LXIII. On the calculations requisite for predicting occultations of stars by the moon

Professor Bessel

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LXIII. On the Calculations requisite for predicting Occultations of Stars by the Moon. By Professor BESSEL.

[Concluded from p. 342.]

7. **THE** quantities referring to the place of observation, viz. u, v, u', v' , depend on the height of the pole and the sidereal time at that place. It is unnecessary to calculate ϕ and ϕ' separately, as we have

$$r \cos \phi' = \frac{\cos \phi}{\sqrt{(1-e^2 \sin^2 \phi)}}; \quad r \sin \phi' = \frac{(1-e^2) \sin \phi}{\sqrt{(1-e^2 \sin^2 \phi)}}$$

where e denotes the eccentricity of the meridians of the earth. For Professor Encke's example, we have $\phi = 52^\circ 31' 15''$, and the oblateness of the earth $= \frac{1}{302.78}$, hence

$$\log . r \cos \phi' = 9.78505$$

$$\log . r \sin \phi' = 9.89752$$

If we denote by μ' the sidereal time, corresponding to the mean time T expressed in degrees, &c., the sidereal time corresponding to $T+t$ will be

$$\mu = \mu' + t.54147''84,$$

and consequently $\sin(\mu - A) =$

$$\sin(\mu' - A) + 2. \sin[t.27073''92] \cdot \cos[\mu' - A + t.27073''92]$$

$$\cos(\mu - A) =$$

$$\cos(\mu' - A) - 2 \sin[t.27073''92] \cdot \sin[\mu' - A + t.27073''92]$$

We have, therefore,

$$u = r \cdot \cos \phi' \cdot \sin(\mu' - A)$$

$$v = r \sin \phi' \cdot \cos D - r \cdot \cos \phi' \sin D \cos(\mu' - A)$$

$$u' = r \cdot \cos \phi' \cdot \frac{2 \sin[t.27073''92]}{t} \cos[\mu' - A + t.27073''92]$$

$$v' = r \cdot \cos \phi' \sin D \cdot \frac{2 \sin[t.27073''92]}{t} \sin[\mu' - A + t.27073''92]$$

For facilitating the calculation of u' and v' there is a Table at the end of this paper which contains the values of

$$\log \frac{2 \sin[t.27073''92]}{t} = \log \lambda$$

and $t.27073''92 = x$ for values of t between 0 and 1.5.

8. I shall now finish the example above adduced. For $T = 7^h$ the sidereal time will be $7^h 54' 7''.264$, and therefore

$$\mu' = 118^\circ 31' 49''.0$$

$$\mu' - A = -50 \quad 42 \quad 17.6$$

We obtain, therefore,

$$u = -0.47177; \quad v = +0.75914$$

and next

$$m \sin M = p - u = -0.19355$$

$$m \cos M = q - v = -0.20674$$

$$M = 223^\circ 6' 46''; \quad \log m = 9.45210.$$

As

As a first approximation we assume the value of t in the quantities $p', q', u', v' = 0$, and obtain

$$\begin{aligned} p' &= +0.5240 & q' &= -0.1679 \\ u' &= +0.1013 & v' &= -0.0091 \\ n \sin N &= +0.4227; & n \cos N &= -0.1588 \\ N &= 110^\circ 35' 26''; & \log n &= 9.65470 \\ t &= +0.2402 \mp 0.1690 \end{aligned}$$

or, Immersion $7^h.0712$; Emersion $7^h.4092$.

We next obtain for the second approximation by the formulæ of section 6.

	Immersion.	Emersion.
$p' \dots \dots$	$+0.52402$	$+0.52404$
$q' \dots \dots$	-0.16790	-0.16791

and by the Table at the end of this paper,

$x \dots \dots$	$+32' 7''.7$	$+3^\circ 4' 38''.6$
$\log \lambda \dots \dots$	9.41915	9.41895

by which we shall find next

$u' = \dots \dots$	$+0.10250$	$+0.10780$
$v' = \dots \dots$	-0.00907	-0.00873
$n \sin N =$	$+0.42152$	$+0.41624$
$n \cos N =$	-0.15883	-0.15918
$N =$	$110^\circ 38' 47''$	$110^\circ 55' 41''$
$\log n =$	9.65365	9.64898
$t = \begin{cases} +0.24026 \\ -0.16857 \end{cases}$		$\begin{cases} +0.23997 \\ +0.16621 \end{cases}$
$= +0.07169$		$+0.40618$

The third approximation gives again the values of p' and q' obtained in the second approximation; and besides

$x \dots \dots$	$+32' 20''.9$	$+3^\circ 3' 16''.9$
$\log \lambda \dots$	9.41915	9.41895
$u' =$	$+0.10251$	$+0.10774$
$v' =$	-0.00907	-0.00873
$n \sin N =$	$+0.42151$	$+0.41630$
$n \cos N =$	-0.15883	-0.15918
$N =$	$110^\circ 38' 49''$	$110^\circ 55' 30''$
$\log n =$	9.65364	9.64904
$t = \begin{cases} +0.24026 \\ -0.16857 \end{cases}$		$\begin{cases} +0.23996 \\ -0.16625 \end{cases}$
$= +0.07169$		$+0.40621$

and as these results differ very little from the preceding ones, we may here conclude the calculation. We have therefore the times of the two phenomena $= 7^h 4' 18''.1$, and $7^h 24' 22''.4$, and the angle denoted by $Q = 36^\circ 49'.6$, and $5^\circ 9'.2$.

9. Such an accurate calculation is, however, not required when the circumstances of the occultation are only wanted for the purpose of making the observation; in that case an error of one minute is of no consequence; and if every thing requisite for this purpose only is wanted, it will be sufficient, provided t falls below, or at least does not much exceed one hour* to apply the following much shorter calculation. In place of

$$\frac{\cos \delta \cdot \sin (\alpha - A)}{\sin \pi} \quad \text{and} \quad \frac{\sin \delta \cdot \cos D - \cos \delta \cdot \sin D \cos (\alpha - A)}{\sin \pi}$$

we put $\frac{\alpha - A}{\pi} \cos \delta$ and $\frac{\delta - D}{\pi}$

and neglecting the variations of $\cos \delta$ and π , so as to designate the right ascension and declination of the moon at the time T by α and δ and their hourly variations by $\Delta \alpha$ and $\Delta \delta$, we have

$$p = \frac{\alpha - A}{\pi} \cos \delta; \quad p' = \frac{\Delta \alpha}{\pi} \cos \delta;$$

$$q = \frac{\delta - D}{\pi}; \quad q' = \frac{\Delta \delta}{\pi}.$$

We next neglect t in the expressions for u' and v' and obtain

$$u = r \cos \phi' \cdot \sin (\mu' - A)$$

$$v = r \sin \phi' \cdot \cos D - r \cos \phi' \cdot \sin D \cdot \cos (\mu' - A)$$

$$u' = r \cos \phi' \cdot \lambda \cos (\mu' - A)$$

$$v' = r \cos \phi' \cdot \lambda \sin (\mu' - A) \sin D$$

For the place to which the calculations refer, the logarithms of $r \cos \phi'$ and $r \sin \phi'$ are to be considered as known. If we write a for $r \cos \phi' \sin (\mu' - A)$; b for $r \cos \phi' \cos (\mu' - A)$; c for $r \sin \phi' \cos D$, we have

$$u = a; \quad u' = b \cdot \lambda; \quad v = c - b \sin D$$

$v' = a \cdot \lambda \sin D$: where $\log \lambda = 9.4192$ and c may be taken from a small table which exhibits the value of this quantity for every degree of D with sufficient accuracy to four figures of decimals. The solution of equation (6) remains the same. In order to have a clear view of the calculation, I subjoin here the whole detail of it:

1830, April 5. 82 Leonis.

$$T = 7^h; \quad \mu' = 118^\circ 32'; \quad \mu' - A = -50^\circ 42'; \quad \pi = 54'.18$$

$$\alpha = 168^\circ 37'.93; \quad \delta = 4^\circ 44'.00; \quad \Delta \alpha = 28'.50$$

$$A = 169 \quad 14.11 \quad D = 4 \quad 14.08 \quad \Delta \delta = -9.08$$

$$-36.18$$

$$+29.92$$

* Should this supposition not be justified at the end of the calculation, or for other reasons a greater accuracy be desired, it may be obtained by a second approximation for u' and v' without changing p' and q' . It is easily seen that the error of the first approximation increases with the distance of the star from the path of the moon's centre, and that its limits cannot be unconditionally assigned.

$$\begin{array}{l|l}
\begin{array}{l}
l.(\alpha-A) = 1.5585n \quad \cos \delta \\
l. \Delta \alpha = 1.4548 \quad -0.0015 \\
l.(\delta-D) = 1.4760 \\
l. \Delta \delta = 0.9481n \\
l. l\pi = 8.2662
\end{array} &
\begin{array}{l}
l. r \cos \phi' = 9.7850 \quad c = 0.7876 \\
l. \sin(\mu'-A) = 9.8887n \quad b \sin D = 0.0285 \\
l. \cos(\mu'-A) = 9.8017 \\
l. a = 9.6737n \\
l. b = 9.5867 \\
l. \lambda = 9.4192 \\
l. \sin D = 8.8683
\end{array}
\end{array}$$

$$\begin{array}{lll}
p = -0.6656 & u = -0.4717 & l. m \sin M = 9.2876n \\
q = +0.5523 & v = +0.7591 & l. m \cos M = 9.3156n \\
p' = +0.5242 & u' = +0.1014 & l. n \sin N = 9.6261 \\
q' = -0.1638 & v' = -0.0092 & l. n \cos N = 9.1892n \\
M = 223^\circ 9'; l m = 9.4525; N = 110^\circ 5'; l n = 9.6533
\end{array}$$

$$\begin{array}{lll}
l. \frac{m}{k} = 0.0171 & l. -\frac{m}{n} = 9.7992n & l. \frac{k}{n} = 9.7821 \\
l. \sin(M-N) = 9.9638 & l. \cos(M-N) = 9.5931n & l. \sin \psi = 9.4626 \\
\psi = 16^\circ 52' & + 0.247 & \mp 0.176
\end{array}$$

$$\begin{array}{ll}
\text{Immersion } 7^h 4^m 3; & \text{Emersion } 7^h 25^m 4 \\
Q \dots 37^\circ 0 & \dots 3^\circ 2
\end{array}$$

10. The method of calculating which Mr. Encke has followed in constructing his Ephemeris for 1830, is perhaps as easy as the one here given; but it requires the previous calculation of some tables. The use of such tables not being required for the method here given, it will be easy for observers in other places to calculate every occultation for their horizons if the values of p, q, p', q' and the hour-angle of the star at the time T , viz. $\mu'-A$, which I will now denote by h , are given.

These data referring to the moon alone remain unchanged in the calculations for other places; only h is to be changed into $h+d$, where the positive d denotes eastern longitude from Berlin. For such a place we have

$$\begin{array}{l}
a = r \cos \phi' \sin(h+d) \\
b = r \cos \phi' \cos(h+d) \\
u = a; \quad u' = b. \lambda \\
v = c - b. \sin D, \quad v' = a. \lambda \sin D;
\end{array}$$

hence m, M, n, N , and lastly, the two values of t , which will show the mean time at Berlin, at the two moments in which the disappearance and reappearance will happen at the place for which the calculation has been made. This process will likewise be illustrated by calculating the occultation of 82 Leonis for Altona.

We assume as given :

$$T = 7^h \left\{ \begin{array}{l} p = -0.6656, p' = +0.5242 \\ q = +0.5523, q' = -0.1638 \end{array} \right\} h = -50^\circ 42'$$

For

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For Altona we have

$$\begin{aligned}
 l. r \cos \phi' &= 9.77485, & l. r \sin \phi' &= 9.90349, & d &= -3^\circ 27' \\
 l. r \cos \phi' &= 9.7749 & u &= -0.4827 & c &= 0.7986 \\
 l. \sin(h+d) &= 9.9088n & v &= +0.7728 & b \sin D &= 0.0258 \\
 l. \cos(h+d) &= 9.7677 & w &= +0.0916 & & \\
 l. a &= 9.6837n & v' &= -0.0094 & & \\
 l. b &= 9.5426 & & & & \\
 l. \lambda &= 9.4192 & & & & \\
 l. \sin D &= 8.8683 & & & & \\
 l. m \sin M &= 9.2622n & M &= 219^\circ 40', & l. m &= 9.4571 \\
 l. m \cos M &= 9.3434n & & & & \\
 l. n \sin N &= 9.6361 & N &= 109^\circ 39', & l. n &= 9.6621 \\
 l. n \cos N &= 9.1887n & & & & \\
 l. \frac{m}{k} &= 0.0217 & l. \frac{m}{n} &= 9.7950n & l. \frac{k}{n} &= 9.7733 \\
 l. \sin(M-N) &= 9.9729 & l. \cos(M-N) &= 9.5344n & l. \sin \psi &= 9.1951 \\
 \psi &= 9^\circ 1' & & + 0.214 & & + 0.093
 \end{aligned}$$

Immersion $7^h 7^m.3$; Emersion $7^h 18^m.4$ Berlin time.

$= 6^h 53^m.5$ $7^h 4^m.6$ Altona time.

$Q = 28^\circ.7$ $10^\circ.6$

Auxiliary TABLE for the Calculations requisite for predicting Occultations of Stars.

t	$\log \lambda$	κ	t	$\log \lambda$	κ
0.00	9.41916	0 0 0.0	0.19	9.41911	1 25' 44".1
0.01	916	0 4 30.7	0.20	911	1 30 14.8
0.02	916	0 9 1.5	0.21	910	1 34 45.5
0.03	915	0 13 32.3	0.22	910	1 39 16.3
0.04	915	0 18 3.0	0.23	909	1 43 47.0
0.05	915	0 22 33.7	0.24	908	1 48 17.7
0.06	915	0 27 4.4	0.25	908	1 52 48.5
0.07	915	0 31 35.2	0.26	907	1 57 19.2
0.08	915	0 36 5.9	0.27	907	2 1 50.0
0.09	915	0 40 36.7	0.28	906	2 6 20.7
0.10	914	0 45 7.4	0.29	905	2 10 51.4
0.11	914	0 49 38.1	0.30	904	2 15 22.2
0.12	914	0 54 8.9	0.31	904	2 19 52.9
0.13	913	0 58 39.6	0.32	903	2 24 23.7
0.14	913	1 3 10.4	0.33	902	2 28 54.4
0.15	913	1 7 41.1	0.34	901	2 33 25.1
0.16	912	1 12 11.8	0.35	900	2 37 55.9
0.17	912	1 16 42.6	0.36	899	2 42 26.6
0.18	912	1 21 13.3	0.37	899	2 46 57.4

Table

Table continued.

<i>t</i>	log λ	α	<i>t</i>	log λ	α
0.38	9.41898	2° 51' 28".1	0.81	9.41834	6° 5' 29".9
0.39	897	2 55 58.8	0.82	832	6 10 0.6
0.40	896	3 0 29.6	0.83	830	6 14 31.4
0.41	895	3 5 0.3	0.84	828	6 19 2.1
0.42	894	3 9 31.1	0.85	825	6 23 32.8
0.43	893	3 14 1.8	0.86	823	6 28 3.6
0.44	891	3 18 32.5	0.87	821	6 32 34.3
0.45	890	3 23 3.3	0.88	819	6 37 5.1
0.46	889	3 27 34.0	0.89	817	6 41 35.8
0.47	888	3 32 4.8	0.90	815	6 46 6.5
0.48	887	3 36 35.5	0.91	812	6 50 37.3
0.49	885	3 41 6.2	0.92	810	6 55 8.0
0.50	884	3 45 37.0	0.93	808	6 59 38.8
0.51	883	3 50 7.7	0.94	805	7 4 9.5
0.52	882	3 54 38.4	0.95	803	7 8 40.2
0.53	881	3 59 9.2	0.96	801	7 13 11.0
0.54	879	4 3 39.9	0.97	798	7 17 41.7
0.55	878	4 8 10.7	0.98	796	7 22 12.5
0.56	876	4 12 41.4	0.99	793	7 26 43.2
0.57	875	4 17 12.1	1.00	791	7 31 13.9
0.58	874	4 21 42.9	1.01	788	7 35 44.7
0.59	872	4 26 13.6	1.02	786	7 40 15.4
0.60	871	4 30 44.4	1.03	783	7 44 46.1
0.61	869	4 35 15.1	1.04	781	7 49 16.9
0.62	868	4 39 45.8	1.05	778	7 53 47.6
0.63	866	4 44 16.6	1.06	775	7 58 18.4
0.64	865	4 48 47.3	1.07	773	8 2 49.1
0.65	863	4 53 18.1	1.08	770	8 7 19.8
0.66	861	4 57 48.8	1.09	767	8 11 50.6
0.67	860	5 2 19.5	1.10	765	8 16 21.3
0.68	858	5 6 50.3	1.11	762	8 20 52.1
0.69	856	5 11 21.0	1.12	759	8 25 22.8
0.70	854	5 15 51.7	1.13	756	8 29 53.5
0.71	853	5 20 22.5	1.14	753	8 34 24.3
0.72	851	5 24 53.2	1.15	751	8 38 55.0
0.73	849	5 29 24.0	1.16	748	8 43 25.8
0.74	847	5 33 54.7	1.17	745	8 47 56.5
0.75	845	5 38 25.4	1.18	742	8 52 27.2
0.76	844	5 42 56.2	1.19	739	8 56 58.0
0.77	842	5 47 26.9	1.20	736	9 1 28.7
0.78	840	5 51 57.7	1.21	733	9 5 59.5
0.79	838	5 56 28.4	1.22	730	9 10 30.2
0.80	836	6 0 59.1	1.23	727	9 15 0.9

Table

Table concluded.

t	$\log \lambda$	z	t	$\log \lambda$	z
1.24	9.41724	9° 19' 31".7	1.38	9.41678	10° 22' 42".0
1.25	721	9 24 2.4	1.39	674	10 27 12.8
1.26	717	9 28 33.2	1.40	671	10 31 43.5
1.27	714	9 33 3.9	1.41	667	10 36 14.2
1.28	711	9 37 34.6	1.42	664	10 40 45.0
1.29	708	9 42 5.4	1.43	660	10 45 15.7
1.30	705	9 46 36.1	1.44	657	10 49 46.5
1.31	701	9 51 6.8	1.45	653	10 54 17.2
1.32	698	9 55 37.6	1.46	649	10 58 47.9
1.33	695	10 0 8.3	1.47	646	11 3 18.7
1.34	691	10 4 39.1	1.48	642	11 7 49.4
1.35	688	10 9 9.8	1.49	638	11 12 20.2
1.36	685	10 13 40.5	1.50	635	11 16 50.9
1.37	681	10 18 11.3			

F. W. BESSEL.

LXIV. On some Optical Phenomena. By Dr. J. STOKES*.

AS facts are the groundwork of science, and the communication of facts to those that love it the only means of its advancement, I shall give a detail of several optical phenomena of very rare occurrence, and also of some experiments on the transmission of light through very small holes.

On the 25th of August in this year, between 7 and 8 A.M., I witnessed the following:

There was a westerly wind, and the sky was thickly strewed with light fleecy clouds, so small as to resemble flocculi, and so numerous as to form a continuous layer. These were moving with great rapidity towards the sun. In fig. 1, let the point a represent the zenith, and $a a''$ a line passing from thence to the sun. This line appeared to me to pass through the points of contact that four arches $b c, d e, f g, h i$ made with one another. They were rather faint, but exhibited prismatic colours; those of $b c$ being the most perfect, those of $h i$ the least: $f g$ cannot be properly called an arch. Its form possessed rather a striking analogy to that of a well-known curve; $b c$ appeared to be an arc of 60° to a circle whose radius was in the zenith a , and its chord I think parallel to the horizon; its two extremities b and c were well defined: but those of $d e$ were not so; one of them, d , extending indefinitely on that side.

* Communicated by the Author.