242. Convenient Rules for Elementary Calculations as to Thin Lenses

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In the latter case it is proposed to show that the two positive roots must lie between 0 and $q^2/4r$.

If α , β , $-(\alpha + \beta)$ are the three roots (α , β both positive) the given equation is equivalent to

$$\{x + (\alpha + \beta)\}\{x^2 - (\alpha + \beta)x + \alpha\beta\} = 0,$$

or

$$x^3 - (\alpha^2 + \alpha\beta + \beta^2)x + \alpha\beta(\alpha + \beta) = 0;$$

so that

$$q = \alpha^2 + \alpha\beta + \beta^2, \quad r = \alpha\beta(\alpha + \beta).$$

Now

$$(\alpha^2 + \alpha\beta + \beta^2)^2/4\alpha\beta(\alpha + \beta) \text{ is } > \alpha \text{ and also } > \beta.$$

For

$$(\alpha^2 + \alpha\beta + \beta^2)^2 - 4\alpha^2\beta(\alpha + \beta) = (\alpha^2 - \alpha\beta - \beta^2)^2, \text{ etc.}$$

Therefore $q^2/4r$ is greater than either α or β .

[It may be noticed that if we substitute $q^2/4r$ for x in $x^3 - qx + r$ we get $[(q^3 - 8r^2)/8r]^2/r$ which is of the same sign as r , so that *a priori* both α and β lie either between 0 and $q^2/4r$ or between $q^2/4r$ and $+\infty$].

The above inequality applied to the quadratic $x^2 - Px + Q = 0$ gives $(P^2 - Q)/4PQ$ as a superior or an inferior limit according as P , Q are of the same or different sign (the roots being of course *real*). R. F. DAVIS.

242. [T. 3. b.] Convenient rules for Elementary Calculations as to Thin Lenses.

The *divergence* of a pencil of rays with reference to a thin lens is defined as the reciprocal of the distance of its vertex from the centre of the lens. If the rays are convergent, their *divergence* is reckoned negative.

The *divergive power* of a lens is the divergence it imparts to parallel rays passing through it, and is equal to the reciprocal of the focal length of the lens, taken negatively if the lens be *convergive*. Its metric unit is the *dioptr*.

RULE for finding the position of the image of a point produced by a thin lens:

$$\text{Divergence after refraction} = \text{divergence before} + \text{divergive power of lens},$$

or

$$d' = d + p.$$

Here, of course, attention must be paid to the *signs* of the quantities dealt with.

Take, for instance, a convergive lens whose focal length is 6 inches and whose divergive power is therefore $-\frac{1}{6}$. If a point be placed 8 inches in front of it, where is the image? $d' = \frac{1}{8} - \frac{1}{6} = -\frac{1}{24}$. Thus the rays have become *convergent* and meet at a point 24 inches *beyond* the lens.

Again, if the point be brought to within 4 inches of the lens we have $d' = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$. Thus the rays remain *divergent* and the image is behind the object, at distance 12 inches from the lens.

RULE for calculating the divergive power of a thin lens when its dimensions and material are given.

$$\text{Divergive power} = \text{refractivity} \times (\text{sum of concavities}),$$

or

$$p = (\mu - 1) \left(\pm \frac{1}{r} \pm \frac{1}{r'} \right).$$

Here the *refractivity* is defined as the excess of the refractive index of the glass over that of air, and by *concavity* is meant the spherical curvature of a face of the lens, taken as positive if it is concave (negative if convex).

The advantage of the rule in words over that in symbols is that there is no $\pm$ sign to remember the interpretation of. We may note that of the two factors of divergive power, the former is a function of its *material*, the latter of its *form*. Again, the divergive power of a combination of two thin lenses placed in contact is the algebraic sum of their divergive powers.

Rules similar to the above might be stated, substituting *convergence*, *convergive power*, and *convexity* for *divergence*, *divergive power*, and *concavity*, but it is simpler to keep to one set of rules.

What precedes is the gist of the method taught by Lord Kelvin when Professor of Natural Philosophy in Glasgow; to a great extent it was anticipated by Coddington in his *Treatise on the Reflexion and Refraction of Light*, Cambridge, 1829. An account of Kelvin's method was published by Dr. Magnus Maclean in the *Phil. Mag.*, Nov., 1889. Another, with an extension to the case of refraction at a single spherical surface, was given by me in the *Proceedings of the Royal Philosophical Society of Glasgow*, April, 1903.

R. F. MUIRHEAD.

REVIEWS.

Theoretical Mechanics. By J. H. JEANS, F.R.S., Professor of Applied Mathematics in Princeton University. Pp. vii, 364. Ginn & Co., Boston and London. Price 10s. 6d.

All teachers of elementary mechanics rejoice when one who has shewn his powers as an original investigator, as Professor Jeans has done, devotes himself to producing an elementary exposition of the science.

From such an author we seek for guidance on points of difficulty, for new light on familiar matters, for perspective and a mode of treatment in which topics shall appear in their due order and magnitude: nor is our search in vain.

The object of the book may be indicated by an extract from the preface:

"All students will best begin the subject by trying to acquire a firm grasp of physical principles, leaving aside at first mathematical developments and practical applications, except in so far as these contribute to the elucidation of the fundamental physical principles."

Among the many features of novelty and interest in Professor Jeans' treatise may be mentioned a striking introductory paragraph on the uniformity of nature as the basis of science; many original and ingenious examples, the use of the calculus in close connection with the work and yet in such a manner that the cardinal principles are presented without the calculus; the mode of obtaining the principle of the transmissibility of force; the reconciliation of the relativity of velocity, and the dependence of v on the frame of reference employed, with the principle of work and the formula $\frac{1}{2}mv^2$ for kinetic energy; and especially the final chapter on generalised coordinates. As is inevitable where so much of the treatment is novel there are a few of those slips which are so invisible in proofsheets and so obvious in a published book, and a few places where the ordinary student will require additional stepping stones.

Thus at page 72 a formula is obtained for the advantage obtained by using a wheeled car instead of a sledge, and it is remarked that the formula shews that the wheel still possesses an advantage even if the coefficient of friction at the axle is infinite. But if a wheel is locked on the axle it can only slide on the ground.

Again, the numerical illustration of the top in section 257 is vitiated by taking B as the moment of inertia about an axis through the c.g. instead of about a parallel axis through the fixed point.

Again, would not many students object to *writing down* $B\sin^2\theta\phi$ as that part of the moment of momentum of a top about a vertical axis which is due to the motion of the axis.

Why is the coefficient B the moment of inertia about a principal axis through the point of contact other than the axis of symmetry?

The proposition on which the result depends, and which is not explicitly stated by Professor Jeans may be indicated thus:— I_1, I_2, I_3 being moments of inertia about principal axes 1, 2, 3; I the moment of inertia about an axis whose direction cosines are l_1, l_2, l_3 , and the angular velocity about the axis l_1, l_2, l_3 being Ω , vectorially compounded of $\Omega_1, \Omega_2, \Omega_3$, we have for the moment of momentum about l_1, l_2, l_3 .

$$\begin{aligned} I\Omega &= [I_1l_1^2 + I_2l_2^2 + I_3l_3^2]\Omega \\ &= I_1\Omega_1l_1 + I_2\Omega_2l_2 + I_3\Omega_3l_3, \end{aligned}$$

so that the moment of momentum is vectorially compounded of $I_1\Omega_1, I_2\Omega_2$, and $I_3\Omega_3$.