Formation of entangled polaritons in doped resonant medium

A.V. Prokhorov*, I.O. Barinov, A.P. Alodjants, S.M. Arakelian
Vladimir State University, Gorkogo Str. 87, Vladimir, 600000, Russia

ABSTRACT

The nonlinear theory of formation of entangled polaritons of probe field in the case of Raman interaction scheme in Y_2SiO_5 crystal doped by ^{59}Pr atoms is developed. The model of polariton waveguide with doped medium inside based on Fabry-Pérot interferometer is suggested. The opportunity of dynamic amplification of entangled polaritons in such a system is shown.

Keywords: Raman interaction, polaritons, waveguides, entanglement, quantum information

1. INTRODUCTION

For many years exploration of quantum statistic and thermodynamic properties of polaritons (bosonic quasi-particles representing a linear superposition of electromagnetic field and the macroscopic (coherent) excitations) is the subject of intense research. Such coupled atom-field states can be used in quantum information tasks. In this case dynamic, dissipative and correlation properties of polaritons in such systems should be taken into account at the same time. Such problems as: controlling polariton dynamics in “slow” light limit in article ¹, opportunity of obtaining negative values of polaritons group velocity in ^{2,3}, formation of entangled coupled polaritons ⁴, effect of coherent amplification of polaritons in ^{5,6} have been well investigated separately. The relevance of the latter task, in particular, is associated with perspective realization and practical application of polariton low-threshold lasers. Great progress has been made in this area. Thus, in article ⁷ the emission of coherent polaritons in GaN sample with generation threshold of 1 mW at room temperature has been produced and in ⁸ model of miniature (diameter of generation range of $400\text{ }\mu\text{m}$) semiconductor polariton diode with 6 mW threshold power has been made. Coherence of radiation caused by such sources is associated with the emission of low dispersion branch polaritons from the main state of high-temperature Bose-Einstein condensation using parametric polariton-polariton scattering in two-dimensional trap. It should be mentioned that in the earlier article ⁹ semiconductor exciton waveguide has been experimentally obtained. It produces laser generation under low-density optical pumping.

However, potential of wide implementation of such compact low threshold lasers in quantum optics make new requirements to properties of radiation caused by such sources. Here we deal with such resources of generation as the use of non-classical both optical ¹⁰ and atomic-optical states ⁴ taking into account dynamic effect in the process of polaritons amplification. Besides, for development of the new generation of quantum optical integrated circuits it is necessary to solve the technical task of integrating (as a single module) of optical medium for transfer of quantum information with layered semi-conducting structures for processing this information ¹¹.

In this paper we show the possibility of generation of non-classical entangled states of polaritons formed in nonlinear Raman interaction scheme, where photons of probe field interact with atom excitations in Y_2SiO_5 crystal, doped by ^{59}Pr atoms ($\text{Pr}:\text{YSO}$). Dynamic model of polariton waveguide based on Fabry-Pérot interferometer with doped medium inside is suggested. The model predicts the amplification of entangled Raman polaritons taking into account their group delay in the medium. Control of predicted processes can be made by intensity change and frequency detuning of pump field at resonance.

* avprokhorov@vlsu.ru

Figure 1. (a) Λ -scheme of atom-field interaction for ^{59}Pr atoms; (b) model of polariton waveguide based on doped ^{59}Pr atoms Y_2SiO_5 crystal.

2. GENERATION OF RAMAN POLARITONS IN THREE-LEVEL MEDIUM: PRINCIPAL RATIOS

Let us consider atom-optical interaction scheme given in Fig.1a, which contains corresponding symbols for characteristic parameters. There are two fields – classical pump wave with frequency ω_p and wave vector $\vec{k}_p$ and weak probe pulse at central frequency ω_s (Stokes component) with wave vector $\vec{k}_s$. These fields take part in stimulated Raman scattering under interaction with three-level (states $|a\rangle$, $|b\rangle$, $|c\rangle$) ^{59}Pr atoms doped in Y_2SiO_5 crystal^{12,13}, which placed between two mirrors of Fabry-Pérot resonator with axial extension z (Fig.1b). Probe field is entered into waveguide via prism and formation of (*Raman*) polaritons, propagated at an angle θ relative to the pump field, takes place as a result of mixing of its photons with atomic excitations at the transition $|a\rangle \rightarrow |b\rangle$. Inverse transformation (re-emission) of polaritons also occurs by prism in which changing wave vector of probing photons under reflection results in radiation decay of polaritons.

We suppose that initially most of the atoms are in the ground state $|a\rangle$ – see¹⁴. In this case we may assume $|\bar{a}_j|^2 \approx 1$, $|\bar{b}_j|^2 \approx 0$ for average populations $|\bar{a}_j|^2$, $|\bar{b}_j|^2$ of levels $|a\rangle$ and $|b\rangle$ in the initial state and introduce Bose annihilation operators $\Phi_{1(2),k}$ for such polaritons in atomic medium as:

$$\Phi_{1,k} = \mu f_k + \nu a_k b_k^+, \quad \Phi_{2,k} = \nu f_k^+ + \mu a_k^+ b_k, \quad (1a,b)$$

where f_k , f_k^+ ; a_k , a_k^+ ; b_k , b_k^+ are annihilation and creation operators for photons of probe field and atoms at levels $|a\rangle$ and $|b\rangle$ in k -mode, respectively. The given ratios correspond to two types of excitation occurring in the system and comply with commutation relation $[\Phi_{i,k}; \Phi_{j,k}^+] = \delta_{ij}$, $i, j = 1, 2$ when coefficients μ , ν (the same as Hopfield ones) define partial contributions of photon and atom components to formation of polaritons consistent with the following condition $\mu^2 - \nu^2 = 1$; in general μ , ν may be complex quantities.

Further we may assume that optical field and atomic excitations are described as Gaussian states, with aid of definitions (1) let us introduce the next group of Hermitian operators for amplitude-phase quadrature¹⁵ in two-mode system of polariton excitations (1):

$$X_{1,k} = \Phi_{1,k} + \Phi_{1,k}^+ \quad (2a)$$

$$Y_{1,k} = -i(\Phi_{1,k} - \Phi_{1,k}^+) \quad (2b)$$

$$X_{2,k} = \Phi_{2,k} + \Phi_{2,k}^+ \quad (2c)$$

$$Y_{2,k} = -i(\Phi_{2,k} - \Phi_{2,k}^+) \quad (2d)$$

For consideration of studying cross-correlation properties of two types of polaritons (1) on the basis of (2) let us determine the following parameters:

$$V_k^0 \equiv V_k^\theta \Big|_{\theta=0} = \frac{1}{4} \{ \Delta^2 (X_{1,k} + X_{2,k}) + \Delta^2 (Y_{1,k} - Y_{2,k}) \} \quad (3a)$$

$$V_k^{\pi/2} \equiv V_k^\theta \Big|_{\theta=\pi/2} = \frac{1}{4} \{ \Delta^2 (Y_{1,k} + Y_{2,k}) + \Delta^2 (X_{1,k} - X_{2,k}) \} \quad (3b)$$

where $\Delta^2 A$ specifies dispersion of respective quantity A . Expression (3) in general corresponds to different combinations of dispersion of sum and difference of measured quadratures and may be experimentally observed in the way of so-called unsqueezed spectrum for (3a) and squeezed one for (3b) by means of the change in phase difference between modes in the process of their measurement¹⁵. In case of detecting these spectra for two types of polaritons (1) such measurements should be made indirectly relating to measurement of corresponding quadratures of optical field upon polariton emission at the output of atomic medium. The following two inequalities may be regarded as sufficient criterion for entanglement of polariton modes:

$$V_k^0 < 1, \quad V_k^{\pi/2} < 1. \quad (4a,b)$$

Fulfillment of the first one (4a) gives the idea of the presence of non-classical amplitude correlations and the second shows the availability of phase correlations in two-mode system.

Let us consider the Hamiltonian of atom+field system, which for atom optical interaction scheme (Fig.1a) has the following view:

$$H = \sum_{\vec{k}} \left(\hbar \omega_s(k) f_{\vec{k}}^+ f_{\vec{k}} + \sum_{j=1}^N \left(E_{ja} a_j^+ a_j + E_{jb} b_j^+ b_j + E_{jc} c_j^+ c_j + \hbar \kappa \left(a_j^+ c_j e^{i\omega_p t} + c_j^+ a_j e^{-i\omega_p t} \right) + \frac{\hbar g}{\sqrt{N}} \left(f_{\vec{k}}^+ b_j^+ c_j + c_j^+ b_j f_{\vec{k}} \right) \right) \right), \quad (5)$$

where operators a_j (a_j^+), b_j (b_j^+) and c_j (c_j^+) describe annihilation (creation) of j -atom with wave vector $\vec{k}$ at the levels of $|a\rangle$, $|b\rangle$ and $|c\rangle$ respectively; $f_{\vec{k}}$ ($f_{\vec{k}}^+$) – the same for photons of probe field; E_{ja} , E_{jb} and E_{jc} – energies of j -atom at these levels; N – total number of atoms, m_{at} – atomic mass; $\hbar$ – Planck constant; g – constant of atom-field interaction at the $|b\rangle \rightarrow |c\rangle$ transition, κ – Rabi frequency for $|a\rangle \rightarrow |c\rangle$ transition. Let us pass to new variables:

$$\langle a_j \rangle = \bar{a}_j e^{-iE_{ja}t/\hbar - i\delta_j t}, \quad \langle b_j \rangle = \bar{b}_j e^{-iE_{jb}t/\hbar}, \quad \langle c_j \rangle = \bar{c}_j e^{-iE_{jc}t/\hbar + i\Delta_j t}, \quad \langle f_{\vec{k}} \rangle = \bar{f}_{\vec{k}} e^{-i\omega_s t}, \quad (6)$$

where coefficients at harmonic factors are average atomic and photonic field amplitudes, respectively; δ_j , Δ_j –

detunings from the conditions of precise resonance (see Fig.1a). Assuming that $\frac{d\bar{c}_j}{dt} = 0$ let us exclude adiabatically populations of atoms at higher $|c\rangle$ level. In this case three-level atom medium may be considered in two-level approximation where the condition $\gamma_a, \gamma_b \ll \gamma_c$ (compare with¹⁶) is solved where γ_i ($i = a, b, c$) characterizes relaxation processes for atoms at corresponding levels.

As a result we can obtain the following system of differential equations for average values:

$$i \frac{d\bar{f}_{\vec{k}}}{dt} = -i\gamma_{ph} \bar{f}_{\vec{k}} - \frac{1}{\sqrt{N}} \sum_j \lambda_j \bar{b}_j^* \bar{a}_j - \frac{1}{N} \sum_j \frac{g^2}{\Delta_j} |\bar{b}_j|^2 \bar{f}_{\vec{k}}, \quad (7a)$$

$$i \frac{d\bar{a}_j}{dt} = -\omega_{ja} \bar{a}_j - \frac{\lambda_j}{\sqrt{N}} \bar{b}_j \bar{f}_{\vec{k}}, \quad (7b)$$

$$i \frac{d\bar{b}_j}{dt} = -i\gamma_b \bar{b}_j - \frac{\lambda_j}{\sqrt{N}} \bar{f}_k^* \bar{a}_j - \frac{g^2}{\Delta_j N} |\bar{f}_k|^2 \bar{b}_j, \quad (7c)$$

where the new effective parameter of interaction of atoms and probe field $\lambda_j = \frac{\kappa g}{\Delta_j}$, effective detuning $\omega_{ja} = \tilde{\delta}_j + \frac{\kappa^2}{\Delta_j}$, rate of relaxation process for the field in resonator is γ_{ph} , as well as $\tilde{\Delta}_j = \Delta_j - i\gamma_c$, $\tilde{\delta}_j = \delta_j + i\gamma_a$ are introduced. While working in the low excitation density limit $N_{ph} \ll N$, where $N_{ph} = \langle f_k^+ f_k^- \rangle$ is average number of photons in probe field, let us pass to new variables $\phi_j = \bar{a}_j^* \bar{b}_j$, $\phi_j^* = \bar{b}_j^* \bar{a}_j$ that characterize perturbation of atomic medium for levels $|a\rangle$ and $|b\rangle$ we will obtain the following system of equations:

$$i \frac{d\bar{f}_k}{dt} = -i\gamma_{ph} \bar{f}_k - \frac{1}{\sqrt{N}} \sum_j \lambda_j \phi_j^*, \quad (8a)$$

$$i \frac{d\phi_j^*}{dt} = -(\omega_{ja} + i\gamma_b) \phi_j^* + \frac{\lambda_j}{\sqrt{N}} \bar{f}_k. \quad (8b)$$

In this paper we select ^{59}Pr atoms for modeling because the working level $|c\rangle$ in Fig.1a has a long life time¹³ which enables to produce further regimes in suggested waveguides even at room temperatures. They also have the small value of dipole moment at $|b\rangle \rightarrow |c\rangle$ transition ($\mu_{bc} = 3 \cdot 10^{-33} \text{ C} \cdot \text{m}$, transition wave length $\lambda \approx 606 \text{ nm}$). This situation does not require taking into account the local response of the medium at small densities of optical pumping.

Taking into account the momentum conservation law for transition frequency between levels $|a\rangle$ and $|b\rangle$ we will have:

$$\omega_{ba} \equiv \frac{E_{jb} - E_{ja}}{\hbar} = \frac{E_b - E_a}{\hbar} + \frac{\hbar k_p^2}{2m_{at}} + \frac{\hbar k_s^2}{2m_{at}} - \frac{\hbar \vec{k}_s \cdot \vec{k}_p}{2m_{at}}, \quad (9)$$

where recoil energy of atoms has been taken into account. At the beginning we assume that there are no spatial restrictions associated with waveguide mirrors and interaction Λ -scheme is realized in three-dimensional space of doped medium. Stationary solutions of equations (8) determine two branches of elementary (microscopic) excitations in the system of probe polaritons with characteristic frequencies:

$$\Omega_{1,2}(k) = \frac{1}{2} \left[\omega_+ + \frac{\kappa^2 \Delta}{\Delta^2 + \gamma_c^2} + i\Gamma_+ \pm \sqrt{(D + i\Gamma_-)^2 - 4|\lambda|^2} \right], \quad (10)$$

where $\omega_{\pm} = \omega_{ba} \pm \omega_{ph}$, $\Gamma_{\pm} = \Gamma_{at} \pm \gamma_{ph}$; $\Gamma_{at} = \gamma_a + \gamma_b + \frac{\kappa^2 \gamma_c}{\Delta^2 + \gamma_c^2}$ is effective atomic rate decay, $D = \omega_- + \frac{\kappa^2 \Delta}{\Delta^2 + \gamma_c^2}$ is

effective detuning, $\omega_{ph} = \omega_p - \omega_s$ is effective frequency of photon field for two-level approximation of atomic system.

Principal difference of dispersion relations (10) for examined polaritons of probe field in Λ -scheme of interaction from corresponding ratios of polaritons dispersion in two-level medium without inversion (cf. ¹⁷) is that in case of strong-coupling regime $|\lambda| \gg \Gamma_-$ Rabi splitting of atomic levels $|a\rangle$, $|b\rangle$ becomes imaginary and losses (amplification) of such polaritons can be observed. The expressions for polariton coefficients μ and ν using equations (8) and taking account (10) may have the following form:

$$\mu = \left(1 - \frac{D + i\Gamma_- + 2\omega_s - \sqrt{(D + i\Gamma_-)^2 - 4\lambda^2}}{\omega_+ + \frac{\kappa^2 \Delta}{\Delta^2 + \gamma_c^2} - i\Gamma_- + \left(\sqrt{(D + i\Gamma_-)^2 - 4\lambda^2} \right)^*} \right)^{-1/2}, \quad \nu = \sqrt{|\mu|^2 - 1} \quad (11)$$

Figure 2. (a) Dispersion relations $\Omega_1(k_s)$ and $\Omega_2(k_s)$ for polaritons of probe field and (b) dependences of correlation parameters V_k^0 and $V_k^{\pi/2}$ on normalized difference of wave vectors $\Delta k_s / k_0$ in Λ -scheme of atom-field interaction. At insertion in (a): dependences of absorption coefficients $\text{Im}\Omega_1(k_s)$ (solid line) and $\text{Im}\Omega_2(k_s)$ (dotted line) in coincidence region of polariton branches.

It should be mentioned that coefficients μ and ν do not only determine the contribution of photon and atom components to polariton formation but also set rate of absorption (amplification) of these components. In general it is analogues to emergence of complex mass in¹⁸.

The spectrum (10) for Raman polaritons via normalized wave vector $\Delta k_s / k_0$ is given in Fig.2a, where $\Delta k_s = k_s - k_0$ and $k_0 = (\omega_p - \omega_{ba} - \Delta) / c$, c is light speed in vacuum. The polaritons are formed in atomic medium (^{59}Pr with relative concentration of 0.007%) in Y_2SiO_5 matrix¹³. Splitting frequency between $|a\rangle$ and $|b\rangle$ amounts to $\omega_{ba} = 6.4 \cdot 10^7 \text{ s}^{-1}$, then we select relaxation speed at optical transition as $\Gamma_{at} \approx \gamma_c = 1.47 \cdot 10^7 \text{ s}^{-1}$, intensity of pump field as $I_p = 0.96 \text{ W/cm}^2$, detuning frequency of pump field at resonance as $\Delta = 6 \cdot 10^4 \text{ s}^{-1}$. When we take the number of atoms in interaction range as $N = 3 \cdot 10^4$, the number of photons in testing pulse at input as $N_{ph} = 100$ and under

selected conditions the optical coupling constant $g = \mu_{bc} \sqrt{\frac{\omega_{bc} n_3}{2\hbar \epsilon_0 N}}$ will be equal to $2.8 \cdot 10^7 \text{ s}^{-1}$, the value of local

response will be only $\chi = 5.3 \cdot 10^3 \text{ s}^{-1}$, where n_3 is atom concentration. Usually, two-level scheme for polaritons of upper branch with grown wave vector is characterized by gradual transition from atom-like excitations at ω_F frequency to photon-like ones with asymptote of $\omega = ck_s / \sqrt{\epsilon}$ but only for low branch. In this case there is forbidden frequency zone between two polariton branches. An experimental results (see¹⁹ for pure quartz) correspond to full reflection of light from medium and impossibility of mixing polaritons of two different types.

In the case under consideration (with Raman polaritons) transition between two polariton branches according to excitation type occurs, actually, by leaps in the narrow range of wave vectors where there is coincidence in the travel

of low and upper polariton branches (see Fig.2a). This is the consequence of the condition $|D^2 - \Gamma_-^2 - 4|\lambda|^2| < 2|D\Gamma_-|$ for which radicand expression in (10) becomes less (or equal) to zero. Thus, the forbidden frequency zone disappears and it is substituted by interval of wave vectors having two types of polaritons with equal frequency. This phenomena is of special interest for quantum information processing.

Generally, expression (10) may be factorized into real part ($\Omega_{1,2}^{(R)} = \text{Re}(\Omega_{1,2})$) and imaginary one ($\Omega_{1,2}^{(I)} = \text{Im}(\Omega_{1,2})$) in the following form:

$$\Omega_{1,2}^{(R)} = \frac{1}{2} \left(\omega_+ + \frac{\kappa^2 \Delta}{\Delta^2 + \gamma_c^2} \pm \sqrt[4]{\alpha^2 + \beta^2} \cos(\Phi) \right), \quad (12a)$$

$$\Omega_{1,2}^{(I)} = \frac{1}{2} \left(\Gamma_+ \pm \sqrt[4]{\alpha^2 + \beta^2} \sin(\Phi) \right), \quad (12b)$$

where parameters $\alpha = D^2 - \Gamma_-^2 - 4\lambda^2$ and $\beta = 2D\Gamma_-$ characterize Rabi splitting with the absorption of probe field; $\Phi = \frac{\varphi}{2} + \pi l$ ($l = 0, 1, \dots$) – phase, specifying the multivalent function (10), that is the possibility of existing two different solutions for $\Omega_{1,2}^{(R)}$ and $\Omega_{1,2}^{(I)}$ with the same parameters. Then for determinacy we assume $\Phi = \frac{\varphi}{2}$. In this case φ is determined as:

$$\varphi = \begin{cases} \arctan(\beta / \alpha), \alpha > 0 \\ \arctan(\beta / \alpha) + \pi, \alpha < 0, \beta \geq 0 \\ \arctan(\beta / \alpha) - \pi, \alpha < 0, \beta < 0 \end{cases} \quad (13)$$

From the physical point of view the quantity $\Omega_{1,2}^{(R)}$ in (12a) determines polariton spectrum while that of $\Omega_{1,2}^{(I)}$ characterizes their absorption (at $\Omega_{1,2}^{(I)} > 0$) or amplification (at $\Omega_{1,2}^{(I)} < 0$). Behavior of polaritons in the coincidence region of corresponding curves is accompanied with such transition from absorption to amplification (it is shown at insertion to Fig.2a).

Another nontrivial feature of dependencies given in Fig.2a is the presence of local minimum energy at point k_0 for upper dispersion branch in curves coincidence range. Two polaritons of equal energy can be found near this minimum in the medium but they will correspond to two different values of wave vectors k_1 and k_2 . In this case polariton group velocities with k_1 becomes negative. The existence of such supplementary light waves has been experimentally observed in ².

Let us study correlation properties of investigated polariton system being under discussion. If we take into account (1) and (2) and make averaging in respect of Gaussian states both for probe optical field and atomic excitations of the medium the following average values of parameters introduced in (3) will be obtained:

$$V_k^0 \equiv (\mu^* + \nu)(\mu + \nu^*) \quad (14a)$$

$$V_k^{\pi/2} \equiv (\mu^* - \nu)(\mu - \nu^*) \quad (14b)$$

Figure 2b gives dependencies V_k^0 and $V_k^{\pi/2}$ directly in the coincidence range of polariton curves. These two parameters do not show any correlations in the system of light and dark polaritons provided the solution of the $V_k^0 > 1$ and $V_k^{\pi/2} > 1$. Strong phase correlations between such polaritons are revealed at local minimum at point k_0 ($V_k^{\pi/2} < 1$) that provides potential for their practical implementation in generation and controlling non-classical polariton states in three-level media with pumping.

With the increase of atomic rate decay studied range of extremum is widening and the trapping polaritons of probe field is carried out here. That is actually microscopic filling of the quantum system state when polaritons with similar values of wave vectors within the potential gap fall down to its bottom that corresponds to stabilization of polariton packets in

the minimum of potential gap. In Fig.2a at point k_0 there is no absorption at $\Omega_1^{(I)} = 0$ and at the same time group velocity of presented non-classical stabilized polaritons becomes equaled zero. Thus, medium becomes transparent to polariton radiation and it may contain dynamically stationary non-classical (polariton) packet. However, the possibility of practical observation of this effect was restricted by spatial diffraction of this packet in the plane that is perpendicular to the direction of propagation. We should remove this restriction by choosing the proper geometry in the considered system.

3. AMPLIFICATION OF NON-CLASSICAL POLARITONS IN WAVEGUIDE

Properties of two-dimensional polaritons of probe field will be of interest for us in this section. For this purpose let us study waveguide structure formed by doped crystal Pr : YSO with two mirror planes parallel to axis z in Fig.1b. With such geometry component of wave vector $k_\perp$ perpendicular to mirrors plane forms a standing wave but component parallel to $k_\parallel$ may take arbitrary value and it is responsible for dispersion effects occurring in the system.

In paraxial approximation for probe field we have:

$$\omega_s = \omega_s(k_s) = ck_s = c\sqrt{k_\perp^2 + k_\parallel^2} = ck_\perp + \frac{\hbar k_\parallel^2}{2m_{ph}}, \quad (15)$$

where $m_{ph} = \frac{\hbar k_z}{c}$ is effective photon mass of probe field in cavity.

Dispersion relation for atomic excitations of frequency ω_{ba} taking into account (9) can be presented as:

$$\omega_{ba} \equiv \omega_{ba}(k_\parallel) = \omega_{ba}^0 + \frac{\hbar k_\parallel^2}{2m_{at}} u, \quad (16)$$

$$\text{where } \omega_{ba}^{(0)} \equiv \frac{E_b - E_a}{\hbar} + \frac{\hbar k_p^2}{2m_{at}} + \frac{\hbar k_\perp^2}{2m_{at}} - \frac{\hbar ck_p k_\perp \cos \theta}{2m_{at}}, \quad u = \left(1 - \frac{\hbar k_p \cos \theta}{cm_{ph}}\right).$$

In the present paper we will emphasize the analysis of conditions far from atom-optical resonance $D > \lambda$ that is $\beta/\alpha \rightarrow 0$, when the concept of group velocity of wave packet remains true. Then we will obtain the following decomposition (12):

$$\Omega_{1,2}^{(R)} = \frac{1}{2} \left(\omega_+ + \frac{k^2 \Delta}{\Delta^2 + \gamma_c^2} \pm \sqrt{D^2 - \Gamma_-^2 - 4\lambda^2} \right), \quad (17a)$$

$$\Omega_{1,2}^{(I)} = \frac{1}{2} \left(\Gamma_+ \pm \frac{D\Gamma_-}{\sqrt{D^2 - \Gamma_-^2 - 4\lambda^2}} \right). \quad (17b)$$

Using expressions (17) and taking into account $\omega_{ph} = \omega_p - ck_\perp - \frac{1}{2} \frac{\hbar k_\parallel^2}{m_{ph}}$ and also (16) we will pass to polaritons formed exclusively in parallel axis of plane z . Dispersion law of such polaritons becomes quadratic that is $\Omega_{1,2,tr} \equiv \Omega_{1,2}^{(R)} = \frac{\hbar k_\parallel^2}{2m_{1,2}^{pol}}$, where $m_{1,2}^{pol}$ is effective mass of polaritons determined by

$$m_{pol} = \hbar \left(\frac{\partial^2 \Omega_{1,2}^{(R)}(k_\parallel)}{\partial k_\parallel^2} \Big|_{k_\parallel=0} \right)^{-1} = \frac{2}{\frac{1}{m_{at}} u + \frac{1}{m_{ph}} \pm \frac{\Delta_0}{\sqrt{\Delta_0^2 - \Gamma_-^2 - 4\lambda^2}} \left(\frac{1}{m_{at}} u - \frac{1}{m_{ph}} \right)}. \quad (18)$$

Here $\Delta_0 = \delta + \frac{k^2 \Delta}{\Delta^2 + \gamma_c^2}$, $\delta = \omega_{ba}^0 - \omega_p + ck_\perp$, δ is precision (controlling) detuning of pump field at resonance (see Fig.1a). In turn, group velocity of two-dimensional packet movement (mass centre) in the area parallel to resonator

Figure 3. (a) Dispersion relations $\Omega_{1(2)}^{(R)}$; (b) curves of absorption/amplification $\Omega_{1(2)}^{(I)}$, (c) group velocity $v_{1,tr}$ along z direction and polariton mass m_1^{pol} ; (d) dependences of correlation parameters V_k^0 and $V_k^{\pi/2}$ on normalized difference of wave vectors $k_{\parallel}/k_z$ in Λ -scheme atom-field interaction in doped waveguide. Calculated parameters correspond to Fig.2. Narrow dotted lines are zero values.

mirror can be produced on the basis of (18) and have the following view

$$v_{1,2,tr} = \frac{1}{2} \hbar k_{\parallel} \left(\frac{u}{m_{at}} - \frac{1}{m_{ph}} \pm \frac{\Delta_0}{\sqrt{\Delta_0^2 - \Gamma_-^2 - 4\lambda^2}} \left(\frac{u}{m_{at}} + \frac{1}{m_{ph}} \right) \right) \quad (19)$$

The weak-coupling regime $\Gamma_- > \lambda$ for the given condition is not of interest because polaritons of the low branch near $k_{\parallel} \rightarrow 0$ reveal purely photon-like properties while polaritons of upper branch correspond to atomic excitations. In this case there is no mixing of atomic excitations. Besides, significant absorption for polaritons of both branches can be observed in the system. The strong-coupling far-off-resonant nonlinear regime in Raman configuration ($\Gamma_- < \lambda < D$) for the condition given in Fig.3 is of special interest because of possibility to observe optically induced trapping for non-classical polaritons of upper branch, examined in section 2. It is in analogy to three-dimensional case but in two-dimensional system of doped waveguide due to the presence of local extremum (energy minimum) at $k_{\parallel} = 0.019k_{\perp}$ (point A in Fig.3a) and lack of absorption (in Fig.3b). Here, taking into account zeroing group velocity of polariton in Fig.3c, there is complete stop of polariton in the direction of an axis z and its dynamics will be set by re-reflection in the direction perpendicular to axis z . It is associated with the formation of standing wave in doped waveguide, when established field configuration perpendicular to axis z becomes self-reproducing with an accuracy up to total loss and phase shift²⁰. Additional losses, in turn, are associated with absorption (amplification) in the matrix Y_2SiO_5 itself. This situation in principle can be taken into account and compensated by the increasing of pump field intensity. Figure 3d represent a view of parameters V_k^0 and $V_k^{\pi/2}$ for atom-optical interaction in polariton waveguide given in Fig.3a-c. For each of these parameters small amplitude squeezing near $k_{\parallel} = 0$ is observed provided the condition $V_k^0 < 1$ is fulfilled. This squeezing increases with the growth of ratio $k_{\parallel}/k_{\perp}$. In particular in the presented case such squeezing can be estimated $(1 - V_k^0) \cdot 100 = 20\%$ provided that $k_{\parallel}/k_s = 0.5$. At selected parameters the depth of potential gap in Fig.3 will

have the amount $\Delta E_{trap} = 0.026 \mu eV$, its width $\Delta k_{pol} = 7.3 \cdot 10^4 m^{-1}$. Using parabolic approximation the effective mass of trapped polarons can be calculated $m_{eff}^{pol} = 1.9 \cdot 10^3 m_{ph} = 7 \cdot 10^{-8} m_{at}$. In this case interaction parameters for Fig.3 are selected in such a way that losses

Figure 4. Time dependence of average polariton number $N_{pol,1}$ normalized on initial number of photons N_{ph}^{in} in probe field at input of waveguide corresponding to different values of pump intensity.

of polariton of upper branch propagated along axis z and those created at the expense of absorption in ^{59}Pr are completely compensated due to the action of pump field that satisfies the condition (see expression (17b)):

$$|D| = \Gamma_+ \sqrt{\frac{\Gamma_-^2 + 4|\lambda|^2}{4\Gamma_{at}\gamma_{ph}}}, \quad (20)$$

Actually, expression (20) determines the threshold of amplification of non-classical polaritons of probe field that at these parameters corresponds to the intensity of pump field of $I_p = 0.96 W/cm^2$. The dynamics of normalized mean number of $N_{pol,1} = |\Phi_{1,k}|^2$ in accordance with equation (8) and expressions (1) and (17) at the threshold point is represented by curve 1 in Fig.4 for threshold value of pump power being only $P_{TH} = 8.7 mW$ (compare with respective values in experiments ^{7, 8}).

With the decrease of intensity of the pump field polariton waveguide runs actually under blocking condition, group velocity has a negative value (at pump intensity of $I_p = 0.58 W/cm^2$) and collapse of a number of polaritons (curve 2 in Fig.4) can be observed. In case of exceeding the threshold group velocity becomes positive, sharp parametric growth of the number of polaritons occurs and waveguide runs as polariton amplifier in the range of intensities of $0.96 \div 0.975 W/cm^2$ – see Fig.5a. Fig.5a represents the diagram of the dependence of mean number of polaritons at the output of doped waveguide of fixed length $L_{eff} = 9 mm$ on pump intensity taking into account delay time $\tau_{del} = L_{eff} / v_{1,tr}$ polariton packet in the medium.

However, further increase of pump power results in rapid decrease of mean number of polaritons at the waveguide output. The amplification of polaritons in the intensities range of $0.975 \div 47 W/cm^2$ halts (see Fig5a). It completely corresponds to transition (observed in such systems) from giant signal of luminescence from the cavity at definite pump

power^[21] to the “gap” of polariton signal with further increase of pump power²². The given phenomenon is accounted for the fact that rise of pump density after passing the threshold results in faster growth of group velocity of polaritons and decrease of their delay time in the medium as compared with the growth rate of

Figure 5. Calculated dependences of (a) average number of polaritons $N_{pol,1}$ normalized on initial number of photons N_{ph}^{in} and (b) correlation parameters V_k^0 at output of waveguide with fixed length L_{eff} on intensity of pump field taking into account delay time τ_{del} polariton wave packet in doped waveguide. At point $I_p = 1.003 W / \text{cm}^2$ scale of graphic is changed. At insertion there are dependences of polariton group velocities $v_{1,tr}$ and average polariton number $N_{pol,1}$ via intensity of pump field with fixed time of interaction.

polaritons mean number (see insertion to Fig.5a). So, point A on the curve 3 of Fig.4 under pump intensity of $0.98 W / \text{cm}^2$ corresponds to the time when polariton packet passes fixed distance L_{eff} and further amplification is impossible due to the medium restriction. With further increase of pump intensity growth of group velocity retards under growing amplification rate of polaritons (at the insertion to Fig.5a) and mean number of them at the output again begins to rise (Fig.5a). Thus, amplification of faster (with large values of group velocities) polaritons with sloping outline of the curve (Fig.5a) takes place in pump intensity range higher than $47 W / \text{cm}^2$. The group velocity of polaritons is $v_{1,tr} = 2.8 \cdot 10^6 \text{ m/s}$ for pump intensity of $I_p = 109 W / \text{cm}^2$ and, the coefficient characterizing polaritons amplification

in doped waveguide becomes about $\frac{N_{pol,1}^{out}}{N_{ph}^{in}} = 2$ that corresponds to curve 4 in Fig.4.

Figure 5b represents the dependence of parameter V_k^0 on pump field intensity under the condition of amplification of Raman polaritons. It is important to note that the quantity V_k^0 is rapidly decrease in this case by increasing of pumping power. Thus, the suggested dynamical model of polariton waveguide is capable to formation entangled

polaritons for probe field in the presence of their amplification. Practically, the structure being under discussion can be realized by using the technique of ion-beam lithography²³ with doping of ⁵⁹Pr atoms into the matrix.

4. CONCLUSION

In the presented work the semi-classical theory of polariton excitations, formed in Raman interaction scheme (Raman polaritons) is developed. The presence of pump field results in modification of polariton spectrum when there is a complete coincidence of low and upper dispersion branches in the system with losses (amplification) under certain condition. In this case non-classical correlations between bright and dark polaritons take place. The suggested model of polariton waveguide based on Fabry-Pérot interferometer with doped Pr:YSO medium inside under certain power range of optical pumping may serve for amplification of non-classical Raman polaritons. The nature of such amplification largely depends on the group velocity of polariton propagating in the medium. Recording and competition of dynamic effects in the amplification process lead to occurring of two different regions with different character of amplification and correlation properties for rapid and slow polaritons.

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