XXIV.—*On the Thermal Conductivity of Iron, Copper, and German Silver.* By
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Professor TAIT. (Plates XXXIV., XXXV.)

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[INTRODUCTION.

Shortly after I read to the Society my paper on "Thermal and Electric Conductivity" (*Trans. R. S. E.*, 1878), in which I stated that the results were "by no means final, even so far as my own work is concerned," I was requested by Sir WYVILLE THOMSON to undertake the examination of the "Pressure Errors of the 'Challenger' Thermometers." This investigation led to another on the "Compression of Sea-Water," and allied subjects, which is not yet finished. Meanwhile, though I had prepared everything for my promised repetition of the experiments on Thermal Conductivity, the bars formerly used having been nickelised, &c., I found that it would be impossible for me to carry out the investigation. I therefore asked Mr MITCHELL, who, as Neil-Arnott Scholar, had already done good and careful work on Thermal Conductivity in my Laboratory, to repeat the experiments under the altered conditions. I put at his disposal all the apparatus which was employed in the former research. The Government Grant Committee allowed a sum for the payment of a computer to reduce the results, and the observations were at once commenced. The results are now laid before the Society, and are probably as good as the method and the thermometers employed can furnish.

As regards the method, one grand defect is the uncertainty as to the relative amounts of surface loss of heat in the two parts of the experiment. The nickelising has, to a very great extent at least, removed the part of this uncertainty which was due to oxidation of the bars; but there remains another part, not at all easy to reckon and allow for, which depends on the fact that each thermometer in the long bar is maintained for hours in a nearly constant state of graduated temperature throughout its stem, while the corresponding state of that in the short bar not only varies rapidly as the cooling proceeds, but probably always materially differs from it. No attempt has been made to correct the results so far as this cause of error (which is

probably of no great importance) is concerned. It is clear that its effect will be to make the rate of cooling a little too small at the lower, as compared with the higher, temperatures.

Another defect, which indeed FORBES pointed out, is due to the very small temperature-gradients towards the colder end of the long bar. Mr MITCHELL has carried out my suggestion of an artificial cooling of the middle of the bar, and it is highly interesting to compare together the results he has obtained with and without this cooling.

ANGSTRÖM expressly stated (*Pogg. Ann.*, cxviii. 1863) that no account need be taken of the change of specific heat with temperature. In my paper above referred to, I said that it appears that, in iron especially, this change produces a very considerable effect on the estimated values of the conductivity. In default of better data, Mr MITCHELL has used those given (after NICOL and others) in a short paper in *Proc. R. S. E.*, p. 126, 1880-81, and in my *Heat*, § 246. The importance of this correction is shown by the comparison of the results obtained from it with those obtained when it is not applied. Mr MITCHELL's experimental results are given in such a form that any subsequent improvement in these data can be taken advantage of without further experiment, and with very little trouble in the matter of calculation. The fact that the various short bars were exactly similar in surface in his experiments has enabled him to make a rough test of the accuracy of these data.

In the short paper above referred to, I showed that the consideration of the rise of specific heat with temperature would destroy if not overcome the apparent fall of conductivity of iron at higher temperatures. But I had not then the means of properly applying the correction without repeating about one-half of the laborious calculations incident to FORBES' method. Mr MITCHELL has in his calculations taken account of this consideration: and it must be regarded as one of the chief features of his paper that he has thus shown that iron does not form an exception to the law that ordinary metals *improve* in thermal conductivity as their temperature is raised.

As I am responsible for the methods employed by Mr MITCHELL in the experiments and calculations, though not for the calculations themselves, I must state here the directions given and the grounds for them, at least in so far as they introduce processes differing (to any considerable extent) from those used by FORBES or by myself.

1. As to the empirical formula (B) for the statical curve, in the special case of the iron bar when there was no artificial cooling.

This I obtained by plotting the logarithms of the temperature excesses as ordinates, the abscissæ being distances along the bar. The curve so obtained was nearly straight at the lower temperatures, and became rapidly more curved at higher temperatures. I therefore treated it as a branch of a hyperbola, and

found its asymptote. Thus the *form* of the empirical expression was suggested at once.

2. The allowance for the unequal heating of the stems of the thermometers was obtained thus :—

Let v be the observed temperature (not the temperature excess), w the true temperature, and in accordance with § 10 of my paper $e=10/250^2=0.00016$. From the result of Mr MITCHELL's comparison of the two thermometers, one partially, the other wholly, immersed in a paraffin bath, I have been confirmed in my assumption of an error of 10° at 250° C. Then we have

$$w = v + ev^2.$$

Thus, for the true temperature-gradient in the statical experiment,

$$\frac{dw}{dx} = (1 + 2ev) \frac{dv}{dx}.$$

Similarly, for the true rate of cooling, we have

$$\frac{dw}{dt} = (1 + 2ev) \frac{dv}{dt}.$$

The quantities on the right-hand sides are given by the experiments, or deduced directly by graphical methods or calculation.

For the statical curve of cooling it is easy to see in this way that each instalment of area must be multiplied by

$$1 + 2e \frac{v_1 + v_2}{2},$$

where v_1 and v_2 are the limiting temperatures of the instalment.

It is clear that this correction increases the gradient at any point of the bar in a greater ratio than that in which it increases the total area of the corresponding part of the curve which expresses the flux of heat ; so that its effect must be to diminish the estimates of conductivity at higher, more than at lower, temperatures.

3. I was much surprised at the first results obtained by Mr MITCHELL for the rates of cooling at high temperatures. At my instance he has repeated this part of the experiment in a form similar to that which I had employed, and certainly less likely to entail error, and the data thus obtained have been incorporated in the paper, in so far as they relate to the specified tables. [The remaining small difference between our results may be due to an over-estimate in my 6 p.c. reduction for oxidation.]

4. There still remains a possible source of error, due to the thermometers themselves:—Kew Standards though they be. This arises from the way in which the 200° C. and 300° C. points were determined at Kew. The

tubes having been carefully calibrated before filling, the standard points 0° C. and 100° C. were directly determined in the usual manner. But the positions of 200° C. and 300° C. were determined by taking successive portions of the tube whose volume (cold) corresponded to that of the portion (also cold) from 0° C. to 100° C. I have not the means of making allowance for this defect, which will probably mar all experiments of the kind until suitable air-thermometers are employed.

5. The fact that the conductivity deduced from experiments on the iron bar, when its full length is employed, differ so considerably from those obtained when it is artificially cooled in the middle, appears to be intimately connected with a remark made in my paper (§ 14) that “in measuring conductivity, at whatever temperature, things ought to be arranged so as to avoid any slow flux of heat.” It seems that, even after the lapse of eight hours, the steady state of temperature has *not* been reached in the colder parts of the long iron bar.

6. As the numerical data, concerning specific gravity and specific heat, which Mr MITCHELL has (in default of better) been obliged to employ, are only rough estimates, I asked him to test them by finding the ratios of the rates of cooling of copper and iron at various common temperatures. The surface material was the same in the two bars, and their dimensions equal, so that the *amount* of heat lost in a given (short) time must have been the same for each at the same temperature. The ratio of the rates of cooling should therefore be constant for all temperatures if, and only if, the rate of change of specific heat with temperature be the same for each of the two materials. The result does not seem to favour the accuracy of the assumed data, but the process employed is not by any means an accurate one.

7. As my determinations of the relative electric conductivities of the bars had been verified by Mr D'ARCY THOMPSON, there is no necessity for their repetition. But, using them, with Mr MITCHELL'S results for thermal conductivity, my comparative table (*Trans. R. S. E.*, 1878, p. 739) should be altered (subject, of course, to correction for improved values of specific gravity and specific heat) to something like the following :—

	Thermal.	Electric.
Copper (Crown),	1.5	1.729
„ (C.),	1.0	1.000
Forbes' Iron,	0.23	0.264
Lead,	0.12	0.149
German Silver,	0.13	0.117

P. G. TAIT.]

I. EXPERIMENTAL METHODS.

As already stated, this work was intended to be a repetition of Prof. TAIT's experiments upon the same bars, and accordingly every care was taken that it should be conducted under conditions as nearly as possible similar; except, of course, the important feature of the nickel films. For a full account of the original experimental details, reference may be made to the papers of FORBES (*Trans. R. S. E.*, 1860-61 and 1864-65); and for subsequent improvements, to that of Prof. TAIT (*Trans. R. S. E.*, 1878). To what is there given concerning the experimental means employed little can be added. The set of thermometers, the position and use of each, the maintenance of regularity in gas pressure, and consequently in the steadiness of temperature during the last few hours of the long bar experiment—all were exactly the same as in the preceding work; and any slight novelty in this part of the work was more by way of addition to, than change in, the method. This was essential in order that a thorough comparison might be made between the two series of results.

The four metallic bars experimented upon were, respectively, wrought iron (that originally employed by FORBES), copper of superior electric conductivity (copper, crown), copper of inferior electric conductivity (copper, C.), and German silver. These, together with the short bar corresponding to each, were all plated with nickel. The plating, although comparatively light, was of good quality. Before beginning work on one of the bars, it was polished with fine rouge and paraffin oil, and afterwards thoroughly dried; while during the progress of the experiments on the long bars these were kept free from dust, &c., or anything likely to interfere with the normal amount of radiation. It was found that, when the distances along the bars, especially between the closer holes, were measured by a dividing engine, some small temperature correction would require to be made for the (slight) inexactness of the position of the holes. In no case was this correction greater than one-third of a degree C., and this, since it was only required at the highest temperatures, was well within the limits of other inevitable and undetermined errors.

The thermometers used in this inquiry were, as already stated, those used by Prof. TAIT in his experiments. They were made under the direction of Dr BALFOUR STEWART, and afterwards carefully corrected at Kew. A fair idea of their quality *before* they had been used for any experimental work of this nature may be obtained from the following table, which gives the corrections according to the Kew certificates:—

Kew Standard, No.	Range Centig.	Correction at			
		0° C.	100° C.	200° C.	350° C.
*431	0°–100°	0·0	–0·1	...	...
432	0°–100°	0·0	–0·1	...	...
433	0°–100°	0·0	0·0	...	...
434	0°–100°	0·0	0·0	...	...
435	0°–100°	0·0	+0·1	...	...
436	0°–200°	0·0	0·0	0·0	...
437	0°–200°	0·0	+0·2	+0·1	...
438	0°–200°	0·0	+0·1	+0·1	...
439	0°–200°	0·0	0·0	0·0	...
440	0°–200°	0·0	+0·2	+0·1	...
441	0°–350°	0·0	+0·15	+0·1	...
*442	0°–350°	0·0	+0·15	+0·25	...
443	0°–350°	0·0	+0·1	+0·25	...
444	0°–350°	0·0	+0·25	...	+0·3
*445	0°–350°	–0·1	–0·1	...	+0·1
446	0°–350°	0·0	+0·2	...	+0·3
447	0°–350°	0·0	+0·15	...	+0·1
*449	0°–350°	0·0	+0·1	...	+0·1
*450	0°–350°	0·0	+0·05	...	+0·05

* Not used in the experiments.

But after several exposures to high temperatures, sometimes nearly 300°, the thermometers began to show an error, and that one of increase, in the indications. The amount of this error in each instrument was found by comparing the reading taken when the long bar had cooled down overnight after the previous day's heating, with the simultaneous reading of a thermometer in the short bar standing in the vicinity of the long bar. This latter thermometer was only used for temperatures which did not exceed that of the air, and had never been subjected to any such treatment as had those it was employed to correct. The difference of the two readings named was taken as the error of the first, and was applied in the reduction of the experiments. In the case of the thermometers used in the hottest hole in the bars, this error amounted to about seven degrees at ordinary temperatures.

A series of corrections for the different thermometers was thus obtained, and in order to test whether the readings obtained from one day's work, and after the application of these corrections, were consistent with those of another day, the thermometers in the first three holes were interchanged among each other, and also with the thermometer which was always used to indicate the temperature in the cooling experiments on the short bar. It was found that the different sets of readings when so tested were consistent with each other.

This, however, is a source of less error than that which arises from the circumstance that the temperature (especially at any point near the hot end) of the long bar during an experiment is given by a thermometer with an

unequally heated stem. In connection with this, Prof. TAIT calculated that for a temperature of 250°C . "the utmost error that can be introduced in the indications of the thermometers used is somewhere about 10°C . That is to say, the highest temperatures were read, at the most, 10° lower than they would have been if the whole thermometer had been exposed to the same temperature. This correction of 10° at 250° diminishes at lower temperatures and increases at higher, nearly as the square of the excess of the temperature above the freezing point" (*Trans. R. S. E.*, 1878).

Wishing, if possible, to obtain some experimental verification of this computation, I had two thermometers constructed, with stem, bulb, capacity, bore, and length of degree similar to each other and to those used in determining the temperature towards the hotter end of the long bars. They were tested in the following manner. One was placed vertically, with *bulb* only immersed, in a bath of melted paraffin wax; the other, placed horizontally, was wholly immersed in the same bath. The paraffin was maintained at a steady temperature throughout. After their indications had become stationary, the thermometers were read, and the excess of the reading of the wholly immersed, over that of the partially immersed, thermometer noted. From several experiments of this kind, it was calculated that the error in question amounts, at 250°C . to $9^{\circ}\cdot 5$, an exceedingly close verification of the previously calculated estimate.

In the reduction of the experimental readings, and in the deduction of conductivity this error has not been taken into account, chiefly because the thermometer used in the experiment on the cooling of the short bar was almost exactly the same in construction as those used in the long bar, and that therefore the difference in this respect between the two experiments must be small. In *Appendix II.*, however, I have endeavoured to correct for this source of error.

Statical Experiment.—The bar to be experimented upon had one end inserted into an iron crucible containing melted solder, which is heated by a powerful Bunsen burner. Heat conducted along the bar raises the temperature of each portion of it, and this is carried on until the flow of heat across each section has become steady, a state indicated by the steadiness of the thermometric indications along the whole length of the bar. This condition was attained after (about) 7 hours in the case of iron, 6 in German silver, and after 5 hours in copper. This steadiness of state is then maintained for some time, generally at least an hour, after which the distribution of temperature along the bar, as well as the temperature of the unheated short bar in the vicinity, is carefully determined.

Initially, therefore, the success of this part of the experimental work depends upon the maintenance of this particular distribution of temperature,

after it has been reached. This means that (besides such minor details as draughts of air, and a steady or a slowly changing temperature in the laboratory) great regularity in gas pressure has to be ensured.

The gas regulator, devised by Prof. CRUM BROWN, was employed in this inquiry also. Its working is as effective as its construction is simple; and its automatic action, hour after hour, relieves the experimenter of what would otherwise be the necessity of constantly noting and regulating the temperature of the melted solder in the crucible.

The following extract from my note-book shows sufficiently how perfect are the results obtained by employing it:—

German Silver.

July 9, 1886.

Gas lit 6.50 a.m.

Hour p.m.	2.30	2.45	3.0	3.15	3.30	3.45	4.0	4.15	4.30
Temp. in hole nearest hot end,	270.85	270.9	270.7	270.6	270.7	270.6	270.7	270.7	270.75

This, showing as it does only an extreme variation in the temperature excess of .09 per cent., bears out what has been said. The room in which the experiments were conducted was lighted from the north alone, the south windows and the doors were kept closed, and the air temperature throughout the day did not change so rapidly as to cause any uncertainty in the results.

A slight modification of the experiment on the long bar was tried, enabling the method to give results with more certainty. Where, as in iron, the flux of heat across any section at the lower temperatures (*i.e.*, below 20°) is small, a small error in the determination of the temperatures makes a comparatively large difference in the estimate of conductivity. In order to avoid this, a suggestion of Prof. TAIT's (first given in *Trans. R. S. E.*, 1878, p. 734) was adopted. While the bar was heated in the ordinary manner, a cold water bath was placed halfway up the bar towards the heated end, and through it a stream of water was continually passed from below. This had the effect of "steepening" the temperature-gradient, and thus allowing the measurement or calculation of the tangent to be made with greater exactness. This process is almost necessary in the case of metals the change in whose conductivity with increased temperature is so small that the ordinary experiment of FORBES is insufficient to detect its sign and amount.

An idea of the effect of this midway cooling on the distribution of temperature along the bar may be obtained from the following table, in which are compared the temperature excesses at the various holes in the bar, both in the ordinary experiment and in that with the cooling bath.

IRON.

Distance from origin in inches.*	Temperature Excess °C.	
	11/6/86. Bar <i>not</i> cooled.	13/4/87. Bar cooled 46 in. from origin.
0	229·3	229·35
3	162·5	163·3
6	118·45	119·85
9	87·6	89·5
15	50·35	51·0
21	29·8	29·95
27	18·25	17·65
33	11·4	9·6
45	4·6	−1·05 †

* The hole in the bar nearest the heated end is taken as origin.

† The − sign indicates that the actual temperature of the bar at this point was *below* that of the short bar standing in the vicinity.

Cooling Experiment.—The chief difficulty hitherto in this part of the experimental method has been the oxidation of the short bar when heated to a high temperature. This introduces an uncertainty into the final results owing to the necessity of applying a value of the “rate of cooling” of the short bar at a particular temperature, to a portion of the long bar whose surface at that temperature is generally in a different state from that of the short bar at the same temperature. And although a correction may be applied to the ordinates in the curve of cooling in order to remove this, its application must in many cases be a matter of doubt.

In this work, however, this uncertainty has been almost entirely removed by the nickel-plating of the bars; the difficulty arising from an oxidised surface never even presented itself, for at the end of each experiment the surface of the short bar on cooling had retained its original brightness, and had, except in a slight manner to be yet noted, in no way been affected. So that the condition which is necessarily involved in the deduction of the conductivity, viz., that the surface of the long bar at any particular temperature should be the same as, or, at least, strictly comparable with, that of the short bar when at the same temperature, was very fully realised.

Each of the short bars experimented upon was so placed, on bearings, in a rack, that it was possible to rotate it, on its long axis, while it was being heated. The temperature of the bar was raised by placing beneath it a row of 50 very small Bunsen burners. The heating was proceeded with as cautiously as possible, and generally (where it finally reached about 270°) occupied about 2 hours. This, so far, guaranteed an equable distribution of temperature through-

out the bar. But, for a reason to be shortly stated, the results thus obtained were not used.

To this method of heating there is an objection. There is always a *slight* smoking of the Bunsen burners, and this, together with moisture deposited at the commencement of the heating, dulls the bright plated surface of the bar to some extent. At the instance of Prof. TAIT, who was surprised to find the rates of cooling greater with the nickelised than with the plain bars, these experiments were repeated in a form calculated to avoid any deposition of moisture, and to avoid the smoking as far as possible. The bars were heated to over 100° C. before a clear fire, and then *as quickly as possible* raised to a high temperature over the row of Bunsen burners. The results of *this* set of experiments have been used in the calculations. In *Appendix I.* a comparison between the results of the two methods is given.

The temperature of the bar during cooling was observed by means of a particular thermometer, which was used almost exclusively (with an exception already stated) for this purpose. As might be expected, the zero of this instrument rose after repeated exposures to such high temperatures as it was employed to indicate. But this error was carefully estimated. As already detailed, the short bar during the progress of the statical experiment was placed near the long bar, while its temperature (practically that of the surrounding air) was recorded by a thoroughly trustworthy thermometer. The thermometer used in the cooling experiment was simultaneously placed in one of the holes in the short bar, and its readings compared with those of the other thermometer in the bar. A continuous process of correction was thus established; and by always placing the thermometer in this position when not used for the cooling experiment, the gradual change which the amount of the error underwent as the experimental work proceeded was carefully noted.

Prof. TAIT, in the paper already referred to, pointed out the importance of raising the short bar to a temperature considerably higher than that *actually* required for the observation of the rate of cooling at any particular temperature. The exact reason for this is, that although the mercury in the bulb of the thermometer is heated almost at once, the column of mercury in the stem is not so heated for some little time after, and that the cooling of the two regularly together also does not ensue for some time. For example, in order to obtain a determination of the rate of cooling at 200° C. (as accurately as the method will allow), it is necessary to raise the bar to a temperature of 250° or 260° , insert the thermometer as quickly as possible, and allow the bar to cool down through the temperature required. For a full experimental proof, see *Trans. R. S. E.*, 1878, pp. 730, 731. This affords a complete explanation of the curious result FORBES arrived at, viz., that the curve showing rate of cooling in terms of temperature excess exhibited a point of flexure about 150° C.

By adopting this precaution I have verified Prof. TAIT's results, and although in some cases the temperature excess was as much as 280° , no such result as that obtained by FORBES was even indicated.

In addition to the usual cooling experiments, a series was conducted at temperatures as high as it was possible to safely observe with a mercurial thermometer, generally about 330° C. These experiments were of great use in accurately estimating the rates of cooling which are necessary for the higher portions of the statical temperature curve. They were not made until all the normal experimental work was over, as they involved some risk of the safety of the thermometers employed.

II. DEDUCTION OF CONDUCTIVITY.

Statical Curve.—Although the manner in which this curve was constructed was similar to that in previous work, it may yet be well to describe it in detail.

From among all the statical experiments on the metal bar for which the curve was required, that one was chosen which, in its approach to, and maintenance of, the final thermal distribution, appeared to be the most steady, and when considered in all respects, the most successful. This was termed the *Standard Experiment*. The readings taken during the steady state of the bar were then examined, and one special set of readings, apparently the most trustworthy, was selected. The appropriate corrections having been applied, the true temperature excess at each hole in the bar was obtained by subtracting from the corrected reading the reading taken simultaneously of the temperature of the short bar. The series of numbers thus obtained were then laid down as points on a curve, in which the abscissæ represented distance along the experimental bar, and the ordinates denoted temperature excesses. The other experiments on the same bar were then taken in the order of their apparent merits, regarded in the same way as in selecting the standard experiment. The temperature excesses were plotted separately for each experiment, on a sheet of tracing paper superposed upon a blank part of the sheet of divided paper on which had been marked the points obtained from the standard experiment. The axes on both sheets were coincident. After each experiment had been so represented on the tracing paper, it was carefully drawn along, keeping the horizontal axes still coincident, over the sheet beneath, until the points marked upon both lay in one smooth curve. Each point on the superposed tracing paper was then transferred to that, immediately covered by it, on the sheet below. In this manner a series of points was obtained, through which a smooth curve was drawn, exhibiting the relation existing between temperature excess and position along the bar—what FORBES termed the statical curve of temperature.

It may be remarked that once the standard experiment, along with, say, two other decidedly trustworthy experiments, is chosen, and the three represented by a curve in the manner already described, we have therein furnished a test of the correctness of the others; indicated by the extent to which they agree with the first. By this means, the calculator is enabled to set aside as less trustworthy, certain of the experiments. The only discrepancy, which falls to be reported in this connection, between the several experiments on the same bar, lies in the uncertainty connected with the representation by the curve of the temperature excess at the cool end of the bar. Here it was somewhat difficult to trace the curve accurately, and to determine the point on the bar where, for all practical purposes, at least, the temperature excess disappears. An error of a tenth of a degree is quite sufficient to cause this doubt. This circumstance argues strongly in favour of the adoption of that modified form, already detailed, of the long bar experiment, viz., where the bar is cooled midway by the application of a cold water bath; for in this case there is no dubiety whatever as to the point where the horizontal axis is crossed by the curve of temperature-excess.

Regarding the equation representing the curve, two formulæ were employed:—

$$\log v = \log A - \frac{bx}{1+cx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

$$\log v = \log A + \frac{b}{c+x} - ex \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

where v = temperature excess,

x = distance along bar, reckoned from any

arbitrary origin; and where

A , b , c , and e are constants.

The formula (A), involving three constants, originally employed by REGNAULT (*Mém. Ac. Sci.*, vol. xxi.) to represent the relation between the temperature and pressure of saturated water-vapour, was that used by FORBES and also by Prof. TAIT, to represent the curve of temperature-excess in iron. But the differences between the *calculated* and *observed* values of v , both as shown by the tables in FORBES' last paper, and also when used by myself, were such as to lead to the construction, for use in this work, of the empirical formula (B). It was constructed by Prof. TAIT to suit as closely as possible, by four disposable constants, the curve plotted from the logarithms of the temperature excesses obtained in one of my earlier experiments on FORBES' iron bar. For the iron curve, it has been found to work very well, and certainly much better than its predecessor. But in the case of both copper bars and German silver, as well as the iron bar cooled midway, the equation (A) has been found to be more applicable, and has accordingly been used. In some cases it was found better

to take the curve in two or more sections, employing for each the proper corresponding values of the constants.

A conception of the relative value, for this purpose, of the two formulæ may be obtained from the two tables subjoined, showing the degree of approximation of the value of v , the temperature excess, as calculated by the formula, to the value as given by the observational curve. The first, headed (F), is extracted from FORBES' paper already quoted; the second, headed (M), is the record of one of my own experiments, along with the values of v calculated from formula (B); while the third table, also headed (M), shows in how far the same experimental results are approximated to by formula (A). These tables refer, of course, to iron.

I. (F).

x , in feet.	v by experimental curve.	v by formula $v = \log A - \frac{bx}{1+cx}$	Difference.
0	°C. ...	°C. 272.7	...
0.25	190.5	191.0	+0.5
0.5	134.7	135.9	+1.2
0.75	97.3	98.23	+0.93
1.0	72.0	72.0	0.0
1.25	53.6	...	...
1.5	40.8	40.21	-0.59
2.0	24.2	23.58	-0.67
2.5	14.8	...	...
3.0	9.33	9.08	-0.25
4.0	4.0	4.0	0.0
5.0	1.8	1.96	+0.16
6.0	0.9	...	...
8.0	0.28	0.36	0.08

II. (M).

x , in feet.	v by experimental curve.	v by formula $\log v = \log A + \frac{b}{c+x} - cx$	Difference.
0.0	247.2	246.6	-0.6
0.25	172.1	173.8	+1.7
0.5	125.25	125.5	+0.25
0.75	92.3	92.2	-0.1
1.25	52.0	52.0	0.0
1.75	30.35	30.35	0.0
2.25	18.2	18.2	0.0
2.75	11.15	11.1	-0.05
3.75	4.3	4.3	0.0
4.75	1.85	1.7	-0.15
5.75	0.7	0.7	0.0

III. (M).

x , in feet.	v by experimental curve.	v by formula $\log v = \log A - \frac{bx}{1+cx}$	Difference.
0.0	247.2	245.4	-1.8
0.25	172.1	172.1	0.0
0.5	125.25	125.45	+0.2
0.75	92.3	92.3	0.0
1.25	52.0	52.0	0.0
1.75	30.35	30.6	+0.25
2.25	18.2	18.72	+0.52
2.75	11.15	11.65	+0.5
3.75	4.3	4.35	+0.05
4.75	1.85	2.53	+0.68
5.75	0.7	1.34	+0.64

The next table shows how far the formula (A), on the other hand, suits the case of the iron bar cooled midway.

x , in feet.	v by experimental curve.	v by formula $\log v = \log A - \frac{bx}{1+cx}$	Difference.
0.0	206.35	206.35	0.0
0.25	150.85	150.75	-0.1
0.5	111.4	111.2	-0.2
0.75	83.45	82.85	-0.6
1.25	47.7	47.25	-0.45
1.75	28.15	27.9	-0.25
2.25	16.5	16.95	+0.45

The agreement shown here, though not so close as that shown in II. above, was much closer than that which the use of formula (B) gave.

LAMBERT (*Pyrometrie*, Berlin, 1779) showed, from experiment, that the curve of stationary temperature along a bar could be represented by an equation of the form

$$v = Ae^{-px}.$$

whence

$$\log v = \log A - px.$$

So that the curve representing, at each point of the bar, the logarithms of the temperature excesses should be a straight line.

This result was shown to hold for the case of copper (C) by Prof. TAIT (*Trans. R. S. E.*, 1878).

Diagram 2 shows the degree of approximation to it in this work. With the exception of that of German silver, the curves for the other bars did not exhibit the result in question to the same extent.

In connection with the consideration of the statical curve, it will be well to compare in this respect the results of this work with those of Principal FORBES and Prof. TAIT. The curves for iron, according to these three different sets of experiments, are shown in Diagram 1. From the disposition of these curves one to another, it will be seen that (1), that given by FORBES (marked F), shows a greater rate of change of temperature along the bar than either of the other two; (2) that my own experiments, *i.e.*, on the nickel-plated bar (marked M), show a slower rate of change than those of Prof. TAIT (marked T); (3) that all three agree with comparative closeness at the lower temperatures.

The first of these remarks is confirmed when the values of dv/dx at different temperature excesses are plotted. The values which FORBES used are then seen to be all greater than those used in the deduction of conductivity in this work; on the average they are about 10 per cent. larger. No details of this kind (*i.e.*, values of tangents used, &c.) are printed in Prof. TAIT's paper, otherwise they would have formed an interesting comparison.

In the calculation of the tangents the values obtained by differentiation of the formula representing the curve have mainly been relied upon, except at the lower temperatures, where graphical measurements have been made and used. But some allowance must be made for the discrepancy between the calculated and observed values of v . It was effected in this way. By the sign and amount of the difference between these two values, it can be determined whether at any given point the curve (by calculation), in bending away (below or above it) from the observational curve, makes the calculated values of dv/dx too small or too great. By taking other values of the constants, a value of dv/dx may be obtained with a probable error, at any given point, of opposite sign to that obtained by the first value of the constants. Thus it is known between what two limits lies the accurate value of the tangent at the given point.

Excepting the remark made that the value of the tangents graphically measured agreed satisfactorily with the value calculated from the formula, FORBES seems to have used *alone*, and without any such modification as is mentioned above, the values of dv/dx as they are given directly by one particular set of values of the constants,—*viz.*, that set which appeared to give the best agreement with the observed temperature excess along the bar. Treating the curve in sections, as he did, partly avoids the difficulty; still, it cannot be doubted that without any allowance of the kind mentioned, the calculated values of the tangent to the curve, constrained as it is to pass through three

points (or four, as the case may be), cannot be received and adopted without involving the possibility of occasional large error.

Some assistance may be obtained from the graphic representation of the value of dv/dx for different values of v . It sometimes gives a means of casting out untrustworthy estimates, and at all events prevents the entrance into the calculations of any great error. Diagram 3 illustrates its use in the case of the iron bar.

Curves of Cooling.—In the reduction of the observations, the same methods were employed as in previous work. The deduction of the rates of cooling was performed mainly by constructing a curve whose ordinates represented successive differences of temperature of the cooling bar (or $\frac{1}{2}$, $\frac{1}{3}$, &c. difference, where the interval of observation was 2, 3, &c. times the time-unit), and whose abscissæ represented the mean temperature excesses during the interval. It is not accurate to say a curve was thus constructed; in reality, there was thus obtained (as is shown in Diagram 4) a number of irregularly disposed points, through which was drawn a smooth curve, the ordinates of which were the rates of cooling at the temperature excess given by the respective abscissæ. In order to check the results derived in this manner, the following method of “grouped” readings was adopted. Six readings of the cooling bar were selected, taken at equal intervals. Correcting for thermometric errors, and subtracting the air temperature, we obtain six successive temperature excesses. Divide their sum by six, and there results the mean temperature excess during the five time-intervals which elapse between the reading of the first and last. Next take the five successive differences of temperature observed by the six readings; divide their sum by five, and we get the mean rate of cooling at the mean temperature excess during the interval. We thus have obtained, by taking many such groups of readings, a series of estimates of the rate of cooling at different excesses of temperature. When so plotted, the points nearly all lie on a smooth curve which may easily be drawn through them.

It was usual to go over each cooling experiment twice in this manner. All the readings were grouped into sets of six, and the results deduced. Going over them a second time, the readings were again placed in groups, so that one group in this second reduction overlapped equally two groups in the first; thus giving a second series of intermediate points on the curve.

In Diagram 5 is shown the curve thus obtained, in the case of one of the cooling experiments on iron at the higher temperatures. In order to show how satisfactory is this method, the following comparison, made between the ordinates (*i.e.*, rates of cooling) of this curve, obtained from “grouped” readings, and those of the curve obtained from “single” readings, may be considered:—

Temperature Excess.	Rate of Cooling.	
	From "Grouped" Readings.	From "Single" Readings.
°C.		
170	3·52	3·52
180	3·87	3·87
190	4·20	4·20
200	4·56	4·56
210	4·92	4·93
220	5·28	5·28
230	5·70	5·69
240	6·10	6·13
250	6·55	6·60
260	7·06	7·10
270	7·54	7·70

The above table is given to illustrate the results of the method of reduction; for a particular reason (connected with the circumstances under which the above experiment was conducted) the above values of the rate of cooling were not finally employed in the deduction of conductivity.

This process of grouping readings was found valuable in the deduction of the rate of cooling at the highest temperatures employed; since the irregular falling of the mercurial column almost necessitated some equalisation of this kind. This irregular descent of the mercury column is observed in the very best thermometers, and is aggravated in the present case by the (necessary) smallness of the bore, and also perhaps by the rapidity of the cooling at the higher temperatures.

FORBES' peculiar result as to the rate of cooling at different temperature excesses has already been alluded to, as also has its explanation. A comparison of results for the case of iron in the same way as has already been given for the statical temperature curve, may now be given. Diagram 6 shows the three final results, as they were given by Principal FORBES (marked F), by Prof. TAIT (marked T, and reduced, according to his statement, by 6 per cent.), and lastly, the results of experiments made in connection with this work (marked M); the last, however, having been modified as explained in *Appendix I*.

The last two mentioned results clearly disprove the result of FORBES' experiments, viz., that at or about 150° C. the curve should exhibit a point of contrary flexure.

The relation of this to the final results will be noted later.

The following table gives the ordinates of the three different curves beyond 150° of temperature excess, up to which they practically agree. The heading of the columns indicates the set of experiments to which they belong, as explained above:—

Temperature Excess.	Rate of Cooling.		
°C.	F.	T.	M.
150	2.99	3.02	3.02
160	3.19	3.27	3.28
170	3.36	3.53	3.56
180	3.50	3.80	3.815
190	3.64	4.08	4.11
200	3.76	4.37	4.4
210	3.89	4.66	4.7
220	4.01	4.96	5.03
230	4.14	5.26	5.37
240	4.28	5.56	5.74
250	4.40	5.87	6.12
260	4.52	6.2	6.55
270	...	...	6.93

The area of the "statical curve of cooling" was arrived at by treating the curve in sections, and estimating what amount had to be subtracted from or added to the ordinates of each section in order to make it a logarithmic. The areas were checked by the use of a quadrature formula. When it was required to correct the areas for the change in specific heat with increase of temperature, the arithmetical mean of the two temperature excesses on the statical curve, and which included the particular section, was taken; multiplied by the increase per degree and the uncorrected area multiplied by this product with unity added to it.* By plotting a curve, as in Diagram 7, whose ordinates represent the area of the statical curve of cooling beyond different values of the temperature excess, represented by the abscissæ, a useful means of checking results arrived at by the methods mentioned is supplied.

In the case of the two copper bars, the cool end was maintained at a steady temperature by means of a cold water bath. In estimating the area of the statical curve of cooling some allowance must be made for the heat so carried off from the end of the bar. This quantity which has to be added is estimated as follows. Let T_1, T_2, T_3 , &c., be the tangents at three successive increasing values of x , viz., x_1, x_2, x_3 , &c., and A_1, A_2, A_3 , &c., the area of the statical curve of cooling beyond the same values of x . Approximately, $k = \frac{A_1 - A_2}{T_1 - T_2}$. When the quantity on the right hand side of this equation is multiplied by

* The following numbers were used in correcting for change of specific heat as representing the rise per degree Centigrade:—

Iron,	.	.	.	.	.	.	0.0014
Copper (Crown),	.	.	.	.	.	.	0.0088
Copper (C.),	.	.	.	.	.	.	0.009
German Silver,	.	.	.	.	.	.	0.009

They are based on the results of experiments made in 1868, by the late Mr J. W. NICOL in Prof. TAIT's Laboratory; the materials used, with the unfortunate exception of the iron, being portions of the bars tested for conductivity.

T_2 , we obtain an estimate of the number of units of heat flowing past the section represented by the next value of x , viz., x_2 . From the area of the statical curve of cooling we derive another estimate, generally less. The difference between the two is the quantity of heat carried off by the cooling bath at the end of the bar. It is to be noted that (as the error, if any, in the values of the tangents, is usually such as to make them alternately too large and too small) where the estimates of this quantity vary, a process of averages is, within certain limits, permissible.

The following table gives the rate of surface loss, or cooling, for the four metals experimented upon. Those for each metal are the mean of several cooling experiments:—

RATES OF COOLING.

Temperature Excess.	Iron.	Copper (Crown).	Copper (C.).	German Silver.
°C.				
5	...	·047	...	...
10	·11	·116	·13	·145
20	·26	·28	·28	·32
30	·43	·465	·46	·51
40	·61	·66	·64	·73
50	·807	·85	·83	·94
60	1·0	1·04	1·03	1·17
70	1·19	1·25	1·235	1·37
80	1·405	1·466	1·45	1·62
90	1·605	1·7	1·68	1·88
100	1·83	1·927	1·91	2·14
110	2·04	2·16	2·15	2·41
120	2·28	2·39	2·39	2·7
130	2·51	2·63	2·63	2·97
140	2·77	2·87	2·88	3·25
150	3·02	3·14	3·15	3·54
160	3·28	3·38	3·43	3·84
170	3·56	3·665	3·72	4·14
180	3·815	3·98	4·04	4·46
190	4·11	4·31	4·4	4·79
200	4·4	4·66	4·75	5·11
210	4·7	4·98	5·11	5·43
220	5·03	5·33	5·465	5·77
230	5·37	5·57	5·84	6·11
240	5·74	5·89	6·2	6·42
250	6·12	6·22	6·57	6·78
260	6·55	6·58	6·95	7·15
270	6·93	6·92	...	...

The following table shows the result of dividing the rate of cooling in copper (Crown), by that of iron at the same temperature. The column of ratios show that the specific heat of copper rises *more quickly* than what has been allowed for in the final deduction of conductivity. This, of course, by increasing the areas, would increase the conductivity at the higher temperatures.

The third column in the table shows the quotient resulting from the division of each of the ratios by the first.

Temperature Excess.	Rate of Cooling.		Ratio of these Rates.	$\frac{\text{Ratios}}{1.054.}$
	Copper (Crown).	Iron.		
10	.116	.11	1.054	1.0
20	.28	.26	1.077	1.021
40	.66	.61	1.082	1.026
60	1.04	1.0	1.04	.987
80	1.466	1.405	1.043	.989
100	1.927	1.83	1.053	.999
120	2.39	2.28	1.048	.994
140	2.87	2.77	1.036	.983
160	3.38	3.28	1.03	.979
180	3.98	3.815	1.044	.989
200	4.66	4.4	1.059	1.005
220	5.33	5.03	1.059	1.005
240	5.89	5.74	1.026	.971

III. FINAL RESULTS.

The units in terms of which the conductivity is given are those of foot, pound, minute ; while the thermometric scale is, of course, Centigrade.

IRON.

(Ordinary Form of Experiment.)

x , in feet.	v .	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to the Succeeding Value of x .	Area of Statical Curve of Cooling beyond x .	Area corrected for change of Spec. Heat.
0.0	247.2	30.1	13.52	43.669	53.049
0.25	172.1	19.54	8.703	30.149	34.598
0.5	125.25	13.22	5.96	21.446	23.885
0.75	92.3	9.2	7.2	15.486	16.877
1.25	52.0	4.79	3.74	8.286	8.785
1.75	30.35	2.92	1.92	5.0	5.38
2.25	18.2	1.52	1.056	2.626	2.708
2.75	11.15	1.0	.949	1.57	1.612
3.75	4.3	...	.381	...	...
4.75	1.85	...	.172	...	...
5.75	0.7	...	* .062	...	...

Hence, thermometric conductivity of Iron.

(1) From areas uncorrected for sp. ht. change.

0°. 100°. 200°. 300°.
.0125 .01156 .01062 .00968

(2) From areas corrected for sp. ht. change.

.0129 .01266 .01243 .0122

* The last number in each of these columns gives the whole area of that part of the curve beyond the corresponding value of x .

IRON.
(Bar cooled midway.)

x , in feet.	v .	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to Succeeding Value of x .	Area of Statical Curve of Cooling beyond x .	Area Corrected for Change in Specific Heat.
0.0	206.35	21.83	11.17	37.557	44.084
0.25	150.85	15.51	7.55	26.384	29.95
0.5	111.4	11.0	5.27	18.834	20.896
0.75	83.45	7.9	6.49	13.564	14.825
1.25	47.7	4.27	3.304	7.074	7.638
1.75	28.15	2.3	1.81	3.77	4.112
2.25	16.5	1.3	.96	1.96	2.192
2.75	9.0	.62	.3	1.0	1.2

Hence, thermometric conductivity of Iron.

(1) From areas uncorrected for sp. ht. change.

0°.	100°.	200°.	300°.
.0112	.0116	.0120	.0124

(2) From areas corrected for sp. ht. change.

.0118	.0129	.0140	.0151
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COPPER (CROWN).

x , in feet.	v .	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to Succeeding Value of x .	Area of Statical Curve of Cooling beyond x .	Area Corrected for Change in Specific Heat.
0.0	238.25	10.58	16.64	114.74	130.151
0.25	211.65	9.12	13.811	98.102	109.91
0.5	185.95	7.85	11.064	84.29	93.42
0.75	162.6	6.83	17.877	73.22	80.47
1.25	126.8	5.2	13.327	55.35	60.003
1.75	99.65	3.94	10.032	42.02	45.123
2.25	78.85	3.02	7.505	31.99	34.123
2.75	62.7	2.3	10.359	24.48	26.021
3.75	39.65	...	5.976	...	...
4.75	24.8	...	3.167	...	...
5.75	13.45	...	1.375	...	...
6.75	6.4	...	.309	...	...

Hence, thermometric conductivity of Copper (Crown).

(1) From areas uncorrected for change in sp. ht.

0°.	100°.	200°.	300°.
.0728	.0736	.0744	.0752

(2) From areas corrected for change in sp. ht.

.0745	.0785	.0825	.0865
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COPPER (C.).

x , in feet.	v .	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to Succeeding Value of x .	Area of Statical Curve of Cooling beyond x .	Area Corrected for Change in Specific Heat.
0.0	238.15	12.21	16.66	98.199	111.357
0.25	202.95	10.27	13.065	81.539	91.121
0.5	175.6	8.8	10.488	68.474	75.619
0.75	149.95	7.48	15.354	57.986	63.519
1.25	111.6	5.46	12.93	42.632	46.119
1.75	83.95	3.9	7.767	29.702	31.844
2.25	62.55	2.9	5.53	21.935	23.439
2.75	48.1	2.17	7.09	16.405	17.545
3.75	27.9	1.26	3.71	9.315	10.398
4.75	16.15	0.66	2.02	5.605	6.555
5.75	8.6	...	.85	...	...
6.75	3.0	...	.135	...	...

Hence, thermometric conductivity of Copper (C.).

(1) From areas uncorrected for sp. ht. change.

0°.	100°.	200°.	300°.
.0512	.053	.0548	.0566

(2) From areas corrected for sp. ht. change.

.0515	.056	.0605	.065
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GERMAN SILVER.

x , in feet.	v .	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to Succeeding Value of x .	Area of Statical Curve of Cooling beyond x .	Area Corrected for Change of Specific Heat.
0.0	243.95	31.425	15.792	42.944	48.998
0.25	169.7	21.17	9.927	27.152	29.863
0.5	116.15	14.35	6.228	17.225	18.487
0.75	80.9	9.79	5.595	10.997	11.597
1.25	39.95	4.63	2.974	5.402	5.591
1.75	20.0	2.24	1.309	2.428	2.484
2.25	10.45	1.128	.624	1.119	1.134
2.75	5.45	0.606	.307	.495	0.495
3.75	1.55	0.16	.144	.188	0.188
4.75	0.5	...	.036	...	...
5.75	0.15	...	.008	...	...

Hence, thermometric conductivity of German Silver.

(1) From areas uncorrected for change in sp. ht.

0°.	100°.	200°.	300°.
.0068	.0079	.0090	.0101

(2) From areas corrected for change in sp. ht.

.0069	.0084	.0099	.0114
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It only now remains to compare briefly the results given above with previous work upon the same bars. The particular values of the rates of cooling, and also of the tangents to the curve of stationary temperature excess, which FORBES used, have been already alluded to. Herein, it would seem, lies the explanation of his result that the thermometric conductivity of iron decreases as temperature increases. Employing, as he did, values of dv/dx , 10 per cent. *larger* than those found in this work, and also values of the rate of cooling which are now known to be much too *small* at any temperature excess beyond 150° C., these two factors are quite sufficient to account for the large decrease in conductivity which he stated for iron.

These remarks are fully confirmed by the results of Prof. TAIT, which are intermediate between those of FORBES and those given by me in the tables above. In Diagram 1 it is shown that the rates of change of temperature excess at different points along the long bar were, for the curve representing the experiments in question, intermediate between those of FORBES and my own. Again, in regard to the rates of cooling, the same may be said, as has already been referred to in connection with Diagram 6. This explains why Prof. TAIT's results for iron (viz., that its thermal conductivity is as nearly as possible constant), should lie between those of FORBES (already stated) and those given in this paper, where the conductivity of iron is shown to increase; for, because of reasons already given, the results derived from the experiments on the iron bar cooled midway, are certainly more trustworthy than those of the ordinary form of the experiment.

With respect to the copper and German silver bars, the results for these differ mainly from those of Prof. TAIT in regard to the rate at which the conductivity rises. In both samples of copper the rise is less, but in German silver more than in the previous work on these bars. But as to the absolute value of the conductivity there is no great difference between the two series of estimates.

In conclusion, I have to tender thanks to Prof. TAIT for the interest he has shown throughout in this inquiry, and for much valuable assistance and guidance both in the experimental part of the work and in the more laborious work of reduction. I have also to thank my friend Mr ROBERT DICKINSON for the care he has bestowed upon the accurate execution of the diagrams published along with this paper.

APPENDIX I.

In the determination of the rate of cooling, when the bar experimented upon was heated to about 100° C. by radiation from an open fire, previous to its being heated rapidly over the small Bunsen burners, it was found that the rates of cooling of the bar so treated were, for temperature excesses above 150° C., in a constantly decreasing ratio to those deduced from the experiments in which the bar had been subjected to a prolonged heating, during which the smoking action of the burners so far altered the brightness of the surface as to produce the effect mentioned. In the final deduction of conductivity these lower rates of cooling have been used. The following table shows, in the case of iron and German silver, the difference between the two series of estimates of the rate of cooling. In both copper bars the difference was noticeable only at the very highest temperatures used in the cooling experiments, and as, besides being comparatively small, such difference was at a temperature excess not reached in the long bar experiment, its effect on the area of the "statical curve of cooling" was almost inappreciable.

It must be remarked, however, that even with the larger differences between the two series of rates of cooling in the cases of iron and German silver, the two corresponding estimates of the conductivity differ only by about 1 per cent.

Temperature Excess.	IRON.			GERMAN SILVER.		
	Lower.	Higher.	Ratio.	Lower.	Higher.	Ratio.
150	3.02	3.02	1.0	3.5	3.54	.988
160	3.28	3.28	1.0	3.79	3.84	.987
170	3.56	3.57	.997	4.14	4.17	.992
180	3.815	3.88	.983	4.46	4.51	.989
190	4.11	4.2	.978	4.79	4.92	.973
200	4.4	4.52	.973	5.11	5.28	.967
210	4.7	4.88	.963	5.48	5.66	.968
220	5.03	5.28	.952	5.77	6.05	.954
230	5.37	5.60	.958	6.11	6.4	.954
240	5.74	6.13	.936	6.42	6.79	.945
250	6.12	6.59	.928	6.78	7.15	.948
260	6.55	7.09	.923	7.15	7.55	.947
270	6.93	7.59	.913	...	...	...

APPENDIX II.

In the text of this paper it is stated that the error due to the circumstance of the thermometric readings being those of a variably heated thermometer is not taken into account in the final results given above. While the paper was being printed, I applied to the tangents and areas the corrections given by

Prof. TAIT in the introduction to this paper. The results of the application of these corrections are given in the following table along with the results given already. The numbers in brackets indicate the conductivity as deduced from data uncorrected for the error in question, those without brackets indicate that from corrected data.

It will be observed from a comparison of the corresponding numbers, that the effect of applying this correction is to make the conductivity more nearly constant at different temperatures.

Temperature, C., . . .		0°.	100°.	200°.	300°.
IRON (Ordinary).	{ Not corrected for specific heat change. }	(·0125) ·0127	(·01156) ·0115	(·01062) ·0103	(·00968) ·0091
	{ Corrected for specific heat change. }	(·0129) ·0131	(·01266) ·0123	(·01243) ·0115	(·0122) ·0107
IRON (Cooled Bar).	{ Not corrected for specific heat change. }	(·0112) ·01154	(·0116) ·01162	(·0120) ·01170	(·0124) ·01178
	{ Corrected for specific heat change. }	(·0118) ·0119	(·0129) ·01274	(·0140) ·01358	(·0151) ·01442
COPPER (Crown).	{ Not corrected for specific heat change. }	(·0728) ·0705	(·0736) ·0710	(·0744) ·0715	(·0752) ·0720
	{ Corrected for specific heat change. }	(·0745) ·0776	(·0785) ·0792	(·0825) ·0808	(·0865) ·0824
COPPER (C.).	{ Not corrected for specific heat change. }	(·0512) ·0509	(·0530) ·052	(·0548) ·0531	(·0566) ·0542
	{ Corrected for specific heat change. }	(·0515) ·0520	(·0560) ·0554	(·0605) ·0588	(·0650) ·0622
GERMAN SILVER.	{ Not corrected for specific heat change. }	(·0068) ·0070	(·0079) ·0078	(·0090) ·0086	(·0101) ·0094
	{ Corrected for specific heat change. }	(·0069) ·0068	(·0084) ·0082	(·0099) ·0096	(·0114) ·0110