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The Accuracy of our Service Field Range-Finders

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THE ACCURACY OF OUR SERVICE FIELD RANGE-FINDERS.

By Major A. W. WHITE, R.A., School of Range-Finding, Aldershot.

THE degree of accuracy attainable in field range-finding has been hitherto commonly regarded as more or less a matter of individual opinion, or at best of individual experience: hence, while a variety of conflicting statements on the subject have been promulgated, no serious attempt has been made to test them by the precise methods proper to scientific investigation.

To remedy this omission in the case of the Service range-finders, I shall in this paper briefly examine the question by the aid of simple formulæ, from which tables of probable error may be calculated, and a standard of accuracy established for each range-finder. Until this has been done, it will be plainly impossible to appraise exactly the tactical value of these instruments, or to apply them with complete advantage in the field.

Our Service field range-finders are at the present time the Watkin for artillery and infantry, and the Weldon for cavalry and infantry. It will be advisable to consider these instruments separately.

THE WELDON RANGE-FINDER.

The Weldon range-finder consists of three prisms, reflecting respectively the following angles within manufacturing limits.

90°, 88° 51' 15", 74° 53' 15".

FIG. 1.

There are three ways of using the instrument:—

1st Method.—A being the *object* (Fig. 1) and AD the range required:—The angle $ADP = 90^\circ$ is found by the first prism, and the angle $ACD = 88^\circ 51' 15''$ by the second prism,¹ the point P being a *direction point* found on the ground, or set out for the purpose. Then CD being actually measured, the range = 50 CD.

FIG. 2.

2nd Method.—The triangle ADC (Fig. 2) is determined as before, but instead of measuring CD, and multiplying it by 50 to get the range, the third prism is made use of to make the angle $AFD = 74^\circ 53' 15''$. Then FC, the sub-base, being measured, the range = 200 FC.

FIG. 3.

3rd Method.—Two observers are required, each with a range-finder. They face one another, and, using the second prism, reflect the *object* on to each other respectively, thus making an isosceles triangle, ADC (Fig. 3), having the interior base angles at D and C, each $= 88^\circ 51' 15''$. Then CD (the *base*) being measured, the range = 25 CD.

Causes of Error.

The principal causes of error of range with the Weldon range-finder are—

¹ The angle C is sometimes determined first and the angle D afterwards.

1. Instrumental errors; that is to say, defects in the prisms.
2. Errors of measurement.
3. Inaccuracy in appreciating the angles.
4. Inaccuracy in appreciating the line.
5. Errors in marking the base points; that is to say, differences between the points as marked on the ground and as determined by the eye.
6. Errors through not holding the instrument at the proper inclination to the horizon.

1. *Instrumental Error.*—The Service regulations permit of a certain amount of variation (plus or minus) in the instruments supplied by contract. As an empirical factor, this may be taken to cause an error not exceeding—

By the 1st and 3rd methods, ± 1.5 per cent. of the range.

„ 2nd method, ± 2.5 per cent. of the range.

The instrumental error may however be ascertained for any given instrument, and its readings corrected by the addition or subtraction of so much per cent. from the range.

2. *Error of Measurement.*—With good measuring tapes, on flat ground and in the absence of high wind, this ought not to be more than $\frac{1}{200}$ of the length measured, so that the error of range due to error of measurement should not generally exceed ± 0.5 per cent.

3. *Inaccuracy in Appreciating the Angles.*—The smallest difference of angle appreciable by the naked eye is about 1 minute; that is to say, the angle subtended by 1 inch at 95.4 yards' distance. From this it follows that a faultless range-taker is liable to an error of 1 minute at each base point owing to imperfect estimation of the reflected angle.

4. *Inaccuracy of Line.*—Errors of line occur in the 1st and 2nd methods only. They can hardly be avoided on uneven ground,

FIG. 4.

where, as in the section, Fig. 4, the base points C, D, and the direction point P, are not in the same straight line.

5. *Errors in Marking the Points.*—These, unless large, are of consequence in the 1st and 2nd methods only.

If pickets are used, such that the instrument can be placed upon them in taking the angles, no error is to be expected; but if the points are roughly marked, important differences may exist between their actual and their true positions.

6. *Incorrect Slope of the Instrument.*—No estimate can be given of the amount of unavoidable error from this cause. It will be generally at a minimum on flat ground, and increase in proportion as the object and the direction point are respectively one above and the other below the level of the observer.

Formule for Errors of Range due to Inaccuracy in the observed Angles irrespective of the Causes of such Inaccuracy.

Referring to Figs. 1, 2, 3, if any errors occur in the angles at D, C, F, the three drill formulæ—

$$\begin{aligned} AD &= 50 CD \dots\dots\dots 1st \text{ method} \\ &= 200 CF \dots\dots\dots 2nd \quad ,, \\ &= 25 CD \dots\dots\dots 3rd \quad ,, \end{aligned}$$

will no longer hold good, and an error of range will be the result.

Let A = the true apex angle (Figs. 1, 2) = $1^{\circ} 8' 45''$,

B = the true angle CDF (Fig. 2) = $13^{\circ} 58'$,

F = the true angle AFD (Fig. 2) = $74^{\circ} 53' 15''$,

δ = error in the right angle ADP (Figs. 1, 2),

e = error in the $88^{\circ} 51' 15''$ angle, i.e., when the second prism is used,

t = error in the third angle,

R = true range,

R' = apparent range as found by the drill.

Then by first method—

$$R' = 50 R (\tan (A \mp e) \pm \tan \delta) \dots\dots 1st \text{ Weldon formula.}$$

By second method—

$$R' = 200 R \frac{\sin (B \pm e \mp t)}{\sin (F \pm t)} \{ \tan (A \mp e) \pm \tan \delta \} \dots\dots 2nd \text{ ditto.}$$

By third method—

$$R' = 25 R \{ \tan (A \mp e) + \tan (A \mp e') \} \dots\dots 3rd \text{ ditto,}$$

where e' = error made by the second observer.

The upper signs are to be taken when the increments δ , e , t are positive, that is, when the errors they express are of *excess*, and *vice versa*.

These formulæ are easily proved. The first covers four general cases which can be examined separately; for example:—

FIG. 5.

Let $AD'P$, Fig. 5, be greater than 90° , and $AC'P$ greater than $88^\circ 51' 15''$, ADC being the range-finding triangle as it ought to be.

$$\begin{aligned}\text{Then } R' &= 50 C'D', \text{ by the drill} \\ &= 50 (DC' + DD') \\ &= 50 \{AD \tan DAC' + AD \tan DAD'\} \\ &= 50 R \{\tan (\Lambda - e) + \tan \delta\}.\end{aligned}$$

The second formula covers eight general cases, which also may be examined separately.

FIG. 6.

For example:—let AFD (Fig. 6) be the true range-finding triangle with reference to the direction point P , and let

$$\begin{aligned}AD'P &\text{ be greater than } 90^\circ, \\ AC'P &\text{ be less than } 88^\circ 51' 15'', \\ AF'D &\text{ be less than } 74^\circ 53' 15'',\end{aligned}$$

so that

$$\begin{aligned}\delta &\text{ is } +, \\ e &\text{ ---}, \\ t &\text{ ---}.\end{aligned}$$

Then

 $R' = 200 F'C'$ by the drill.

But by trigonometry $\frac{F'C'}{C'D'} = \frac{\sin F'D'C'}{\sin C'F'D'}$

Now $F'D'C' = AC'D' - C'F'D'$
 $= (ACD - e) - (CFD - t)$
 $= (ACD - CFD) - e + t$
 $= CDF - e + t$
 $= B - e + t.$

Again $C'F'D' = CFD - t = F - t.$

Therefore $\frac{F'C'}{C'D'} = \frac{\sin (B - e + t)}{\sin (F - t)}.$

But $C'D' = R (\tan (A + e) + \tan \delta).$ (See proof of Formula 1.)

Therefore

$$R' = 200 F'C' = 200 R \frac{\sin (B - e + t)}{\sin (F - t)} \{ \tan (A + e) + \tan \delta \}.$$

The proof of the third formula is obvious.

Extension of the Formulæ.

The three general formulæ just discussed can be made to cover the whole of the six causes of error enumerated at p. 691 by giving the proper value to δ , e , and t , but it will be convenient to treat the following separately.

Instrumental Error.—This will be best dealt with by adding to or subtracting from the error in range a quantity equal to the full allowable error per cent. of range, if the *liability* to error is under consideration, or half that amount if the *average* error of a number of instruments is in question.

Error of Measurement.—The proportion of the error of measurement to the length measured, be it base or sub-base, will express the percentage of the resulting error of range: thus a yard error in a base of 100 yards will cause an error of $\frac{1}{100}$, i.e., 1 per cent. of the range; 1 inch in a sub-base of 2 yards will cause an error of $\frac{1}{12 \times 36} = \frac{1}{72}$, i.e., 1.39 per cent. of the range; and so on; so that the effect of an error of measurement can be best treated independently, and, as will be seen further on, may be made to include certain errors of marking equivalent to errors of measurement.

Error through Wrong Inclination of the Instrument.—This is often very serious, but as it is an extremely variable factor it will not be taken into account at all in what follows.

There remain then three distinct forms of error, viz.:—

Inaccuracy in appreciating the angle.

Errors of line.

Errors of marking.

The first is directly dealt with by the formulæ. The other two require to be translated into their angular equivalents.

Angular Equivalents of Small Errors of Line.

1. If an error in direction be made at right angles to the true primary base, and this error being measured at the right angle point amounts to a length a , D being the distance to the direction point, then the resulting errors of range will be given by adding—

$$\pm \tan^{-1} \frac{a}{D} \text{ to } \delta \text{ in Formula 1;}$$

$$\mp \tan^{-1} \frac{a}{D} \text{ to } e \text{ in Formula 2;}$$

the upper sign being taken when the deviation is towards the *object*, and *vice versa*, provided the right angle is determined first. If the right angle is determined second, the rule for the signs will be reversed.

2. If an error in direction be made in laying out the sub-base at right angles to it, and this error, being measured at the 88° point, amounts to a length b , then the resulting error of range will be given by adding $\pm \tan^{-1} \frac{b}{R}$ to e in Formula 2 (independent of other corrections), the upper sign being taken when the deviation is towards the right angle point, and *vice versa*.

Angular Equivalents of Small Errors of Marking.

Errors of *marking* must not be confounded with errors of *line*, though the effect may sometimes be the same.

1. *Errors at the First and Second Points.*—In Formulæ 1 and 2—let c be the linear deviation at the first point, at right angles to the primary base, D and R having their previous signification.

Then if the right angle point be the first determined,

$$\text{add } \pm \tan^{-1} \frac{c}{D} \text{ to } \delta;$$

but if the 88° angle point be the first,

$$\text{add } \pm \tan^{-1} \frac{c}{D + \frac{R}{50}} \text{ to } e.$$

The upper sign is to be taken when the deviation is towards the *object*, and *vice versa*.

Again, if m and n are the linear errors in the direction of the base line at the 90° and 88° points respectively,

$$\text{add } \pm \tan^{-1} \frac{m}{R} \text{ to } \delta,$$

$$\text{and } \pm \tan^{-1} \frac{n}{R} \text{ to } e,$$

the upper or lower signs being taken according as m and n are made towards the *direction point* or from it.¹

2. *Errors at the Third Point.*—Small errors in marking the third points are of no consequence, except such as are in the line of the sub-base, and these are best treated as simple errors of measurement, and multiplied by 200 for the resulting error in range.

Application of the Formulæ.

We are now in a position to tabulate general results or work out the error of range in particular instances, and it may be useful to observe that for ordinary values of δ we may write $\tan(\Lambda - e + \delta)$ instead of $\tan(\Lambda - e) + \tan \delta$ in Formulæ 1 and 2.

The following will serve as examples.

Example 1.—A range-taker adopting Method 1 has marked his right angle point by placing his forage cap between his feet. In observing the angle, however, he has held the instrument exactly over his right foot.

The cap is 6 inches across, the range is 500 yards, and the direction point is a telegraph post about 48 yards from the right angle point.

The instrument is perfect, and all other operations are correctly performed. What is the error of range due to the inaccuracy of marking the first point?

Here $e = 0$

$$\delta = +\tan^{-1} \frac{3 \text{ inches}}{48 \times 36} = 6 \text{ minutes.}$$

$$\text{Therefore } R' = 50 \times 500 \{ \tan 1^\circ 8' 45'' + \tan 6' \} \\ = 544 \text{ yards.}$$

Hence the error amounts to 44 yards, or 8.8 per cent.

Example 2.—The direction point is a tree, the stem of which slopes 1 in 5 to the vertical. There are several knots upon it, and a range-taker makes his second and third points on a knot 2 feet 6 inches higher up than the one he looked at when making his right angle.

No other mistake is made. The range is 1,500 yards. The direction point is 95 yards from the second (88°) point. What will the error be?

Here in Formula 1, put $\delta = 0$, and since $\frac{1}{5}$ th of 2 feet 6 inches = 6 inches,

$$\text{take } e = \tan^{-1} \frac{6}{36 \times 95} = \pm 6 \text{ minutes.}$$

¹ Instead of adding $\pm \tan^{-1} \frac{m}{R}, \pm \tan^{-1} \frac{n}{R}$ to δ and e in Formula 1, the error of range resulting from the errors m and n can be found independently; thus—

$$\text{Error} = 50(\pm m \mp n),$$

the upper sign being taken when m and n are towards the direction point.

Then $R' = 50 \times 1500 \tan 1^\circ 2' 45''$, or if e be negative $50 \times 1500 \tan 1^\circ 14' 45''$.

$$= 1,369 \text{ yards or } 1,631 \text{ yards.}$$

Therefore the error = 131 yards.

If the second method be adopted, put in Formula 2

$$\delta = 0,$$

$$t = 0,$$

$$e = \pm 6', \text{ as before.}$$

$$\text{Then } R' = 200 \times 1500 \frac{\sin (B \pm 6)}{\sin I'} (\tan A \mp 6),$$

$$= 1,379 \text{ or } 1,620 \text{ yards,}$$

and the error will be 121 yards.¹

Example 3.—A range-taker marks his *third* point with his foot. He measures from his heel, whereas he holds the instrument exactly over his toe. His foot is 11 inches long, what is the error in range from this cause alone?

$$\text{Here the error} = 200 \times 11 \text{ inches}$$

$$= 61 \text{ yards.}$$

Example 4.—Two range-takers working together by Method 3, take the range to an Officer crossing their front at a quick walk. In making the observation one range-taker completes coincidence 1 second in time sooner than the other.

What is the error if the range be 1,000 yards?

At 4 miles an hour the distance walked by the Officer in one second would be 70.4 inches.

Hence the error can be obtained by multiplying by 25.

$$25 \times 70.4 \text{ inches} = 49 \text{ yards.}$$

Or it may be found by putting in Formula 3—

$$e = \tan^{-1} \frac{7.4}{36 \times 1000} = 6' 42'',$$

$$e' = 0.$$

$$\text{Thus } R' = 25,000 \{ \tan 1^\circ 15' 27'' + \tan 1^\circ 8' 45'' \}$$

$$= 1,049 \text{ yards.}$$

Error, 49 yards, as before.

¹ It will be noticed that the error here is diminished by the use of the third prism; that, however, is not invariably the case.

The following may be of interest:—

TABLE A.—*Greatest Variation in the observed Range attributable to the Optical Limit.*

Error = 1'	Method 1.	Method 2.	Method 3.
Error per cent. of range will not exceed			
Occurs—			
In one angle only	1.45	1.54	0.73
In two angles	2.01	2.79	1.46
In three angles	..	2.92	..

In the above table it is assumed that the *instrument* is perfect, and that the *measurement*, the *line*, and the *marking* are absolutely correct.

The figures are obtained by making δ , c , t successively 0 and $\pm 1'$ in Formulæ 1, 2, 3, and tabulating the highest readings. It is evident of course that every value of error less than that expressed by the figures in the table may occur from zero in both directions.

Example 5.—A *faultless* range-taker with a *perfect* instrument takes the range of an object 1,000 yards away. What difference may possibly occur between any two readings by Method 1 or 2. Here by Table A the extreme error is $\pm 10 \times 2.92 = \pm 29.2$ yards, therefore 58.4 yards difference may occur in two readings.

Example 6.—A good range-taker is employed to find the distance of an object 1,500 yards away. Between what extreme limits ought his readings to be?

By Table B—

1st method 1,365 to 1,635.
2nd method 1,350 to 1,650.

Theory and Practice Compared.

It may be asked how far the conclusions arrived at theoretically tally with the results of experiment.

Now it happens that trials of a tolerably exhaustive kind were made at the School of Range Finding in 1884. Two instruments were used and there were three independent observers. Twelve observations were made at every range, three with each instrument by the first method, and three with each by the second, the mean being recorded in every instance. The country traversed was partly open, partly enclosed. The ground was in places rough and hilly, in others flat. Every kind of direction point was made use of.

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The following are the figures of the series in which pickets and measuring tapes were made use of:—

Range in yards.	Average error per cent. of range by 1st method.
Under 1,000	11.0
1,000 to 2,000	5.84

By the second method the error was about 1 per cent. greater than by the first.

Comparing these figures with those given in Table B, it would appear that the *actual average error* under *ordinary conditions* is not much less than the *theoretical extreme error* in *favourable circumstances*, but it must be remembered that in drawing up the table, no notice has been taken of errors from holding the instruments incorrectly.

THE WATKIN FIELD RANGE-FINDER.

The Watkin field range-finder is an instrument of the sextant type, measuring distances by the mechanical solution of a right angled triangle, and giving the result, without calculation, on a graduated cylinder. It can be used with any unit of length, so that there are no limits to the ranges taken, or to the bases made use of, except that the instrument is so constructed that the base must not be more than $\frac{1}{4}$ th, nor less than $\frac{1}{8}$ rd the range. It is fitted with a telescope giving an optical advantage of from 3 to 1 to 4 to 1.

The physical conditions required are that (Figs. 7, 8), O being the object, two accessible points, A and C, may be found, such that the angle ACO may be a right angle, and the points A and O visible from C, and C and O from A.

No direction point is needed.

FIG. 7.

FIG. 8.

There are many ways of using the instrument, but the authorized drill for ordinary occasions is as follows:—

The observer having marked a point A (Figs. 7, 8), finds C at a convenient distance from it, the right angle, OCA, being determined by the instrument.

A sub-base, AP, having been set out, 3, 6, or 12 yards long, the observer standing at C reads off the length of AC (reflecting P on to A), and sets the instrument to that base.

He then proceeds to the point A, and from there reads the range on a graduated cylinder, by reflecting C on to O, which is equivalent to measuring the instrumental right angle less the angle OAC.

Causes of Error.

The common causes of error of range with the Watkin range-finder are—

1. Instrumental error, due to defective workmanship or to deterioration from wear.
2. Error in the sub-base.
3. Inaccuracy in appreciating the angles.
4. Difference between the base points as marked on the ground, and as determined by the eye.

Instrumental Error.—At present a total error, not exceeding 1 per cent. of the range, is permitted in new instruments, tested under standard conditions—that is, when the range is 20 times the base.

If from wear and tear the error increases to 2 per cent., the instrument is considered to be out of repair and is dealt with accordingly.

Error in the Sub-base.—The sub-base is laid out by a metallic cord, any alteration in which is corrected by comparison with a steel tape forming part of the equipment; the error in the sub-base should, therefore, in no circumstances, exceed $\frac{1}{200}$ th of the length laid out.

Unavoidable Inaccuracy in estimating the angles.—With the naked eye this may amount to one minute at each angle (see p. 691); with the telescope it should not exceed 20".

Difference between the Base Points as Marked and as Determined.—This error will occur if the instrument is held incorrectly with reference to the pickets.

When the Service artillery range-finding equipment is used, the error should be inconsiderable, because the large pattern range-finder is marked with a ring the size of the picket-head, and it is a part of the drill to rest the instrument on the picket and verify the angles, with the ring in exact position.

Formulae for Errors of Range.

Although a general formula can be given embracing every factor of error, it will be found convenient to deal separately with the errors of range arising from specific causes, as follows:—

1. Error in the sub-base.

Let b = the correct sub-base.

n = error in the sub-base expressed in the same units as b .

R = true range.

Then the error in range will be $-R \frac{n}{b+n}$, the proper sign being given to n Formula 1.

Thus if n be negative, that is, if the sub-base be too short, the error will be positive, or, in other words, the range read will be too long.¹

2. Error in appreciating the angle subtended by the sub-base in the secondary triangle.

Let t = error in the angle, considered to be positive when increasing the angle, and *vice versa*.

B = the correct base.

The error in the range = $-\frac{R}{B} \times \frac{B^2 + b^2}{B + b \cot t}$, the proper sign being given to t in each case.². Formula 2.

Formula 2 can be proved thus:—

Let ϕ be the real angle,
 B' the base as read,

then, by trigonometry $B = b \cot \phi$,

and by the construction of the instrument,

$$B' = b \cot (\phi + t).$$

Therefore
$$B' = b \frac{\cot \phi \cot t - 1}{\cot \phi + \cot t}.$$

$$\begin{aligned} B' - B &= \frac{b^2 \cot \phi \cot t - b^2}{b \cot \phi + b \cot t} - b \cot \phi \\ &= -\frac{b^2 + b^2 \cot^2 \phi}{b \cot \phi + b \cot t} = -\frac{B^2 + b^2}{B + b \cot t}. \end{aligned}$$

But
$$R' - R = (B' - B) \frac{R}{B}.$$

Therefore
$$R' - R = -\frac{R}{B} \times \frac{B^2 + b^2}{B + b \cot t}.$$

3. Error in appreciating the right angle and variable angle read at the base points of the primary triangle.

Let δ = error in the right angle, considered as positive when increasing the angle, and *vice versa*.

e = error in the variable angle, with the same convention.

¹ The corresponding error in the base itself will be $-B \frac{n}{b+n}$.

² The corresponding error in the base will be

$$-\frac{B^2 + b^2}{B + b \cot t}.$$

Then the error in range caused by these errors

$$= -\frac{R^2 + B^2}{R + B \cot(\delta + e)},$$

the proper signs being given to δ and e Formula 3.

Formula 3 can be proved in a similar manner to Formula 2. The variable angle referred to is the complement of the variable angle at the base of the primary triangle.

Corollary.—If i be the error in the instrumental right angle as distinguished from any error of appreciation—

FIG. 9.

FIG. 10.

and $\triangle OAC$ the true right angle (Figs. 9 and 10), it is evident that the variable angle OAP is increased or diminished by the same quantity, i , when the instrument is reversed.

Therefore in this case the error in range

$$= -\frac{R^2 + B^2}{R + B \cot 2i},$$

where i is to be taken as positive when it increases the right angle, and *vice versa*.

4. Error in the position of the instrument.

Let the instrument be held a length, a , out of position at the right angle in a direction towards or from the object, and similarly a length a' at the other point of the base, and let a or a' be considered positive when the picket-head is nearer to the object than the ring on the instrument.

Then if a , a' , are small, the error will be included in Formula 3 by adding a quantity k to $\delta + e$, where $k = \cot^{-1} \frac{B}{a + a'}$, the proper signs being given to a and a' .

Application of the Formulæ.

The formulæ we have established are:—

Watkin Formula 1.—Error of range due to error n in the sub-base

$$= -R \frac{n}{b + n}.$$

Watkin Formula 2.—Error of range due to error in appreciating the apex angle of the subordinate triangle

$$= -\frac{R}{B} \times \frac{B^2 + b^2}{B + b \cot t}.$$

Watkin Formula 3.—Error of range due to errors δ and e in the angles at the primary base points, together with incorrectness of position of instrument, a a' , at those points

$$= -\frac{R^2 + B^2}{R + B \cot(\delta + e + k)}, \text{ where } k = \cot^{-1} \frac{B}{a + a'},$$

the proper signs being given to n , t , δ , e , a , a' respectively.

A variety of examples may be worked out by means of the above as in the case of the Weldon formulæ, and it may be observed that for ordinary values of the angular errors we may substitute for the cotangent of the error the value of 1 divided by the error itself expressed in circular measure; thus for $\cot 1'$ we may write $\frac{180 \times 60}{\pi}$.

Example 1.—A 6-yard sub-base is laid out 1 inch too short. What is the effect on the observed range?

Put
$$n = -\frac{1}{36} \text{ in Formula 1,}$$

then the error
$$= \frac{1}{215} R,$$

that is to say, the error is 0.46 per cent. of the range, and has the effect of making the apparent range too long.

Example 2.—A careless range-taker holds the instrument $3\frac{1}{2}$ inches too far back when reading a range of 1,200 yards with a base of 60 yards. What error will this cause?

In Formula 3 put
$$\delta = 0,$$

$$e = 0,$$

$$k = \cot^{-1} \frac{60 \times 36}{3.66}.$$

$$\begin{aligned} \text{Then the error} &= -\frac{1440000 + 3600}{1200 + 60 \times 590.1} = -\frac{1443600}{36606} \\ &= -39\frac{1}{2} \text{ yards.} \end{aligned}$$

That is to say, the apparent range will be $39\frac{1}{2}$ yards too short.

The following tables correspond to those given at pp. 698, 699 for the Weldon instrument.

TABLE C.—*Greatest Variation in the observed Range attributable to the Optical Limit.*

Without telescope.

Error = 1'	Base 10 times the sub-base. Range 10 times the base.	Base 20 times the sub-base. Range 20 times the base.	Base 30 times the sub-base. Range 30 times the base.
	Error per cent. of range		
Occurs in—			
One angle only.....	0·29	0·59	0·86
Two angles.....	0·59	1·17	1·72
Three angles.....	0·88	1·77	2·27

If the telescope is used divide the figures in the table by 3.

Halving the figures in Table D for a working average, they agree closely with practice; the error of a fairly trained range-taker being found by experience to be about 1 per cent. after the instrumental error has been allowed for.

CONCLUDING REMARKS.

From the foregoing investigation into the common causes of inaccuracy with the service range-finders, it will be seen that it is quite practicable to obtain a precision sufficient for the requirements of active service. On the other hand, it is easy, by want of instruction or neglect of practice, to lose all the tactical advantage which the range-finder confers.

In the case of musketry and of riding we know that failure is not uncommon; so with range-finding; but surely the *best*, not the *worst* record, is the practical standard of merit. Want of skill in the handling furnishes no valid argument against the utility of any appliance, and the sooner this is generally acknowledged in respect to range-finders the better for the interests of the Service, and the less we shall hear of such unhappy performances as the over-estimation of an artillery range by 1,000 yards,¹ incidents more worthy of notice by court-martial than of record with the statistics of a campaign.

¹ See Military Prize Essay, R.U.S.I., 1897, Journal, vol. xxi, p. 409. It should perhaps be mentioned that until the end of 1883 there were no trained range-takers in the British Army, and but few unbroken range-finders.

TABLE D.—*Maximum admissible Error in Range under ordinary Service Conditions.*

Relation of range to base, &c.				Separate errors.			Totals.		Conditions.		
How taken.	Range.	Base.	Sub-base.	Angles and position.	Measurement.		Instrumental.	Including instrumental error.		Excluding instrumental error. ¹	
					yards.	yards.		yards.			Per cent. of range.
Without telescope.	1,000	60	G	19.9	5.0	12.6	37.9	3.8	21.9	2.5	
	2,000	100	"	45.3	10.0	30.0	85.3	4.2	55.3	2.7	
	3,000	120	"	80.7	15.0	58.1	153.8	5.1	95.7	3.1	
With telescope.	3,000	120	G	34.2	15.0	58.1	107.3	3.6	49.2	1.6	
	4,000	125	"	54.4	20.0	95	169.4	4.2	74.4	1.8	
	5,000	125	"	81.6	25.0	156.7	266.3	5.3	109.6	2.2	
With telescope and double units.	5,000	200	12	50.7	25.0	96.9	172.6	3.3	75.7	1.5	
	6,000	"	"	69.1	30.0	140.5	239.6	3.9	99.1	1.6	

1. Instrumental error not to exceed $1\frac{1}{2}$ per cent. when $R=20D$.
2. Error of sub-base = $\frac{1}{200}R$.
3. Error in appreciating each angle not to exceed —
 Without telescope..... 1' 0"
 " 0 20.
4. Error in position of instrument on the picket-head not to exceed $\frac{1}{2}$ inch at each angle.

1. Instrumental error not to exceed $\frac{1}{4}$ per cent. when $R=20B$.
2. Error of sub-base = $\frac{1}{100}B$.
3. Error in appreciating each angle not to exceed—
Without telescope..... 1' 0"
With "..... 0 20.
4. Error in position of instrument on the picket-head not to exceed $\frac{1}{4}$ inch at each angle.

¹ The instrumental error of every artillery range-finder is ascertained periodically, and recorded in the history sheet of the instrument kept by the battery to which it belongs.