JOURNAL

OF THE

FRANKLIN INSTITUTE.

OF THE STATE OF PENNSYLVANIA,
FOR THE PROMOTION OF THE MECHANIC ARTS.

VOL. CXVII.

MAY, 1884.

No. 5.

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THE MOST ECONOMICAL POINT OF CUT-OFF.

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In complying with Professor Marks' request in the last February number of this JOURNAL, I hope to remove the grounds for the imputation of any unfairness which he has been pleased to impute to me. In his communication of December last, equation (1) is

$$y = \frac{\text{Mean effective pressure} \times V}{\text{Cost steam} + \text{constant charges} - \text{value saved by compression}} \quad (1)$$

and equation (6) deduced from (1) is

$$y = \frac{eP_b \left[1 + \log. \frac{1}{e} - \frac{k}{e} \right] - B \left[1 - b \left(1 + \log. \frac{b}{k} \right) \right]}{e + \frac{C}{D} \left\{ eP_b \left[1 + \log. \frac{1}{e} - \frac{k}{e} \right] - \left[B - b \left(1 + \log. \frac{b}{k} \right) \right] - b \frac{B'}{P_b} \right\}} \quad (6)$$

The second term of the denominator of equation (1) is *constant charges*, while the second term of the denomination of equation (6) is a function of *e*, a variable. This apparently changes the character of equation (1) and requires explanation. We find, however, if he explains that point, and if we admit his assumption that his "so called" constant charges are a fraction only of the cost of steam, that they

will disappear in determining the maximum of his expression as he claims; for it is equivalent to a function of the form

$$y = \frac{f(\varphi)}{F(\varphi) + af(\varphi) + c},$$

from which we find for a maximum, the relation,

$$[F(\varphi) + af(\varphi) + c]f'(\varphi) - f(\varphi)[F'(\varphi) + af'(\varphi)] = 0;$$

$$\text{or} \quad F(\varphi).f'(\varphi) + cf'(\varphi) - f(\varphi).F'(\varphi) = 0;$$

from which $af(\varphi)$ has disappeared; and it is apparent from the process that the only law which will cause the second term of the denominator of the fraction to disappear, is to make the ratio between it and the numerator constant. If this, then, were the only law, there would be no other solution. If there be any variation from this law, my conclusion, that the solution is *special*, is correct.

In the last February number, Professor Marks states: "That the constant charges are dependent solely on wages, cost of oil, interest, insurance, taxes and probable deterioration of machinery, and independent of all else."

"The person designing an engine knows what horse-power is expected of it."

On page 82, he says: "How is John Doe to get 150 horse-powers for the least sum of money?" "There is no answer save this: *by using the least possible steam per horse-power per hour.*"

On page 84: "I have assumed John Doe to be a manufacturer, who wishes to do that thing which at the end of one year or ten years is most profitable, and not a person cramped for capital."

The problem before us, then, is the *designer's* problem, and not the *owner's* problem.* John Doe having confidence in the Professor, secures his services in selecting the engine which will be the most economical for delivering 150 horse-powers for ten years. From the known initial pressure (say 90 pounds), the back pressure (say 5 pounds), and the clearance (which he assumes to be small), he finds by means of equation (8)

$$e = \frac{B}{P_b} = \frac{5}{90} = \frac{1}{18},$$

* The *designer's* problem consists in making an engine which will deliver a given number of horse-powers most economically; the *owner's* problem consists in delivering the greatest number of horse-powers from a given plant with the most profit to the owner. When the designer has properly solved his problem, it is merged into that of the owner's.

for the approximate value of the cut-off; and for a more accurate value

$$e = \frac{B \left[1 - b \left(1 + \log. \frac{b}{k} \right) \right]}{P_b} + k - b \frac{B}{P_b} \log. \frac{1}{e}, \quad (7)$$

which for want of data we cannot reduce numerically, and hence will assume it as $\frac{1}{17}$, or even $\frac{1}{18}$. These men go into the market, and with the given data, the Professor selects the engine that will produce the 150 horse-powers "*by using the least possible steam per horse-power per hour*;" excepting that, for commercial reasons, he quietly uses a cut-off of $\frac{1}{10}$ * as the basis of selection.

Doe learns from Smith, the proprietor, that the price is \$4,000. Doe questions the wisdom of putting so much money into an engine for his business—at any rate, he intends to see if he cannot do better. Smith, desirous of securing a customer, after learning the capacity of the engine desired, offers one for \$3,000, which he guarantees will deliver the required power. Mr. Doe then enters into a conversation with these men, which we may imagine runs as follows:

Doe. Do you think, Professor, that the smaller engine will do my work?

Professor. I have no doubt of it; but it will cost you more for steam, every day you use it, than the larger one; and I suppose you want to get your steam for the least money; and I have shown in my publications that the cheapest engine in the long run is one "*that will use the least possible steam per horse-power per hour*," and the larger engine will do it, and the smaller one will not.

Doe. Steam! I have my boilers set, and they will furnish the steam; it is an engine I am now after.

Professor. Yes, my good sir, but do you not understand that to make steam requires fuel, and fuel costs money; and an engine that uses more steam than is necessary, wastes money; and although this appears evident, some writers "have deceived themselves, and perpetrated the absurdity of saying that you can save money by using more steam than is really necessary to do the work." (Feb., 1884, p. 83.)

Doe. My good friend, you may yet regret criticising those writers in this manner; for sometimes it is *economy to waste*. For instance, I could have saved \$1,000 in building and stocking my shop had I been six months longer about it; but the interest alone would amount to

* JOURNAL FRANKLIN INSTITUTE, Dec., 1883, and Jan., 1884, p. 41.

more than that, and I expect to make twice that amount in that time. It is barely possible that those writers saw that by *losing* in the cost of steam they could *gain more* at some other point in this problem. But never mind that now, for we must select our engine. Please tell me how the smaller engine can do the work.

Professor. By cutting off later in the stroke. By following the piston with the full pressure of the steam further in the stroke, more power will be secured; but by so doing you will reject more heat in the steam at the exhaust, thus requiring a larger amount of steam, and as we have said, more fuel to do the same work; and I suppose you want to get your 150 horse-powers "*by using the least possible steam per horse-power per hour*;" and this will govern the point of cut-off, and consequently the proper size of the engine to be selected.

Doe. But this \$1,000 difference in the first cost is too great for delivering the same power. Mr. Smith, I think your price of the larger engine is too high. Our Professor says: "The cost of an engine is not proportional to the size of the cylinder, but rather to the amount of labor put upon the engine as a whole. (February 1884, p. 84.)

Smith. The Professor is dealing in "general assertions;"* he ought to give "some law, equation, or example." He does not even intimate whether the work put upon an engine as a whole, in practice, bears any relation to the volume of the cylinder. Should I charge you \$5,000 for the larger engine, it might not conflict with his statement. The fact is he denies the existence of one element of cost, but in the same sentence admits another element, which opposes his own denial. My experience in the business, and observation of general practice, enables me to say, confidently, that the cost of well-made engines varies with† the volume of the cylinder; that is, an engine with a large cylinder costs more than with a small one. I am able to refer you to some tabulated results on this point by Professor Trowbridge, in the Transactions of the American Society of Mechanical Engineers for 1882, page 231, and of remarks thereon by Professor Thurston, *ibid*, 1882, Vol. III, p. 289, showing that for engines above 50 horse-power, the cost is approximately as the volumes of the cylinders. The fact is, Professor Marks advocates a short cut-off, and requires a correspondingly large cylinder. You could probably get 200 horse-

* February, 1882, p. 81.

† In the January number I used *with* in the sense of a functional relation, without deciding whether the relation was direct or inverse. D. W.

powers out of the one he has selected, other things being properly proportioned.

Professor. All the dealers charge in that way, and I suppose we must submit to it.

Doe. The question of the cut-off grows in interest. I see our Professor says: "The point of cut-off has, practically, nothing to do with the constant charges, save so far as it determines the volume of the cylinder required."

Smith. I do not presume to understand all that is written about the cut-off; but there is something amusing in this statement of the Professor. As before, he denies one proposition, and then, in my opinion, admits another element which opposes his denial; for in my opinion, the size of the cylinder, involving as it does the cost of the engine, does affect the economical point of cut-off by making the constant charges, so called, different from what they otherwise would be; for the truth of which I appeal to you in the case of the two engines now before you. The consideration of the theoretical effect of constant charges, I leave to the Professor. There is another peculiar expression, "the volume of the cylinder is determined by the constant charges." Does the Professor come here with the constant charges known, and select a cylinder such that the interest, repairs, etc., thereon will produce these charges?

Professor. I should have said that the constant charges are dependent upon the volume of the cylinder.

Doe. Exactly. You have now touched a practical point; for I desire to make those constant charges, as you call them, as small as I can, for I will be obliged to pay for them in some way.

Professor. But remember that the 150 horse-powers will cost you more for fuel with the smaller engine.

Doe. How much more? I am willing to *lose* on the cost of steam, if I can save more on these constant charges. This \$1,000 in first cost ought to have some influence on the ultimate economy.

Professor. It would seem so at first sight, but I have considered a case in my discussion of this subject. I observed that by saving \$1,000 in first cost of the engine, you could afford to expend more money in producing steam; but to raise more steam, economically, would require larger boilers, and so there would be nothing saved. (Feb. 1884, p. 84.)

Professor (figuring). At $\frac{1}{10}$ cut-off, $13.8 \left(1 + \frac{34}{100-34}\right) = 20.91$ pounds; at $\frac{1}{5}$ cut-off, $15.6 \left(1 + \frac{26}{100-26}\right) = 21.07$ pounds. This

gives 20.91 pounds for the larger engine, and 21.07 pounds for the smaller; figures very different from those produced by Mr. Smith, and favorable to the larger engine.

Doe. How much will the difference amount to per year?

Professor. $(21.07 - 20.97 = 0.16) \times 150 \text{ IP} \times 10 \text{ hours} \times 300 \text{ days} \div 1,000 = 72$ thousand pounds of steam, which, at 25 cents per thousand, will be \$18 per year. But the authors differ considerably in regard to the results of condensation. I see in Barrus' book, from which you took the above figures, that the difference in favor of the larger engine is for $\frac{1}{10}$ and $\frac{1}{5}$ cut-off, respectively, 0.97 of a pound of steam per horse-power per hour, making a difference of about \$110 per year from this cause only.

Doe. I see that there is a possibility of considerable loss from this cause, but the most unfavorable showing does not equal the \$160 saved in "constant charges," and I have decided to order the smaller engine.

Professor. But does not your view of the case warrant you in ordering a still smaller one?

Doe. Possibly; but the practical result is this: you selected an engine according to your theory; Mr. Smith selected another according to his general practice, and, by the help of both of you, I have satisfied myself that Smith's is the more economical engine of the two for me to purchase for my business. If another size were selected, we would apply to it the same mode of reasoning, and ascertain if the money *saved* by the interest, repairs, etc., equaled that *lost* by fuel and other conditions; and, by trying a few engines in this way, we would at last find one which, for such a cut-off as would deliver the required horse-powers with the given steam pressure, would be the most economical for me to purchase, considering first cost, cost of fuel and constant charges. Such an engine is said to have "the most economical point of cut-off." Your method, apparently, does not contemplate this process, and evidently needs revision.

The Professor retires, disappointed at the result. His rule had not purchased the engine, and his theory had received some fatal blows.

On his way he met a friend who had given some attention to the problem of the cut-off, and they entered into a conversation.

Professor. Would you believe it? John Doe has just purchased an engine for his shop that will cost him, say \$100 per year more for steam than it would for a certain other engine which I selected for him.

Friend. I thought John Doe was a shrewd business man. How did it happen?

Professor. Smith convinced him that it would be more economical for him in the long run.

Friend. These salesmen are sharp; but what arguments did he use?

Professor. Why, he convinced him that his saving in interest, repairs, deterioration, oil and wages would exceed the extra cost for fuel.

Friend. Was not that correct?

Professor. Really, I could not gainsay it, with the conditions and figures they gave; but it gives a very different value for the cut-off than found by my theory. I have insisted that the "constant charges" disappear in determining the proper point of cut-off; but according to their showing they have considerable influence. Doe's boilers are already set, and, fortunately for him, they prove to be large enough to furnish the required steam economically. Had they been much smaller, he would have been obliged to have taken the engine of my selection, or expended all he saved on the engine in putting in larger boilers.

Friend. Exactly, and I have noticed your difficulty on this point. The problem is one of maxima and minima, and the fact is you have assumed, beforehand, certain quantities as constant, which, in the process of designing, are variables; and these you enter in your function before seeking the maximum, which, as a mathematical operation, you know, even better than I, to be erroneous, and will fail to give the true maximum. Your admission just made in regard to the boilers is fatal to your theory; for it shows that, in designing an entire plant, the most economical point of cut-off will involve the first cost of the boilers while you declare it to be independent of all such considerations. You have just seen, also, that after they are set, it has had its influence in selecting the engine. Again, you say " C = the constant charges in dollars and cents, and is assumed to remain constant after once being fixed upon." (Feb., 1884, p. 85.) Please inform me

how you could fix the charge for interest before you knew the cost of the engine.

Professor. I assumed that "the constant charges may be determined regardless of the cost of steam." (Feb., 1884, pp. 83 and 84.)

Friend. The answer is rather evasive; for I referred to the cost of engine—not of steam; but, as they are in a measure related, I will say that your assumption is not admissible. The smaller the engine, the less the first cost, and the interest on the money correspondingly less; while the cost for steam will be more so that this quantity—which you have considered constant—*varies* inversely with the cost of steam. Similarly, the cost for repairs and deterioration *vary* in the same sense but not in the same ratio, nor necessarily according to the same law. After the engine is set, the interest is *fixed*, whether the engine be run 200 or 300 days, or if it be run night and day; while the repairs will depend chiefly upon the amount it is used. I am aware that the problem involving such contingencies cannot be solved, but I mention them to show more clearly, if possible, that the proper point of cut-off is not independent of these things.

Professor. But you must admit that after the engine is set, all the charges which can be considered constant are fixed, and must be some "fraction of the mean effective pressure." That John Doe, *after* "his purchase and arrangements are made, has done with constant charges, and must look to economy of steam as the only means of saving money." (Feb., 1884, p. 83.)

Friend. I think our friend, John Doe, will not agree with you. He will inform you, even after his engine is set, that he "has not done with constant charges," that he finds a continual draft on his purse for interest, repairs, oil, insurance and taxes; and that he has no commendation for the man who fixes one dollar upon him unnecessarily. True, if he has been misled in his purchases, and been made to pay an unnecessary amount in first cost, he will be obliged to endure it; but it is wiser to consider all these elements *before* the purchase. But your plea, that they become fixed *after* the purchase, is fallacious from a mathematical point of view; you might as well argue that the cut-off becomes *fixed* after you have found it, or that any other element is fixed. It is unnecessary to argue this point; it is only necessary to convince you that these so-called "constant charges" are variables in the process of designing, to cause you to abandon your rule, giving as it does a cut-off of $\frac{1}{10}$ to $\frac{1}{20}$, which you have insisted up to the present

time to be "practically accurate within the widest range," after establishing certain relations in regard to the steam. (Jan., 1884, p. 1.) When thus convinced, you will admit that "economy of fluid" is only one element of the general problem of economy; that by saving steam *only*, you will lose money. You will then see that your arraignment of living and deceased writers is premature, to say the least. You will also find that you expose yourself to attacks by critics; for instance, you stated that "the volume of the cylinder is dependent upon the constant charges," that "the cut-off is independent of those charges"—conditions incompatible; that whenever it is possible to fix the constant charges on and to design an engine regardless of its horse-power, then will the proof lack in generality" (p. 82), an admission which is fatal to your theory. For in the next paragraph you assert that several of the charges are independent of the horse-power within certain limits, and that for these at least the proportion on page 86 cannot be true; but, even if true for these, the same proportion cannot be true for the other charges, as interest, for instance.

Finally, you are already aware that Rankine made the most general solution yet attained, in which the several elements above mentioned are treated as variables; a special case of which includes your problem of "economy of steam only," and of which the general case includes both the designer's and owner's problems. You are also aware that Denton and Wolff unearthed this solution, so to speak, enlarged upon and amplified it, by discussing it in their papers and applying the method to a large number of existing engines;* and, although it is not claimed by these writers that the method possesses mathematical exactness, yet it is claimed that the defect in the process lies chiefly in the data rather than in the method. In view of these facts, you may at last consider it advisable to take to yourself the advice you have given to another, and "give the most careful consideration to the point at issue."

[NOTE.—Professor Rankine stated his problem thus: "By increasing the ratio of expansion in a Cornish engine, the quantity of steam required to perform a given duty is diminished; and the cost of fuel and of the boil-

† Transactions of the American Society of Mechanical Engineers, 1881, page 147 and page 281; republished in the *American Engineer*, 1881, June, July, August and November; and in the Transactions of the Institution of Civil Engineers (London), 1881-82, Vol. LXVIII, Part II, p. 75; Part III, p. 44.

ers is lowered. But at the same time, as the cylinders and every part of the engine is increased, must be made larger to admit of a greater expansion, the cost of the engine is increased. It thus becomes a problem of maxima and minima to determine what ratio of expansion ought to be adopted under given circumstances, in order that the sum of the annual cost of fuel, and the interest of the capital employed in construction may be the least possible as compared with the work done."—*Phil. Mag.*, 1854, (2), p. 345.

This problem Rankine solves in his usual masterly manner, and then shows how to design a 100 horse-power engine, according to his method, making the cost of engine and of boilers variable; deducing the point of cut-off, the area of the piston, and mass of the boilers. In this he uses the graphical method. In the owner's problem, the cost of the plant being known, the interest will be known, and the cost of fuel and point of cut-off are the variables, but the *method* of finding the point of cut-off is the same as before.]

Stevens' Institute of Technology, Hoboken,

February, 15, 1884.

Critical Point of Liquefiable Gases.—Jamin believes that gases are liquefiable at every temperature, when the pressure is sufficient. He defines the critical point as the temperature at which a liquid and its saturated vapor have the same density; but the general law of vaporization is not suddenly interrupted; the liquid continues to be at its point of ebullition and maximum tension. If it ceases to be visible it is because it is mixed with the gas, in which it swims on account of the equal densities, and when the temperature continues to increase the tension also increases until the liquid is entirely volatilized. Then the space ceases to be saturated and there is only a dry vapor, or gas, far removed from its point of liquefaction. In general, a saturated vapor is distinguished from its generating liquid by its smaller density and its latent heat; in the present case the densities are equal and there is no latent heat, since the volume does not change and there is no work of expansion. Hence, at the critical point, nothing distinguishes the liquid from its vapor, neither the tension, the density, the constituent heat, the appearance, nor any other known property. Jamin submitted these views to Cailletet, with the statement that if his hypothesis was true the liquefaction would be retarded by replacing the air in his mixture of air and carbonic acid by some lighter gas, for instance, hydrogen. Cailletet performed the experiment and found that the results were such as had been predicted.—*Comptes Rendus*, May 21, 1883.