merely the hieroglyphic symbol for a 'hundred' partially straightened.²

The only numerical signs therefore, of the ancient Indian system, to which no positive derivation can be assigned, are those for the powers of ten above twenty, viz. the 'thirty,' the 'forty,' the 'fifty,' the 'sixty,' the 'seventy,' the 'eighty,' and the 'ninety.' It seems, however, probable that these did

¹ This fact may perhaps explain the following quotation from Sibth ul Máridini (Woepcke, *Sur l'Introduction de l'Arithmétique Indienne en Occident*, p. 67): "Sachez que les ordres elementaires des nombres sont au nombre de trois: unités, dixaines, et centaines, dont chacun comprend neuf nœuds."

² The term "latest" is used with the knowledge that an apparently new symbol for a hundred (which Gen. Cunningham supposed to be a Bactrian letter) occurs among the Indo-Scythian and Gupta numerals, but this seems rather a cursive modification of the 's' shaped, or second oldest, form of the symbol (of which it has been suggested that it came from the Bactro-Phœnician form), the 'crook' on the left side only being omitted.

not come from any single source. The forms of the double signs for 'forty' at any rate clearly indicate a derivation from the double signs for 'four,' and they were therefore probably invented in India, after the adoption of the latter. The sign for 'fifty,' is used either turned to the left hand or the right indiscriminately, and it may be suggested on this account, that it was probably borrowed at a late stage, from some Semitic form of cursive writing, and that it was written sometimes in one direction with reference to its Semitic origin, sometimes in another, according to the direction of the Sanskrit writing. The Egyptian hieratic fifty might indeed possibly be the model of the early Indian fifty. There is also another hieratic form for thirty besides that which is shown on Plate I. (*see* Pihan, *Signes de Numeration*, p. 27). On my silver coin of Skanda Gupta, dated in the year 134 of the Gupta era, the thirty is almost identical with this Egyptian hieratic form for thirty. The hieratic was written always in Semitic fashion from right to left.

Again, there seems some probability that the signs for sixty and seventy are connected. The Cingalese numeral system, which preserves faithfully to the present day nearly all the principles of the old Indian system, forms the sign for seventy by the addition of the sign for ten to that for sixty. The ancient Indian form for seventy is evidently only that for sixty, with a spur-shaped stroke added to the right, which may perhaps give the power of an added ten.¹ In connection with this fact may be remembered the Akkadian Υ^2 for 'sixty,' which is converted into $\Upsilon\angle=70$ by the addition of $\angle=10$. So also the Assyrian $\Upsilon\angle=60$ becomes $\Upsilon\angle\angle=70$ by the addition of the $\angle=10$.³

The eighty also seems to have been converted into ninety by the addition of a similar augmenting stroke, and may,

¹ This has already been suggested by Pandit Bhagwán Lál (*Indian Antiquary*, vol. vi. for 1877, p. 46).

² I am indebted for these signs to the kindness of Mr. Pinches, of the British Museum.

³ The Himyaritic '50' is also of the same form as the Akkadian '60,' and is also augmented by signs for '10' (*Ind. Ant.* vol. iv. p. 27).

perhaps, be borrowed from the same source as those for sixty and seventy.¹

It is not practicable, however, with the information at present available, to do more than thus conjecture the process by which this last group of signs was obtained, or the principles on which they were formed. Indeed, as has been pointed out, except of the sixty and eighty, no examples exist in the Nánághát inscriptions, and those of Ásoka only add that for fifty, and it is therefore impossible to be certain even of the exact shape of most of the oldest forms.

At any rate it would seem clear that the early Indians were unacquainted, when they adopted these signs for the powers of ten, with any alphabetical or other system, which would have furnished them with such signs ready made, as, for example, the Greek or Phœnician *alphabetical* systems, which possessed a complete series of symbols for the powers of ten.²

Finding themselves, however, with a perfect system of counting by separate symbols, from unity up to twenty, on the one hand, and on the other with a similar set provided for the numbers between one hundred and twenty thousand, the Indians apparently set themselves to fill the intervening gap, and provided themselves with the missing signs, probably by a process of mixed borrowing and adaptation.³

¹ The Indian sign for eighty $\ominus$ *might* perhaps be taken from the Akkadian sign for sixty placed between two Akkadian signs for "ten," thus $\langle \text{V} \rangle$. Cf. the Assyrian hieratic numerals as given by Menant, *New Assyrian Grammar* (1882).

² It is a corollary of this conclusion that at the time when these indents were made on their alphabet, the Bactrians possessed no regular alphabetical system of notation. It has been suggested that certain letters occurring on the coins of later Bactrian kings, *e.g.* Hippostratus, Azas and Azilisas, express numbers and dates. If so, the idea, or even the system, must have been obtained from the Greeks, and this is rather rendered probable by the fact that these signs often seem differentiated by the vowel *i*, which was used by the Greeks to express 'ten.' If these figures represent numbers at all, therefore, they are probably low numbers, and if dates, regnal dates only.

³ Of course they were not altogether debarred from expressing the intermediate numbers, for they could have used the Phœnician and Bactrian mode by which the highest of these were expressed by groups compounded of the signs for 20 and for 10, and in which 20+10 stood for 30; 20+20 for 40, etc. This Phœnician method of counting by twenty and tens together, must apparently have been the origin of the Modern French "soixante-dix," "quatre-vingt," etc., coming down from the usages of the early Phœnician colonists of Marseilles and other seaports. It is curious that this awkward and antiquated method should have superseded the far more convenient and expressive "septante," "octante," and "nonnante."

Leaving, however, this portion of the subject, it is necessary to consider how far such long series of suggested borrowings from foreign sources was either possible or probable.

The possibility turns of course mainly on the degree of intercourse which existed in ancient times between the Indians and the older Phœnicians, the Bactrians, the Egyptians and other neighbouring nations. This question opens out an enormous subject, into the discussion of which it is impossible to enter at full length, and which has been exhaustively treated by many writers of high authority, such as Lassen (*Indische Alterthumskunde*, specially in vol. iii.), Reinaud (*Memoire sur l'Inde*, and in other papers¹), Heeren, Weber, and others. It will suffice, therefore, merely to cite very briefly a few leading facts, which will show that the connection between the Indians and other neighbouring nations, was both more ancient and more extensive than is ordinarily believed; and that their intercourse with the nations of the west was chiefly carried on through the ports and cities on the *Western Coast of India*.

As regards the Phœnicians, evidence has been brought to light by recent archæological enquirers, which renders at least possible the view² that their original seat was on the coasts of the Persian Gulf, as explained by Sir H. Rawlinson, *J.R.A.S.* Vol. XII. n.s. p. 218, and in the same place, pp. 212–215, will be found a discussion on the early commerce between the ports on the Red Sea on the one hand, and of the Indian Coasts on the other—through the intermediate ports of the Persian Gulf. For a sketch of the actual trade carried on by the Phœnicians, both by sea and land, see Humboldt's *Cosmos*, vol. ii. pp. 128 to 133 (Murray's edition).³ Solomon⁴ no doubt employed the aid of the Phœnicians, because of their already recognized skill in navigation and knowledge of Eastern commerce, when he desired to procure

¹ Specially that in *J. A.* vol. i. series 6.

² Maspero, "*Histoire Ancienne des Peuples de l'Orient*," pp. 146–148, 168–170; F. Lenormant, "*Manuel d'Histoire Ancienne de l'Orient*," vol. ii. pp. 240–244.

³ Cf. also Heeren (*Asiatic Nations*), vol. i. Chapters on Babylonian and Phœnician commerce.

⁴ See 1 *Kings* xxii. 45.

the valuable commodities of the East; and the intercourse of the Phœnicians with Eastern nations must therefore, even at that early date, have been thoroughly established. It is now beyond doubt that, whatever may have been the exact locality of Ophir, the goods which Solomon imported thence were distinctly Indian productions. Nor is there any greater improbability in the supposition, that the Phœnician traders were in frequent intercourse, from very remote ages, with the coasts of India by the Persian Gulf and Red Sea, than with the coasts (to them more difficult of access) of Britain and even of Northern Europe, as it has long since been acknowledged that they were. It was of course with Western India, and indeed with the upper part of the coast of Western India, that this intercourse, whether by sea or land, must have commenced. By land, the route lay through Sind. By sea, the favouring monsoons to which alone the ancient vessels would trust themselves blew from the S.W., and took all ships, whether from the Red Sea or Persian Gulf, naturally to the upper coast. Besides, below Bombay on that coast there is no harbour which can usually be made with safety by sailing ships during the S.W. monsoon, at least till as far South as Aleppee. It was no doubt for this reason that, as Reinaud shows,¹ the direct intercourse with Europe was up to a late date confined to the Western Coast, though afterwards it extended to Ceylon. Neither is there any reason to believe that this intercourse diminished in more modern times, as the demands of increasing civilization in the West afforded growing markets for the luxuries of the East;—Egypt no doubt was one of the earliest and largest of these markets—and there can be little doubt also that it was supplied at first mainly through the agency of Phœnician traders—whether by land or sea. In the Inscription of Cittium, as translated by the Duc de Luynes,² mention is made of the Phœnicians “residing in Egypt,” who can hardly have been anything but resident traders, and this inscription is attributed to the sixth century B.C. It may be uncertain how far at this period

¹ *Journal Asiatique*, series 6, vol. i. p. 354.

² *Numismatique des Satrapies et de la langue de la Phénicie*, pp. 112, 114.

any portion of this Eastern trade fell into the hands of the Egyptians themselves—though at a very early date some of their kings seem to have navigated at least the coasts of the Red Sea. Later on, events occurred which drew still closer the connection between the nations of the East and West. Political events sometimes brought the rulers of Assyria and Babylon into hostile contact with the races of Canaan and of Egypt, and sometimes into friendly or tributary relations. Under the Persian Empire, which reached Westward to Egypt, and to the borders of Greece, and which overlapped the frontiers of India in the East—it seems certain that intercourse between the extremes of that Empire, powerful, rich, and luxurious as it long was, was fostered rather than checked by their mutual relations to its central authority—Greek adventurers in no insignificant numbers passed into Persia and resided there, and it may be presumed that Egyptian fortune-hunters were hardly less numerous. The Greeks through these means gained considerable information of India and Indian affairs, and it may be taken almost for granted that the Egyptians were in relations with India and the East at least as close, and enjoyed knowledge of Indian matters equally intimate. Moreover, it is hardly to be supposed, on the other hand, that the quick-witted natives of India were slow to avail themselves of the opportunities thus afforded to them of acquiring some knowledge of the advantages possessed by Western nations.

Later on, the marvellous fortune which led Macedonian troops, not only to Persia, but across the Indus to the banks of the Bias, and which planted in Central and Western Asia, several dynasties all partaking more or less of Greek origin, added another great stimulus to the progress of relations between Europe and the East. The Courts of these kings became centres of a new culture, where Greek customs and the Greek language prevailed—as was also the case in Egypt. Without doubt these circumstances all added largely to the knowledge which Asiatics possessed of European and Egyptian matters, and to the more perfect knowledge of Asiatic affairs in the West. Moreover, after the great trading cities of the

Phœnicians were ruined by the attacks of the Macedonian and Syrian kings, the direct traffic, which had become thus stimulated and enlarged, fell in a great measure into the hands of those who traded through Egypt and the Red Sea. Later still, the Romans came on the scene, both as traders and politicians; and so early as the latter half of the last century B.C., the Romans (to say nothing of other nations) maintained mercantile establishments in all the chief ports of the Eastern seas.¹ With the favouring monsoon of every year, two thousand persons sailed from the ports of Egypt to those of the Red Sea, the Persian Gulf and of the Indian Peninsula; while the reverse monsoon brought to Egypt a similar influx of visitors or returning voyagers from the same quarters. These facts alone may be held sufficient to show the enormous intercourse which by that time had grown up between the West and the East, and which, considering the difficulties of ancient navigation, it must have taken a long course of years to establish. Intercourse of this nature must too have given to the Indians opportunities of learning, amongst other things, the Egyptian numeral systems, and it is to be observed, that the Indian borrowings from this source have on other grounds been placed amongst the latest obtained from any traceable origin.

Nor, while habits of mercantile enterprize were thus created in India, especially on the western coast, is it probable that the facilities afforded by the passes of Western Afghanistan for intercourse with Bactria, Persia, and Central Asia, were wholly neglected. Many of the products most valuable for western commerce, such as musk, saffron, assafoetida and other drugs and dyes, were obtained in this direction, or from Kashmir through the Panjab, where the Bactrian alphabet and numerals were also in use.

Under these circumstances, it is hardly likely that the

¹ Reinaud, in *Journal Asiatique* for 1863, vol. i. series vi. p. 97; see also rest of memoir, pp. 93 to 234. If we compare this Indian traffic to that with Europe before the employment of steam navigation, and deduct from the latter the demand for military and civil organization, the result will show that the purely commercial intercourse of modern times was not very greatly in advance of that of the last century B.C.

Indians should have long remained in ignorance of the systems of notation employed in these several foreign countries, or that they should have hesitated to avail themselves of such convenient inventions. If the theory offered be accepted, they would have naturally adopted entire the earliest with which they came in contact, the archaic Phœnician. Nor is there any reason to suppose that they would decline to avail themselves of any improvements which they came to know later on; on the contrary, it is abundantly clear from many facts in their history, that the Hindu people have always possessed a great facility for appropriating and assimilating anything which it was advantageous to make their own, blending even into their religion the traditions and superstitions of tribes and sects with whom they found it expedient to amalgamate; and they would hardly have been less ready to adopt from abroad, any signs which would render their method of numeration more complete or more convenient.

The Bactrian numerals would thus have given them a very convenient sign for four, in lieu of the clumsy group of four separate strokes, and a very distinct sign for twenty, which was always, in the archaic Phœnician, liable to be confounded with some of the forms of ten, and which was actually identical with the 'ten' of the Himyaritic (or Sabæan) of later times. The Bactrian hundred, too, was far more easily written than the old Phœnician, being composed of two strokes inclined to each other, instead of three at right angles, as in the Phœnician hundred. Again, if the introduction of the 'abacus' was, as has been suggested, accompanied by a knowledge of the special alphabetic signs already employed with it in Bactria, the Indians would hardly have rejected the convenience which these signs afforded. And thus, accustomed to improvements, they would not have neglected to adopt the neat mode of differentiation which they found in the Egyptian hieratic, when at a later date they became acquainted with it, or have failed to adopt, later still, the more easily written and easily distinguished signs for 100 and 1000 which they saw in the hieroglyphic.

The method in which the Indian numeral system grew up having been thus suggested, it is to be remembered that what has been said above applies, of course, only to the original formation of the Indian numeral system, as it stood at the commencement of the Christian era ; and the table in Plate I. is intended only to show this, together with some of the leading shapes through which it subsequently passed before the simplification of the mode of notation.

This last reform caused the disuse of the special signs for the higher numbers, and brought other influences to bear, which superseded the influence of aksharas, and which determined by degrees the modern forms even of the signs for the units which remained in use ; these points will be considered in their proper place. It will not be attempted, however, to trace all the variety of shapes (in a great measure determined by local accidents), which grew up in various parts of India, and which indeed, to some extent, prevail even now. On this point, Woepcke (*Journal Asiatique*, series 6, vol. i. p. 275) quotes the words of Albiruni (writing in the beginning of the eleventh century), which are here reproduced in their translated form :—
 “De même que les figures des lettres sont différentes dans (les différentes parties de) leur pays, de même aussi les signes de calcul (varient) : Ceux-ci sont appelés *anka* (اَنكَ). Ce que nous employons (en fait de chiffres), est choisi parmi ce qu’il y’en a de mieux chez les Indiens ; et peu important les formes pourvu que l’on connaisse les significations qu’elles renferment. Les Cachmiriens¹ numérotent les feuillets à moyen de chiffres qui ressemblent à des dessins d’ornements, ou aux lettres des Chinois, que l’on n’apprend à connaître que par longue habitude et par des efforts constants, et que l’on n’emploie pas dans le calcul (exécuté) sur la poussière.”

Passing from Northern to Southern India, and from East to West, we still find numerals employed, which it is difficult

¹ Dr. Leitner has recently collected a number of forms of numerals used in Kashmir by shawl-weavers and others, which exactly answer the description here given.—“*Linguistic Fragments*,” Sec. I. Lahore, 1882.

to believe can be in any way traced to the same originals ; for example, the ordinary Hindi numerals, which are certainly directly descended from the older forms, seem at first to bear no resemblance whatever to the Tamil or Malayalam forms, or to those still in use in Ceylon. Much of this difference has arisen from the fact, that the former have been brought into their present shapes by a new influence, the latter by that of the *aksharas* ; and this opportunity may be taken for saying that the continued efforts of Brahman teachers to bring the shape of the Indian numerals into accordance with the alphabetical signs which express the *aksharas*, though they will not be dealt with in this paper, constitute an essential part of the history of Indian numerals ; for this operation was most important, and its ultimate results are, as in the case of the Tamil numerals for example, and in many other instances, still visible. But this part of the subject in itself deserves detailed treatment in a separate essay, and this has been given in Professor Bühler's public lectures, which have been already delivered and are in course of publication. Moreover, the *aksharas*, as has been said, lost their command over the forms of numerals when the simplified notation was introduced, and therefore their history does not bear directly on the object of the present paper, which deals more directly with the question of the descent of our *modern* numerals, which were little affected by this influence.

It is, however, necessary to say a few words on the important share which the *aksharas* once bore in the general history of numeration,¹ which cannot be altogether passed over.

It may be taken as almost certain that the *aksharas* (of course not necessarily the particular Sanskrit *aksharas* of which Dr. Bühler writes, but other equivalents), *phonetic* numerals, in short, of some kind or another, were of immense antiquity and very wide diffusion.

¹ For a general description of the phonetic equivalents used in Sanskrit and cognate languages, see *Nouveau Journal Asiatique*, vol. xvi. pp. 1-42 (Jacquet) ; series 6, vol. i. pp. 284-90 (Woepcke) ; and series 7, vol. xvii. p. 47-130 (Rodet).

So far as actual proof exists,¹ no numerals can be referred to an earlier date than those connected with the hieroglyphics on the tombs of the Egyptian kings of the fourth dynasty, whose period by a moderate computation is placed about 2900 to 3000 B.C. It may not be safe to assume that no numeral signs existed at a prior date; but even at that period the hieroglyphic alphabet had barely passed out of its earlier syllabic form; and the early Egyptian and other similar archaic systems of numeration, were certainly too clumsy to have been employed as instruments for making calculations, or for teaching sciences which involved the use of such calculations. Nevertheless, it is pretty certain that at the period of the fourth Egyptian dynasty the people of that country, (and indeed several other races of mankind) had obtained a fairly high degree of civilization and of knowledge.

Geometry,² for example, must early have risen into importance in Egypt, by reason of the obliteration of land marks during the inundations of the Nile. The connection of the heavenly bodies and their motion with the divisions of time was certainly very early noticed, and the worship of the "hosts of heaven" not only necessitated, but gave a sacred character to the science of astronomy and to its concomitant, astrology. For the pursuit and communication of these sciences, the rude modes of notation by groups of signs, such as those which constituted the earlier Egyptian and Phœnician systems were manifestly wholly unfit, and even still more so the purely pictorial form of the oldest hieroglyphics. No other known mode of dealing with numbers, except that by phonetic forms, would have sufficed the needs of the human race under these circumstances; and these had the advantage of lending themselves with perfect facility to the construction of metrical formulæ, which could be easily taught and committed to memory. That such a method prevailed in India long before the Christian era,

¹ These facts are given on the authority mainly of my friend Mr. R. S. Poole; of the British Museum.

² Sir Gardner Wilkinson's *Ancient Egyptians*, vol. ii. p. 367, ed. 1878.

Dr. Bühler's researches show. It exists¹ to this day, not only in Sanskrit writings, but in a living form in those Sanskrit schools and colleges which are conducted on the old native plan, and it is still used for the conveyance of all forms of knowledge.² Of course, when the *abacus*³ came into use, it would at least be capable of superseding the use of *aksharas* for making calculations, though not necessarily for teaching their results. Indeed, even long after this invention

¹ For example, in the Sanskrit (Native) Colleges at Kishnagur in Bengal—and the following amusing note, borrowed from Dr. Burnell's South Indian Palæography (p. 65, 1st ed.), illustrates this state of things from the early Arab point of view. Albiruni (Reinaud, Memoire, p. 234) gives a remarkable instance of the Indian tendency this way: "Les livres des Indiens sont rédigés en vers, les indigènes croient par là, les rendre plus aisés à retenir dans la mémoire, ils ne recourent pas aux livres qu'à la dernière extrémité. On les voit même s'attacher à apprendre des vers dont ils ignorent tout a fait le sens. J'ai reconnu à mes dépens l'inconvénient de cet usage. J'avais fait pour les indigènes des extraits du traité d'Euclide et de l'Almageste; j'avais composé un traité de l'Astrolabe afin de les initier aux méthodes Arabes, mais aussitôt ils mirent ces morceaux en 'slokas,' de manière qu'il était devenu peu facile de s'y reconnaître." Dr. Burnell adds, "I have myself seen the Penal code put into Tamil verse."

² It does not follow that these were always merely syllables—they were in many cases doubtless, as they even now are in Sanskrit, *words* bearing other significations—see the papers on the subject already cited in a previous note at p. 33.

³ Perhaps in the Greek form of the name of this instrument some trace exists of the use of the 'aksharas.' Admitting that it was probably in its origin, the Semitic term for the material on which the signs were traced, 'fine dust' or 'abak,' yet it was an occasional practice of the Greeks to adapt foreign terms and even names, so as to bear a signification in their own tongue. "Αβαξ, the Greek form of the term, is identical with an adjective ἄβαξ, given by Eustathius as the base of the word ἀβάκησαν, which is found in the Odyssey—in the sense of being 'unconscious' or 'helplessly ignorant,' 'like infants'; the word occurs in a speech of Helen to Menelaus, who, speaking of the visit of Ulysses to Troy, says:

Τῷ ἱκέλῳ κατέδω Τρώων πόλιν, οἱ δ' ἀβακήσαν
πάντες.—ODYSSEY, δ. 249.

"Like unto this (sc. a beggar) he entered the city of the Trojans and they (other people) were unaware" (or 'like babies').

The derivation given is from the verb βαζω 'speak,' with the negative 'alpha,' that is, 'speechless (like a baby),' quā 'in-fans'; other words with the same derivation, such as ἀβακης (adjective), ἀβακέως (adverb), ἀβακίζομαι, ἀβακήμων, all with the same general sense, are also quoted in Liddell and Scott's dictionary (see ἄβαξ). "Αβαξ would thus in Greek mean 'speechless,' 'wordless,' or 'non-phonetic'—surely a very appropriate term for a 'silent' mode of calculation which superseded the phonetic 'aksharas.' The common Sanskrit term for the instrument seems to be *pāṭhi*, which signifies 'a board' or 'calculating board'; but the exact derivation is not given with certainty in any dictionary which it has been possible to consult. The Hindi word seems also to signify primarily 'a board,' though it may have reference to lines or divisions. But this derivation is not quite clear. In Russia (where the introduction of the instrument is attributed to the Mongols) its name signifies a 'counting' or 'computing' board. These later etymologies, however, do not throw much light on the original character of the instrument.

came into use, the teachers of the sciences to which the use of numbers mainly pertained, always in ancient days more or less belonged to peculiar classes, whose interest it was not to vulgarize their knowledge, or needlessly to facilitate its acquisition by the outside public.

While admitting, however, the direct influence, up to the seventh century, of the *aksharas*, on the forms which the Indian numerals assumed, this did not (as has been before said), probably, altogether exclude the influences of other and especially of local circumstances. As a general rule, moreover, the forms borrowed last in point of time will be found to have been employed first in the west, and to have penetrated more slowly eastwards. Thus, the two contemporary dated edicts of Asóka, found in the East and West, exhibit differing forms for each one of the three ciphers of which the date consists; and in the Gupta period the figures on the silver coins intended for circulation in the West, differed materially from those found on the inscriptions, which occur chiefly in the East. Possibly, too, something may be due to dynastic or national influences, for the Gupta and Indo-Scythic forms introduced a few marked peculiarities.

Before quitting this part of the subject, which deals with the question of the origin of the Indian numeral signs, it will be expedient to mention one special set of symbols,¹ which, though of comparatively recent formation, is yet of peculiar interest, both as illustrating the bold method in which the Indians could 'manufacture' signs, and also because their existence explains certain anomalous forms still found in some of the modern systems of numerals.

These are a set of signs for 'one,' 'two,' 'three,' and 'four,' which have hitherto been found only on the coins of a dynasty of Hindu Kings of Kabul and of the Northern Panjab, who were contemporary with the earlier Mahomedan invaders of India.

These signs will be found set out in Fig. 6, Pl. II. The

¹ It was the recognition of the old mode of differentiation as employed in these which led to an examination of the principle of that method, which was the commencement of the inquiry on which this paper is founded.

whole set of numerals to which they belong is of importance, as regards the chain of descent of certain of the modern forms, in which they form an important link. The first four numerals alone have a special origin, which requires fuller elucidation in this place. It will be seen by Fig. 6, Pl. II., that they were obtained by differentiating the upright stroke, which was the ancient Bactrian symbol for 'one,' in order to obtain signs for 'two,' 'three,' and 'four,' exactly on the principle of the Indo-Egyptian method employed in the old Indian numeral system to distinguish the signs for two, three, and four *hundred*, from the sign of *one hundred*.¹ That is to say, the sign for 'one' took one spur mark on the right side to convert it into 'two'; a second spur stroke (not, however, joined to the body of the sign, but superimposed, probably for convenience of cursive writing, on the end of the first side stroke) converted the sign for two, into one for three. The addition of the old Sanskrit unit symbol for 'four' to the extremity of the last side stroke of the three, as used to express four hundred, converted this last into the sign for 'four.' These last two symbols exist in their original form only on one coin each respectively, but the 'four' is found passing through several stages, till it approaches the form of the earlier Arabic 'four,' of which it was the palpable prototype, as the others were of the one, two, and three of the Arabic system.²

It is not easy to say when these symbols were invented. The original Bactrian system of numerals certainly remained in use with the Bactrian alphabet even in the Punjab through the whole of the first century B.C., though how much later it lasted is not so evident. The novel system must have been invented when the Indian principle of notation replaced the Bactrian, for not only the mode of notation, but the actual symbols for the higher numbers are clearly taken from Indian originals; and it would seem probable that this importation

¹ The evidence (from coins) on which this statement is based will be the subject of a separate paper in the *Numismatic Chronicle*, in connection with the era to which the dates belong, which these figures are used to denote.

² There can be little doubt as will be explained in the *Numismatic Chronicle* that the Arabs obtained their numerals from *Kabul*.

took place while the Indians still employed the cumbrous group of single strokes for one, two and three. If the simple and cursive forms, later on adopted in India, had been then in use, the northern people would scarcely have been at the pains to invent less simple signs for themselves; and yet, as will be shown hereafter, the new signs for one, two and three were probably the earliest of the cursive signs employed in India. Possibly the invention just described may be assigned to a period between the beginning of the second century at the earliest, and the middle of the sixth century A.D., at latest.

REFERENCES TO AUTHORITIES FOR THE FIGURES ON PLATE I.

Column A. The whole of this column is taken from Rossi's *Grammatica Copto-Geroglifica*, except the cipher for 100, copied from Pihan, *Signes de Numeration*, and Kminek Szeddo, *Saggio Filologico*.

Column B. This column is taken from Leon Rodet's transcript of the Rhind Papyrus (*Journal Asiatique*, series 7. vol. xvii.), which follows Eisenlohr's work on the mathematical papyri in the British Museum, except the last figure, which is taken from Pihan's *Signes de Numeration*.

Column C. Pihan, *Signes de Numeration*; Gesenius, *Phœn. Monumenta*, pp. 85, 89, and pp. 164-5; De Luynes, *Satrapies et Phénicie*, Paris, 1846, p. 112, and p. 42.

Column D. Dowson on the Taxila Inscription, *J.R.A.S. o.s. Vol. XX.*; Cunningham, *Arch. Survey of India*, vol. v. pl. xvi.

Column E. Cunningham, *Corpus Inscript. Ind.* Kapurdigiri Inscription, 1st Edict. *chh*, lines 1 and 2; *p*, line 1; *g*, lines 9 and 10; *a*, line 1; *b*, line 7; *h*, line 7.

Column F. Pandit Bhagwánlál Indrajī, *Indian Antiquary*, vol. vi. pp. 44, 45.

Columns G. and H. Cunningham, *Corpus Inscript. Ind.* Sasseram and Rupnath Inscriptions; also photograph of Rupnath Inscript. Bühler, "Three new Edicts of Asóka."

Figure 1

Borrowing from the Archaic Phœnician	Indian		
	The Archaic Phœnician	The same Reversed	Nana Ghat Maurya Cave
1	I	—	—
2	II	—	—
3	III	—	—
10	—O—O—	—	—
10	—O—O—	—	—
20	O O O	—	—
100	H H H	—	—
200	H H H	—	—

Figure 2

Borrowing from Bactrian Numerals	Indian		
	Bactrian Numerals	The same Reversed	Maurya Cave
4	X	X	X
20	3	3	3
100	Y	Y	Y
200	Y	Y	Y

Figure 3.

Borrowing from Bactrian Alphabet	Letters of the Bactrian Alphabet		
	Nana Ghat	Letters of the Bactrian Alphabet	Transliteration of the same
4	¥	¥	chh.
5	u	u	p.
6	q	q	g.
7	7	7	a.
8	7	7	h.
9	2	2	h.

Figure 4

Borrowing from Egyptian Hieratic and Hieroglyphic	Indian		
	Hieratic	Nana Ghat	Maurya Cave.
100	9	9	9
200	99	99	99
300	999	999	999
400	9999	9999	9999
700	9999	9999	9999
900	9999	9999	9999
1000	9999	9999	9999
1000	9999	9999	9999

Figure 5

Conjectural Derivation of the signs for the higher powers of ten		Successive Indian forms				Remarks
Suggested Originals	Barly forms	Nana Ghat	Maurya	Cave	Later forms	
30	X	X	X	X	X	No Remarks
40	X	X	X	X	X	Derived from Early four.
50	X	X	X	X	X	Derived from Later four.
60	Y	Y	Y	Y	Y	Derived from Semitic original and written sometimes according to original, sometimes to Indian usage.
70	Y	Y	Y	Y	Y	Some cursive forms of Akkadian Numerals.
80	Y	Y	Y	Y	Y	
90	Y	Y	Y	Y	Y	

Figure 6

Derivation of the Hindu-Kabul and Arabic signs for 'one', 'two', 'three', and 'four'.

Vala bhi.	100 = 7	200 = 7	300 = 7	400 = 7
Hindu-Kabul	1 = 1	2 = 1	3 = 1	4 = 1
Arabic.	1 = 1	2 = 1	3 = 1	4 = 1

Column I.

1. Karlé cave, No. 15. Burgess, Memorandum, No. x. on Cave Temple Inscription, p. 30. Photograph.
2. do. do. p. 36, Karle caves, No. 20. Photograph.
3. As for '2.'
4. do.
5. do. do. p. 44, Junár Inscription, No. 9. do.
6. do. do. do. do. No. 25. do.
7. As for '1.'
8. J.R.A.S. Bombay, vol. vii. Nasik Ins., No. 17, also 25, line 6.
9. do. do. vol. v. do. do. No. 2, line 1.
10. As for 5.
20. Burgess's Memorand. No. x. p. 34 ; Karlé Caves Ins. No. 34, Photograph.
40. do. do. Junár Ins. No. 25, and Nasik Ins. No. 16 ; J.R.A.S. Bombay, vol. viii. pp. 228-29, and vol. vii. p. 52.
70. do. Ushwadatta's Ins. Nasik Cave, No. 16.
100. J.R.A.S. Bo. vol. v. Nasik cave 2 ; vol. vii. Ins. Cave, No. 21.
200. do. do. do. do. do. No. 26.
1000. do. do. do. No. 16. do. do. No. 16.

Column J. Earlier Kshatrapah.¹

1. My coin of Rudra Sinha, son of Rudra Dama, dat. 141.
2. do. do. do. dat. 112 (also Junagarh Ins. J.R.A.S. Bomb. vol. vii. p. 118).
3. do. do. dat. 103.
4. Coin of Damajáta Sriyah, Thomas's facsimile from Dr. Stevenson's coin, dat. 154.
5. My coin of Damajáta Sriyah, dat. 155.
6. Coin of Vijaya Séna, Thomas's facsimile from Stevenson's coin, dat. 166.
7. Jasdan Ins. J.R.A.S. Bomb. vol. vii. p. 234.
8. My coin of Rudra Séna, son of Rudra Sinha, dat. 138.
9. do. do. of Vijaya Séna, dat. 169.
10. My coin of Rudra Séna, son of Rudra Sinha, dat. 112.
20. do. do. do. dat. 122.
30. As '8.'
40. My coin of Rudra Séna, son of Rudra Sinha, dat. 140.
50. do. do. of Damajata Sriyah, dat. 155.
100. As '5.'

¹ The division between the earlier and later Kshatrapahs is taken, somewhat arbitrarily perhaps, at the close of the reign of Rudra Séna, son of Vira Dama, whose dates extend to 198.

Column K. Later Kshatrapah.

1. My coin of Swami Rudra Séna, son of Swami Rudra Damna,
2. do. do. do. dat. 272.
3. do. do. Rudra Séna, son of Vira Damna, dat. 183.
4. Coin belonging to Capt. Temple of do. do. dat. 184.
5. My coin of Swami Rudra Séna, son of Swami Rudra Damna,
dat. 295.
6. My coin of Rudra Séna, son of Vira Damna, dat. 186.
7. do. Swami Rudra Séna, son of Swami Rudra Damna.
8. do. do. do. do. dat. 298.
9. do. Atri Damna, son of Rudra Séna.
10. My coin of Atri Damna, son of Rudra Séna, dat. 213.
20. do. Viswa Sinha, son of Atri Damna, dat. 22.
40. do. Asa Damna, son of Rudra Séna, dat. 243.
70. do. Swami Rudra Séna, son of Swami Rudra Damna,
dat. 272.
80. do. do. do. do. dat. 286.
90. do. do. do. do. dat. 298.
200. do. do. do. do. do.
300. do. do. do. do. dat. 300.

Column L.

1. Cunningham, Arch. Survey of India, vol. iii. pl. xiii. figs. 2, 4, 6, and pl. xiv. fig. 9.
2. Cunningham, Arch. Survey of India, vol. iii. pl. xiv. fig. 9.
3. do. do. do. vol. iii. pl. xiv. fig. 9, pl. xv. fig. 17.
4. do. do. do. do. pl. xv. fig. 8.
5. do. do. do. do. pl. xiii. figs. 2, 3.
6. do. do. do. do. pl. xiv. fig. 14.
7. do. do. do. do. pl. xiv. figs. 10, 11, 12, 13, 14.
8. do. do. do. do. pl. xv. fig. 20.
9. do. do. do. do. pl. xiv. fig. 23.
10. do. do. do. pl. xiii. fig. 2 and 6, and pl. xiv. fig. 9.
20. do. do. do. do. pl. xiii. figs. 3 and 7.
30. do. do. do. do. pl. xv. fig. 8.
40. do. do. vol. iii. pl. xv. fig. 8, and pl. xiv. fig. 11, 12, 13, 14.
50. Growse on Mathura Ins. Ind. Antiquary, vol. vi. p. 219, fig. 11.
80. Cunningham, Arch. Survey of India, vol. iii. pl. xv. figs. 16, 17, 18.
90. do. do. do. vol. iii. pl. xv. figs. 19 and 20.
100. do. do. do. do. pl. xiv. fig. 14.

Column M.

1. Chandra Gupta Ins. Udayagiri, Cunningham, Arch. Survey,
vol. x. p. 19.
2. do. do. fig. 1.

- 3, 4, and 5, as 1.
 6 and 7. From Pandit Bhagwán Lál Indrají's facsimiles in Indian Antiquary, vol. vi. pp. 44 and 45.
 8. Ins. at Bitha, Cunningham, Arch. Survey, vol. xi. pl. iv. p. 2 ; also at Garhwa, vol. x. pl. v.
 9. Pandit Bhagwán Lál's facsimiles, Ind. Ant. vol. vi. pp. 44 and 45 ; also Cunningham, Arch. Survey, vol. x. pl. iv. fig. 2.
 10. Cunningham, Arch. Survey, vol. x. pl. iv. ; also vol. iii. pl. iv. fig. 1.
 20. do. do. vol. x. pl. iv. fig. 2.
 30. do. do. vol. iii. pl. xx. fig. 1.
 80. do. do. do. do.
 90. do. do. vol. x. pl. v. fig. 1.
 100. do. do. do. pl. iv. fig. 2.

Column N.

1. Coin of Kumára Gupta, Thomas's transcription.
 2. My coin of Skanda Gupta, dat. 142.
 4. Coin of Skanda Gupta, Cunningham, Arch. Survey, vol. ix. pl. v. fig. 9, dat. 144 ; second from coin of Buddha Gupta, dat. 174, vol. ix. pl. v. fig. 13.
 5. Thomas's facsimile.
 8. do. do.
 9. do. do.
 20. do. do. as No. 1.
 40. My coin of Skanda Gupta, dat. 142.
 50. Coin of Toramana, Cunningham, Arch. Survey, vol. ix. pl. v. fig. 18.
 70. do. of Buddha Gupta, as 2nd form of 4.
 90. My coin of Bakra (Chandra?) Gupta. Indian Antiquary vol. vi. for 1877, p. 57.
 100. As 4.

Column O. Early Valabhi.¹

2. Grant of Dháraséna II. Ind. Ant. vol. vi. (photograph made by Dr. Bühler in my possession).
 5. do. do. Ind. Ant. vol. viii. for 1879, p. 301. Photograph.
 6. do. Guhaséna, Ind. Ant. vol. iv. for 1875, p. 174. Photograph by Dr. Bühler in my possession, also grant of Siladitya I., J.R.A.S. Bombay, vol. xi. p. 311.

¹ The Valabhi dates extend over a period of about 240 years. These dates give three nearly equal periods of about 80 years, say from 206 to 290 (Valabhi), 290 to 365, and 365 to 447 ; the first period terminating with the reign of Siladitya I., and the third commencing with that of Siladitya III.

7.	Grant of Dhruvaséna I.,	Ind. Ant. vol. v. 1876, p. 204.	Phot.
8.	do. Siladitya I.	do. ix. 1881, p. 239.	do.
9.	do. Dháraséna I.	do. do.	do.
10.	do. do.	do. do.	do.
40.	do. Guhaséna	do. v. 1876, p. 207.	do.
60.	do. do.	do. vi. 1877, p. 9.	do.
70.	do. do.	do. vii. 1878, p. 73.	do.
80.	do. Siladitya I.,	J.R.A.S. Bombay, vol. ii. end.	
90.	do. do.	Ind. Ant. vol. ix. 1880, p. 239.	Phot.
200.	do. Dháraséna I.	do. v. 1876, p. 204.	do.

Column P. Middle Valabhi.

3.	Grant of Dháraséna IV.,	Ind. Ant. vii. 1878, p. 900.	Phot.
5.	do. Dhruvaséna II.	do. vi. 1877, p. 15.	do.
6.	do. Dháraséna IV.	do. 1872, pp. 16 and 64.	do.
7.	do. Kháragriha II.	do. vii. 1878, p. 78.	do.
8.	do. do.	do. do.	do.
10.	do. Dhruvaséna II.	do. vi. 1877, p. 17.	do.
20.	do. Dháraséna IV.	do. i. 1872, pp. 16, 64.	do.
30.	do. Dháraséna IV.	do. vii. 1878, p. 94.	do.
200.	do. Dhruvaséna II.	do. vi. 1877, p. 15.	do.

Column Q. Later Valabhi.

1.	Grant of Siladitya V.,	Ind. Ant. vol. vi. 1877, p. 19.	do.
2.	do. Siladitya III.	do. v. 1876, p. 211.	do.
3.	do. Siladitya IV.,	J.R.A.S. Bombay, vol. xi. p. 311.	
4.	do. Siladitya V.,	Ind. Ant. vi. 1877, p. 19.	Phot.
5.	do. Siladitya V.	do. vi. 1877, p. 19.	do.
10.	do. Siladitya IV.	J.R.A.S. Bombay, vol. xi. p. 311.	
40.	do. Siladitya V.	Ind. Ant. vi. 1877, p. 19.	Phot.
70.	do. Siladitya III.	do. v. 1876, p. 211.	do.
300.	do. Siladitya III.	do. do. do.	do.
400.	do. Siladitya IV.	J.R.A.S. Bombay, vol. xi. p. 311.	