Secular perturbations of (7) Iris

arising from the actions of the eight major planets of the solar system.

By *C. J. Merfield.*

Introduction.

In the determination of the secular perturbations of this minor planet, the method of computation devised by Mr. R. T. A. Innes¹⁾ has been adopted. This method is now easy of application as tables of the two hypergeometrical functions²⁾ involved are available.

As soon as the components of the disturbing force were obtained, I adopted the process of mechanical quadrature to the variation of the elements instead of to the functions $aR_0:r$ etc., as suggested by Mr. Innes. Tables of the functions involved in this procedure are appended, and will be useful to compare with similar ones found from any future investigation.

Throughout the numerical work seven figure logarithms have been adopted, and when necessary addition and subtraction logarithms have been used.

All available controls have been applied as the work proceeded. The final test of the values of the perturbations in the plane of the orbit has been deduced from the formula

$$\sin \varphi \frac{1}{2} A_1^{(S)} + \cos \varphi B_0^{(C)} = 0.$$

The residual, after introducing the numerical values of the periodic series, is appended in each case to the value of the secular perturbations of the orbit elements.

Orbit elements.

The orbit elements of the major planets, referred to the epoch 1850, have been taken from Dr. Hill's memoir³⁾ on, »A new theory of Jupiter and Saturn«.

The elements of the orbit of Iris⁴⁾, adopted in this investigation are by Brünnow. These elements are mean ones, referred to the same epoch and are given below. The values of the true anomalies and logarithms of the radii vectores, corresponding to the several values of the eccentric anomalies into which the orbit of Iris has been divided, are appended.

Orbit elements of Iris.

$$\begin{aligned} \pi &= 41^\circ 23' 21''.1 \\ \Omega &= 259 \ 47 \ 55.8 \\ i &= 5 \ 28 \ 3.0 \end{aligned} \left. \vphantom{\begin{aligned} \pi &= 41^\circ 23' 21''.1 \\ \Omega &= 259 \ 47 \ 55.8 \\ i &= 5 \ 28 \ 3.0 \end{aligned}} \right\} 1850.0$$

$$\begin{aligned} \log e &= 9.3633349 \\ \log a &= 0.3777130 \\ n &= 351582''.56415 \\ m &= 0 \end{aligned}$$

<i>E</i>	<i>v</i>	<i>log r</i>
0°.00	0° 0' 0°.00	0.2637225
11.25	14 12 15.14	0.2662200
22.50	28 14 53.06	0.2735331
33.75	41 59 15.55	0.2851528
45.00	55 18 29.27	0.3003152
56.25	68 7 50.33	0.3181025
67.50	80 24 47.04	0.3375442
78.75	92 8 46.21	0.3576994
90.00	103 20 50.20	0.3777130
101.25	114 3 11.93	0.3968447
112.50	124 18 51.93	0.4144790
123.75	134 11 19.98	0.4301203
135.00	143 44 21.89	0.4433814
146.25	153 1 50.50	0.4539699
157.50	162 7 40.56	0.4616740
168.75	171 5 46.17	0.4663511
180.00	180 0 0.00	0.4679191
191.25	188 54 13.83	0.4663511
202.50	197 52 19.44	0.4616740
213.75	206 58 9.50	0.4539699
225.00	216 15 38.11	0.4433814
236.25	225 48 40.02	0.4301203
247.50	235 41 8.07	0.4144790
258.75	245 56 48.07	0.3968447
270.00	256 39 9.80	0.3777130
281.25	267 51 13.79	0.3576994

¹⁾ Computation of secular perturbations. Monthly Notices of the Royal Astr. Soc. Vol. 67 p. 427.

²⁾ Tables for the application of Mr. Innes' method (appended to the preceding paper). Reprinted as an appendix to a paper by Mr. R. T. A. Innes, »On the periods of the elliptic functions of Weierstrass«. Proceedings of the Royal Society of Edinburgh. Vol. 27 p. 357. See also, »Tables of the two hypergeometrical functions ... between the limits ι equals 90 and 180 degrees« by C. J. Merfield. Monthly Notices of the Royal Astr. Soc. Vol. 68 p. 605.

³⁾ Astronomical Papers prepared for the use of the American Ephemeris and Nautical Almanac Vol. 4. See also Astr. Nachr. 4178-79.

⁴⁾ Berliner Astr. Jahrbuch für 1901.

E	ν	$\log r$
292.50	279° 35' 12.96	0.3375442
303.75	291 52 9.67	0.3181025
315.00	304 41 30.73	0.3003152
326.25	318 0 44.45	0.2851528
337.50	331 45 6.94	0.2735331
348.75	345 47 44.86	0.2662200

Secular perturbations.

Iris — Mercury.

(Table I.)

$$\begin{aligned} II &= 340^\circ 24' 50.20 & K &= 326^\circ 44' 38.57 \\ II' &= 13 57 40.37 & K' &= 326 9 20.57 \\ I &= 11 57 16.84 & \log k &= 9.9994570 \\ \log \alpha &= 0.7898913 & \log k' &= 9.9910431 \end{aligned}$$

$$m' = 1:7500000$$

$$\left[\frac{de}{dt}\right]_{\infty} = +0.000022 \quad \left[\frac{d\Omega}{dt}\right]_{\infty} = -0.002057$$

$$\left[\frac{d\chi}{dt}\right]_{\infty} = +0.000888 \quad \left[\frac{d\pi}{dt}\right]_{\infty} = +0.000879$$

$$\left[\frac{di}{dt}\right]_{\infty} = -0.000079 \quad \left[\frac{dL}{dt}\right]_{\infty} = +0.095771$$

$$\sin \varphi^{1/2} A_1^{(S)} + \cos \varphi B_0^{(C)} = +0.00000003.$$

Iris — Venus.

(Table II.)

$$\begin{aligned} II &= 323^\circ 18' 24.08 & K &= 92^\circ 16' 31.36 \\ II' &= 321 22 0.27 & K' &= 91 36 19.45 \\ I &= 8 51 15.05 & \log k &= 9.9968378 \\ \log \alpha &= 0.5183752 & \log k' &= 9.9979854 \end{aligned}$$

$$m' = 1:425000$$

$$\left[\frac{de}{dt}\right]_{\infty} = +0.000008 \quad \left[\frac{d\Omega}{dt}\right]_{\infty} = -0.007730$$

$$\left[\frac{d\chi}{dt}\right]_{\infty} = +0.001956 \quad \left[\frac{d\pi}{dt}\right]_{\infty} = +0.001921$$

$$\left[\frac{di}{dt}\right]_{\infty} = -0.000021 \quad \left[\frac{dL}{dt}\right]_{\infty} = +0.294276$$

$$\sin \varphi^{1/2} A_1^{(S)} + \cos \varphi B_0^{(C)} = -0.00000008.$$

Iris — Earth.

(Table III.)

$$\begin{aligned} II &= 321^\circ 35' 25.30 & K &= 301^\circ 6' 50.18 \\ II' &= 20 33 43.90 & K' &= 300 56 31.57 \\ I &= 5 28 3.00 & \log k &= 9.9997567 \\ \log \alpha &= 0.3777130 & \log k' &= 9.9982649 \end{aligned}$$

$$m' = 1:322800$$

$$\left[\frac{de}{dt}\right]_{\infty} = +0.001494 \quad \left[\frac{d\Omega}{dt}\right]_{\infty} = -0.249486$$

$$\left[\frac{d\chi}{dt}\right]_{\infty} = +0.233689 \quad \left[\frac{d\pi}{dt}\right]_{\infty} = +0.232554$$

$$\left[\frac{di}{dt}\right]_{\infty} = +0.000537 \quad \left[\frac{dL}{dt}\right]_{\infty} = +2.557725$$

$$\sin \varphi^{1/2} A_1^{(S)} + \cos \varphi B_0^{(C)} = -0.00000012.$$

Iris — Mars.

(Table IV.)

$$\begin{aligned} II &= 329^\circ 24' 1.31 & K &= 68^\circ 12' 15.40 \\ II' &= 261 15 46.03 & K' &= 68 4 16.93 \\ I &= 7 6 45.19 & \log k &= 9.9967230 \\ \log \alpha &= 0.1948159 & \log k' &= 9.9999231 \end{aligned}$$

$$m' = 1:3093500$$

$$\left[\frac{de}{dt}\right]_{\infty} = -0.007488 \quad \left[\frac{d\Omega}{dt}\right]_{\infty} = -0.161445$$

$$\left[\frac{d\chi}{dt}\right]_{\infty} = +0.108617 \quad \left[\frac{d\pi}{dt}\right]_{\infty} = +0.107883$$

$$\left[\frac{di}{dt}\right]_{\infty} = -0.003100 \quad \left[\frac{dL}{dt}\right]_{\infty} = +0.364441$$

$$\sin \varphi^{1/2} A_1^{(S)} + \cos \varphi B_0^{(C)} = -0.00000042.$$

Iris — Jupiter.

(Table V.)

$$\begin{aligned} II &= 317^\circ 54' 47.08 & K &= 29^\circ 20' 27.28 \\ II' &= 288 27 11.64 & K' &= 29 34 41.28 \\ I &= 6 43 13.50 & \log k &= 9.9973077 \\ \log \alpha &= 9.6614756 & \log k' &= 9.9997019 \end{aligned}$$

$$m' = 1:1047.879$$

$$\left[\frac{de}{dt}\right]_{\infty} = -0.920384 \quad \left[\frac{d\Omega}{dt}\right]_{\infty} = -51.415659$$

$$\left[\frac{d\chi}{dt}\right]_{\infty} = +32.173222 \quad \left[\frac{d\pi}{dt}\right]_{\infty} = +31.939301$$

$$\left[\frac{di}{dt}\right]_{\infty} = +1.152955 \quad \left[\frac{dL}{dt}\right]_{\infty} = -44.506227$$

$$\sin \varphi^{1/2} A_1^{(S)} + \cos \varphi B_0^{(C)} = -0.00000001.$$

Iris — Saturn.

(Table VI.)

$$\begin{aligned} II &= 311^\circ 30' 34.27 & K &= 311^\circ 12' 58.33 \\ II' &= 0 17 45.52 & K' &= 311 12 39.07 \\ I &= 7 41 16.42 & \log k &= 9.9999999 \\ \log \alpha &= 9.3982174 & \log k' &= 9.9960788 \end{aligned}$$

$$m' = 1:3501.6$$

$$\left[\frac{de}{dt}\right]_{\infty} = +0.002063 \quad \left[\frac{d\Omega}{dt}\right]_{\infty} = -2.043325$$

$$\left[\frac{d\chi}{dt}\right]_{\infty} = +1.183675 \quad \left[\frac{d\pi}{dt}\right]_{\infty} = +1.174379$$

$$\left[\frac{di}{dt}\right]_{\infty} = +0.060782 \quad \left[\frac{dL}{dt}\right]_{\infty} = -1.753369$$

$$\sin \varphi^{1/2} A_1^{(S)} + \cos \varphi B_0^{(C)} = +0.000000003.$$

Iris—Uranus.

(Table VII.)

$$\begin{aligned} II &= 322^\circ 24' 11''.42 & K &= 233^\circ 8' 46''.16 \\ II' &= 89 \ 15 \ 41.85 & K' &= 233 \ 8 \ 14.61 \\ I &= 6 \ 14 \ 7.55 & \log k &= 9.9974235 \\ \log \alpha &= 9.0946086 & \log k' &= 9.9999996 \end{aligned}$$

$$m' = 1:22800$$

$$\left[\frac{de}{dt}\right]_{00} = -0''.000023 \quad \left[\frac{d\Omega}{dt}\right]_{00} = -0''.028378$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0''.022584 \quad \left[\frac{d\pi}{dt}\right]_{00} = +0''.022455$$

$$\left[\frac{di}{dt}\right]_{00} = +0''.000314 \quad \left[\frac{dL}{dt}\right]_{00} = -0''.031874$$

$$\sin \varphi \frac{1}{2} A_1^{(S)} + \cos \varphi B_0^{(C)} = -0.000000005.$$

Iris—Neptune.

(Table VIII.)

$$\begin{aligned} II &= 309^\circ 49' 24''.00 & K &= 75^\circ 28' 14''.46 \\ II' &= 234 \ 32 \ 27.66 & K' &= 75 \ 5 \ 39.76 \\ I &= 6 \ 44 \ 49.33 & \log k &= 9.9980022 \\ \log \alpha &= 8.8995716 & \log k' &= 9.9989889 \end{aligned}$$

$$m' = 1:19700$$

$$\left[\frac{de}{dt}\right]_{00} = -0''.000058 \quad \left[\frac{d\Omega}{dt}\right]_{00} = -0''.008987$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +0''.006401 \quad \left[\frac{d\pi}{dt}\right]_{00} = +0''.006361$$

$$\left[\frac{di}{dt}\right]_{00} = +0''.000288 \quad \left[\frac{dL}{dt}\right]_{00} = -0''.009370$$

$$\sin \varphi \frac{1}{2} A_1^{(S)} + \cos \varphi B_0^{(C)} = +0.000000057.$$

Secular perturbations of Iris

arising from the actions of the eight major planets of the Solar system.

Planet	$\left[\frac{de}{dt}\right]$	$\left[\frac{d\chi}{dt}\right]$	$\left[\frac{di}{dt}\right]$	$\left[\frac{d\Omega}{dt}\right]$	$\left[\frac{d\pi}{dt}\right]$	$\left[\frac{dL}{dt}\right]$
Mercury	+0''.000022	+ 0''.000888	-0''.000079	- 0''.002057	+ 0''.000879	+ 0''.095771
Venus	+0.000008	+ 0.001956	-0.000021	- 0.007730	+ 0.001921	+ 0.294276
Earth	+0.001494	+ 0.233689	+0.000537	- 0.249486	+ 0.232554	+ 2.557725
Mars	-0.007488	+ 0.108617	-0.003100	- 0.161445	+ 0.107883	+ 0.364441
Jupiter	-0.920384	+32.173222	+1.152955	-51.415659	+31.939301	-44.506227
Saturn	+0.002063	+ 1.183675	+0.060782	- 2.043325	+ 1.174379	- 1.753369
Uranus	-0.000023	+ 0.022584	+0.000314	- 0.028378	+ 0.022455	- 0.031874
Neptune	-0.000058	+ 0.006401	+0.000288	- 0.008987	+ 0.006361	- 0.009370
$\Sigma =$	-0.924366	+33.731032	+1.211676	-53.917067	+33.485733	-42.988627

$$\left[\frac{de}{dt}\right]_{00} = - 0''.92437$$

$$\left[\frac{d\Omega}{dt}\right]_{00} = -53''.91707$$

$$\left[\frac{d\chi}{dt}\right]_{00} = +33''.73103$$

$$\left[\frac{d\pi}{dt}\right]_{00} = +33''.48573$$

$$\left[\frac{di}{dt}\right]_{00} = + 1''.21168$$

$$\left[\frac{dL}{dt}\right]_{00} = -42''.98863$$

Table I. Mercury.

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin \nu + S_0 (\cos \nu + \cos E)$
0°.0	0.0806789n	8.5888910n	7.8161190n	-0.0776106
22.5	0.0638559n	7.7999154n	7.0420796n	-0.5596410
45.0	0.0413604n	8.3372200	7.4610587	-0.8766366
67.5	0.0161621n	8.5879492	7.6811820	-1.0021534
90.0	9.9909549n	8.6494836	7.6983269	-0.9632333
112.5	9.9685844n	8.6324313	7.6195983	-0.8089176
135.0	9.9519650n	8.5701369	7.4574796	-0.5857761
157.5	9.9436130n	8.4711078	7.1373727	-0.3250202
180.0	9.9452116n	8.3187251	6.4480708n	-0.0416634
202.5	9.9573246n	8.0197665	7.3417280n	+0.2585409

E	$\log R_0$	$\log S_0$	$\log H'_0$	$R_0 \sin \nu + S_0 (\cos \nu + \cos E)$
225°0	9.9791200n	7.4189207n	7.6578405n	+0.5676661
247.5	0.0079850n	8.2922494n	7.8722923n	+0.8598326
270.0	0.0392328n	8.6043890n	8.0297838n	+1.0742617
292.5	0.0665536n	8.7806673n	8.1283530n	+1.1161869
315.0	0.0839520n	8.8516189n	8.1491640n	+0.9068779
337.5	0.0883961n	8.8045919n	8.0637390n	+0.4650428
Σ_1	0.1124756n	-0.0283529	-0.0292972	+0.0038857
Σ_2	0.1124747n	-0.0283512	-0.0292973	+0.0038710

E	$-R_0 \cos \nu + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin \nu$	$H'_0 \cos u$	$H'_0 \sin u$	$-2R_0 \frac{r}{a}$
0°0	+1.2041453	+0.0051311	-0.0040682	1.8523302
22.5	+1.0149693	+0.0010845	-0.0001944	1.8226613
45.0	+0.6597039	-0.0027662	-0.0008404	1.8407407
67.5	+0.2478059	-0.0035664	-0.0032116	1.8924452
90.0	-0.1368304	-0.0021149	-0.0045225	1.9587766
112.5	-0.4482315	-0.0002974	-0.0041542	2.0247921
135.0	-0.6729150	+0.0007581	-0.0027653	2.0828760
157.5	-0.8151423	+0.0007616	-0.0011413	2.1311009
180.0	-0.8814782	-0.0002199	+0.0001743	2.1699400
202.5	-0.8699982	-0.0020569	+0.0007706	2.1994576
225.0	-0.7650268	-0.0045450	+0.0001706	2.2172689
247.5	-0.5393959	-0.0071161	-0.0022132	2.2170781
270.0	-0.1722178	-0.0084114	-0.0066294	2.1890859
292.5	+0.3109339	-0.0064787	-0.0117737	2.1252735
315.0	+0.8006072	-0.0009142	-0.0140685	2.0304122
337.5	+1.1350167	+0.0045886	-0.0106330	1.9286178
Σ_1	+0.0359882	-0.0130824	-0.0325494	+16.3414305
Σ_2	+0.0359579	-0.0130808	-0.0325508	+16.3414265

Table II. Venus.

E	$\log R_0$	$\log S_0$	$\log H'_0$	$R_0 \sin \nu + S_0 (\cos \nu + \cos E)$
0°0	9.3674041n	6.7007702	7.1911666n	+0.0010042
22.5	9.3580933n	6.5493115	6.4783509	-0.1073102
45.0	9.3306675n	6.0890074	6.9130829	-0.1759023
67.5	9.2921474n	5.5293868n	7.1494434	-0.1932329
90.0	9.2506463n	5.9998411n	7.1853689	-0.1732592
112.5	9.2127887n	6.0754453n	7.1460092	-0.1347048
135.0	9.1830991n	6.0869608n	7.0694436	-0.0899771
157.5	9.1642824n	6.0886100n	6.9669022	-0.0445690
180.0	9.1577230n	6.0951552n	6.8287501	+0.0002490
202.5	9.1638571n	6.1080482n	6.6010123	+0.0449958
225.0	9.1822098n	6.1192895n	5.8166153	+0.0901768
247.5	9.2113639n	6.1034475n	6.5709087n	+0.1344959
270.0	9.2486070n	5.9763471n	6.9773786n	+0.1726891
292.5	9.2895479n	4.0332088	7.2118715n	+0.1920619

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
315.0	9.3279315n	6.2804329	7.3431172n	+0.1751967
337.5	9.3562118n	6.6204373	7.3570401n	+0.1082359
Σ_1	8.0955315n ¹⁾	+0.0005134 ¹⁾	-0.0014782 ¹⁾	+0.0001772
Σ_2	8.0955359n	+0.0005121	-0.0014772	-0.0000274

¹⁾ Contains the factor $\frac{a}{r}$.

E	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$-2R_0 \frac{r}{a}$
0.0	+0.2330259	+0.0012169	-0.0009648	0.3584624
22.5	+0.2012270	+0.0002961	-0.0000531	0.3588751
45.0	+0.1220622	-0.0007833	-0.0002380	0.3583436
67.5	+0.0325689	-0.0010483	-0.0009440	0.3572799
90.0	-0.0413132	-0.0006491	-0.0013881	0.3561856
112.5	-0.0922271	-0.0001000	-0.0013960	0.3552914
135.0	-0.1230788	+0.0003102	-0.0011316	0.3546480
157.5	-0.1390180	+0.0005144	-0.0007707	0.3542202
180.0	-0.1437881	+0.0005283	-0.0004188	0.3539640
202.5	-0.1387063	+0.0003737	-0.0001400	0.3538736
225.0	-0.1224929	+0.0000655	-0.0000025	0.3539225
247.5	-0.0914893	-0.0003555	-0.0001106	0.3541277
270.0	-0.0407311	-0.0007455	-0.0005876	0.3545170
292.5	+0.0324376	-0.0007853	-0.0014270	0.3551478
315.0	+0.1208113	-0.0001429	-0.0021989	0.3560932
337.5	+0.1996898	+0.0009015	-0.0020891	0.3573238
Σ_1	+0.0044953	-0.0001999	-0.0069303	+2.8461363
Σ_2	+0.0044826	-0.0002034	-0.0069305	+2.8461395

Table III. Earth.

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
0.00	0.2214051n	8.1378804n	8.6927141n	-0.0274733
11.25	0.2158876n	8.0550077n	8.5023397n	-0.4255246
22.50	0.2029486n	7.9434810n	8.1181488n	-0.7710696
33.75	0.1837587n	7.7747325n	7.5489383	-1.0307034
45.00	0.1600914n	7.4644826n	8.2119844	-1.1924456
56.25	0.1338347n	6.0742376	8.3924812	-1.2628782
67.50	0.1066585n	7.4575047	8.4664777	-1.2589470
78.75	0.0798755n	7.7098319	8.4912731	-1.2002688
90.00	0.0544546n	7.8332763	8.4883076	-1.1045394
101.25	0.0310941n	7.9000508	8.4682244	-0.9857316
112.50	0.0102908n	7.9346640	8.4369951	-0.8539037
123.75	9.9924040n	7.9489863	8.3980836	-0.7157510
135.00	9.9776975n	7.9494619	8.3533498	-0.5753237
146.25	9.9663634n	7.9399376	8.3034054	-0.4347160
157.50	9.9585383n	7.9224481	8.2476557	-0.2946380

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
168°75	9.9543166n	7.8976242	8.1840227	-0.1548756
180.00	9.9537542n	7.8647012	8.1081023	-0.0146464
191.25	9.9568693n	7.8211647	8.0109739	+0.1271018
202.50	9.9636421n	7.7616923	7.8725558	+0.2714113
213.75	9.9740073n	7.6754093	7.6326521	+0.4190078
225.00	9.9878413n	7.5369637	6.7730679	+0.5699184
236.25	0.0049505n	7.2623061	7.5841037n	+0.7229747
247.50	0.0250439n	6.2429395n	7.9647478n	+0.8751514
258.75	0.0477127n	7.4181998n	8.1977893n	+1.0207862
270.00	0.0723914n	7.7403078n	8.3735586n	+1.1507437
281.25	0.0983639n	7.9399779n	8.5160390n	+1.2519392
292.50	0.1246971n	8.0790204n	8.6332355n	+1.3073925
303.75	0.1502848n	8.1745031n	8.7268164n	+1.2978751
315.00	0.1738918n	8.2319234n	8.7950234n	+1.2053364
326.25	0.1941609n	8.2534140n	8.8336708n	+1.0178625
337.50	0.2096992n	8.2414737n	8.8364329n	+0.7355847
348.75	0.2191470n	8.2008828n	8.7942117n	+0.3754563
Σ_1	1.2975344n ¹⁾	-0.0477999 ¹⁾	-0.1632311 ¹⁾	+0.0225517
Σ_2	1.2975176n	-0.0477885	-0.1632289	+0.0225544

¹⁾ Contains the factor $\frac{a}{r}$.

E	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$-2R_0 \frac{r}{a}$
0°00	+1.6649650	+0.0386191	-0.0306197	2.5612065
11.25	+1.5886253	+0.0289984	-0.0130357	2.5434582
22.50	+1.3980458	+0.0129206	-0.0023158	2.5107226
33.75	+1.1274112	-0.0035326	-0.0002209	2.4673369
45.00	+0.8183502	-0.0155889	-0.0047358	2.4194888
56.25	+0.5071449	-0.0214401	-0.0122396	2.3727623
67.50	+0.2184559	-0.0217534	-0.0195892	2.3308790
78.75	-0.0347199	-0.0183327	-0.0249904	2.2955772
90.00	-0.2480623	-0.0130397	-0.0278845	2.2671728
101.25	-0.4225762	-0.0072877	-0.0284738	2.2452041
112.50	-0.5619765	-0.0019533	-0.0272826	2.2288805
123.75	-0.6709665	+0.0025182	-0.0248812	2.2173862
135.00	-0.7542386	+0.0059644	-0.0217578	2.2100184
146.25	-0.8159048	+0.0083780	-0.0182814	2.2062274
157.50	-0.8592276	+0.0098183	-0.0147117	2.2056128
168.75	-0.8864998	+0.0103572	-0.0112293	2.2079269
180.00	-0.8989888	+0.0100506	-0.0079687	2.2130454
191.25	-0.8969025	+0.0089257	-0.0050511	2.2209428
202.50	-0.8793560	+0.0069829	-0.0026160	2.2316861
213.75	-0.8443247	+0.0042067	-0.0008513	2.2454036
225.00	-0.7886127	+0.0005926	-0.0000222	2.2622453
236.25	-0.7078918	-0.0038060	-0.0004944	2.2823791
247.50	-0.5968871	-0.0088044	-0.0027382	2.3058968
258.75	-0.4498850	-0.0139821	-0.0072901	2.3327833
270.00	-0.2617231	-0.0185628	-0.0146302	2.3627696
281.25	-0.0294861	-0.0213343	-0.0249300	2.3954126
292.50	+0.2451531	-0.0207192	-0.0376528	2.4297315

E	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$-2R_0 \frac{r}{a}$
303.75	+0.5531445	-0.0151771	-0.0511049	2.4643608
315.00	+0.8758536	-0.0040446	-0.0622456	2.4976063
326.25	+1.1845254	+0.0113738	-0.0672268	2.5271482
337.50	+1.4427850	+0.0271877	-0.0630012	2.5500535
348.75	+1.6127732	+0.0378035	-0.0494697	2.5626188
Σ_1	+0.8145359	+0.0076699	-0.3397720	+37.5870159
Σ_2	+0.8144672	+0.0076689	-0.3397706	+37.5869284

Table IV. Mars.

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
0.00	0.4045290 _n	9.4374385	9.5186355 _n	+0.5476064
11.25	0.4542313 _n	9.4956910	9.3391508 _n	-0.0877189
22.50	0.4813082 _n	9.4852076	8.0318236 _n	-0.8820133
33.75	0.4763706 _n	9.4059826	9.3381424	-1.6023971
45.00	0.4450310 _n	9.2748430	9.5715574	-2.0506635
56.25	0.4003719 _n	9.1085171	9.6338885	-2.2139672
67.50	0.3519826 _n	8.9051754	9.6234013	-2.1734059
78.75	0.3042515 _n	8.6311781	9.5760735	-2.0067348
90.00	0.2588230 _n	8.1282082	9.5089840	-1.7688577
101.25	0.2164276 _n	7.9625136 _n	9.4312370	-1.4975360
112.50	0.1775720 _n	8.4195479 _n	9.3480047	-1.2182985
123.75	0.1426813 _n	8.5910429 _n	9.2621873	-0.9470798
135.00	0.1120652 _n	8.6845934 _n	9.1751127	-0.6923702
146.25	0.0859275 _n	8.7426145 _n	9.0868514	-0.4574935
157.50	0.0643662 _n	8.7814344 _n	8.9962457	-0.2425288
168.75	0.0473934 _n	8.8090094 _n	8.9006150	-0.0458005
180.00	0.0349656 _n	8.8296331 _n	8.7949045	+0.1351024
191.25	0.0269750 _n	8.8456217 _n	8.6693526	+0.3026727
202.50	0.0233190 _n	8.8580623 _n	8.5024007	+0.4590933
213.75	0.0238583 _n	8.8670804 _n	8.2283786	+0.6059755
225.00	0.0284541 _n	8.8719306 _n	7.1369593	+0.7442003
236.25	0.0369701 _n	8.8708715 _n	8.1915912 _n	+0.8738017
247.50	0.0492781 _n	8.8607748 _n	8.5383990 _n	+0.9938835
258.75	0.0652816 _n	8.8362224 _n	8.7511085 _n	+1.1026180
270.00	0.0849235 _n	8.7872322 _n	8.9127646 _n	+1.1972705
281.25	0.1082337 _n	8.6924684 _n	9.0475281 _n	+1.2743561
292.50	0.1353757 _n	8.4894913 _n	9.1653052 _n	+1.3297369
303.75	0.1666999 _n	7.6209334 _n	9.2705792 _n	+1.3583971
315.00	0.2028346 _n	8.8027599	9.3649109 _n	+1.3534486
326.25	0.2446439 _n	8.9101325	9.4472154 _n	+1.3030753
337.50	0.2929216 _n	9.1506241	9.5123207 _n	+1.1843705
348.75	0.3474193 _n	9.3209667	9.5468567 _n	+0.9544446
Σ_1	3.2422360 ¹⁾	+0.8343456 ¹⁾	+0.3151861 ¹⁾	-1.0834255
Σ_2	3.2422235 _n	+0.8342755	+0.3152486	-1.0833867

¹ Contains the factor $\frac{a}{r}$.

E	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$-2R_0 \frac{r}{a}$
0°00	+2.5382187	+0.2586567	-0.2050797	3.9045279
11.25	+2.8985760	+0.1991518	-0.0895251	4.4031990
22.50	+2.9331861	+0.0105915	-0.0018984	4.7660467
33.75	+2.5418015	-0.2174178	-0.0135950	4.8399444
45.00	+1.8775352	-0.3567699	-0.1083847	4.6629763
56.25	+1.1653169	-0.3737947	-0.2133904	4.3832010
67.50	+0.5301416	-0.3122133	-0.2811520	4.1005660
78.75	+0.0104048	-0.2228577	-0.3037898	3.8482912
90.00	-0.3920676	-0.1367554	-0.2924415	3.6295517
101.25	-0.6885071	-0.0669274	-0.2614922	3.4402444
112.50	-0.8951406	-0.0159143	-0.2222769	3.2761820
123.75	-1.0294117	+0.0184162	-0.1819593	3.1341417
135.00	-1.1074763	+0.0395669	-0.1443374	3.0113667
146.25	-1.1428833	+0.0508846	-0.1110338	2.9054567
157.50	-1.1461206	+0.0692827	-0.0824618	2.8142143
168.75	-1.1247512	+0.0539307	-0.0584718	2.7356560
180.00	-1.0838411	+0.0488644	-0.0387429	2.6680975
191.25	-1.0263594	+0.0406466	-0.0230022	2.6100157
202.50	-0.9537447	+0.0297771	-0.0111554	2.5604129
213.75	-0.8661420	+0.0165830	-0.0033559	2.5185186
225.00	-0.7627795	+0.0013698	-0.0000514	2.4840051
236.25	-0.6422222	-0.0154155	-0.0020025	2.4570125
247.50	-0.5026175	-0.0329876	-0.0102593	2.4382270
258.75	-0.3419352	-0.0499909	-0.0260647	2.4290883
270.00	-0.1581288	-0.0642467	-0.0506355	2.4319436
281.25	+0.0508265	-0.0725383	-0.0847641	2.4504740
292.50	+0.2872043	-0.0705412	-0.1281937	2.4902154
303.75	+0.5542315	-0.0530824	-0.1787416	2.5592894
315.00	+0.8572663	-0.0150233	-0.2312044	2.6697265
326.25	+1.2047559	+0.0467285	-0.2761132	2.8386621
337.50	+1.6066390	+0.1289023	-0.2987008	3.0886714
348.75	+2.0640528	+0.2138833	-0.2798879	3.4431532
Σ_1	+3.6282745	-0.4174403	-2.1069758	+50.9967310
Σ_2	+3.6277538	-0.4318000	-2.1071895	+50.9963482

Table V. Jupiter.

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
0°0	8.5268147	6.7585999	7.8409908n	+0.0011472
22.5	8.5537888	5.2366943n	7.4272467n	+0.0169090
45.0	8.6086261	6.9960011n	7.4797837	+0.0321255
67.5	8.6795528	7.2570905n	8.0420382	+0.0461532
90.0	8.7580771	7.2827506n	8.3333189	+0.0561850
112.5	8.8371796	7.0128224n	8.5226515	+0.0577471
135.0	8.9096343	6.8439007	8.6333595	+0.0469786
157.5	8.9672139	7.4169157	8.6615101	+0.0235592
180.0	9.0007022	7.5773816	8.5868744	-0.0075581
202.5	9.0015186	7.5516003	8.3566592	-0.0374763
225.0	8.9650537	7.3360020	7.6663487	-0.0578538

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
247°5	8.8936571	6.7244179	7.9495843n	-0.0651586
270.0	8.7971655	6.6648091n	8.1855123n	-0.0608854
292.5	8.6919150	6.7290605n	8.2065530n	-0.0488016
315.0	8.5992471	5.1075411n	8.1418659n	-0.0326930
337.5	8.5404931	6.7181521	8.0243337n	-0.0154862
Σ_1	0.1653207	+0.0027005 ¹⁾	+0.0565638	-0.0225540
Σ_2	0.1653189	+0.0027004	+0.0565640	-0.0225542

¹⁾ Contains the factor $\frac{a}{r}$.

E	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$2R_0 \frac{r}{a}$
0°0	-0.0336368	+0.0054335	-0.0043080	0.0517433
22.5	-0.0315446	+0.0026326	-0.0004719	0.0563169
45.0	-0.0246481	-0.0028881	-0.0008774	0.0679608
67.5	-0.0114617	-0.0081863	-0.0073719	0.0871794
90.0	+0.0093889	-0.0091260	-0.0195152	0.1145796
112.5	+0.0369197	-0.0023792	-0.0332308	0.1496152
135.0	+0.0664064	+0.0113652	-0.0414597	0.1889438
157.5	+0.0900826	+0.0254618	-0.0381520	0.2250116
180.0	+0.1001618	+0.0302665	-0.0239972	0.2465689
202.5	+0.0930142	+0.0212882	-0.0079752	0.2435060
225.0	+0.0715419	+0.0046349	-0.0001739	0.2146605
247.5	+0.0431884	-0.0085023	-0.0026443	0.1703935
270.0	+0.0153958	-0.0120392	-0.0094886	0.1253706
292.5	-0.0071558	-0.0077570	-0.0140966	0.0896966
315.0	-0.0225997	-0.0008989	-0.0138341	0.0665088
337.5	-0.0310318	+0.0041906	-0.0097106	0.0546189
Σ_1	+0.1820102	+0.0267479	-0.1136541	+1.0763363
Σ_2	+0.1820110	+0.0267484	-0.1136533	+1.0763381

Table VI. Saturn.

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
0°0	7.6786372	6.2151429	7.0661010n	+0.00032823
22.5	7.7141347	6.0377016	6.7660689n	+0.00264739
45.0	7.7745155	5.6547703n	6.3880939	+0.00483458
67.5	7.8451642	6.3315321n	7.1543068	+0.00678561
90.0	7.9168473	6.4750657n	7.4691329	+0.00810336
112.5	7.9833795	6.3833871n	7.6573007	+0.00817825
135.0	8.0391554	5.8359866n	7.7579991	+0.00657636
157.5	8.0784551	6.1135252	7.7768948	+0.00343296
180.0	8.0960050	6.3962343	7.7056374	-0.00049804
202.5	8.0878559	6.3666464	7.5142738	-0.00419332
225.0	8.0523119	6.0145112	7.0710836	-0.00682818

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
247°5	7.9907175	5.7429029n	6.7479868n	-0.00803254
270.0	7.9085154	6.1813631n	7.2068380n	-0.00784672
292.5	7.8170024	6.1372549n	7.2989779n	-0.00654519
315.0	7.7347255	5.4589664n	7.2822479n	-0.00450064
337.5	7.6840006	6.0038390	7.2049065n	-0.00210419
Σ_1	3.2007142	-0.0000931 ¹⁾	+0.0078862 ¹⁾	+0.00016895
Σ_2	3.2007099	-0.0000931	+0.0078862	+0.00016897

¹⁾ Contains the factor $\frac{a}{r}$.

E	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$2 R_0 \frac{r}{a}$
0°0	-0.00477131	+0.00091241	-0.00072342	0.0073397
22.5	-0.00446653	+0.00057439	-0.00010295	0.0081468
45.0	-0.00345646	-0.00023384	-0.00007104	0.0099574
67.5	-0.00158126	-0.00106012	-0.00095465	0.0127651
90.0	+0.00130887	-0.00124765	-0.00266801	0.0165150
112.5	+0.00499643	-0.00032440	-0.00453096	0.0209496
135.0	+0.00873375	+0.00151432	-0.00552415	0.0254597
157.5	+0.01149279	+0.00332104	-0.00497624	0.0290701
180.0	+0.01247398	+0.00397856	-0.00315446	0.0307073
202.5	+0.01148847	+0.00306024	-0.00114646	0.0297062
225.0	+0.00895922	+0.00117700	-0.00004417	0.0262428
247.5	+0.00561635	-0.00053449	-0.00016623	0.0213066
270.0	+0.00217381	-0.00126452	-0.00099662	0.0162011
292.5	-0.00082728	-0.00095965	-0.00174397	0.0119637
315.0	-0.00304546	-0.00012419	-0.00191132	0.0090857
337.5	-0.00434273	+0.00063511	-0.00147171	0.0076006
Σ_1	+0.02237640	+0.00471209	-0.01509319	+0.1415087
Σ_2	+0.02237624	+0.00471212	-0.01509317	+0.1415087

Table VII. Uranus.

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
0°	6.7522424	5.0555885	5.9418004n	+0.00002273
45	6.8348267	4.6749849n	5.7579772	+0.00055607
90	6.9835481	5.0677927n	6.4736045	+0.00093952
135	7.1170091	5.0471411	6.6990558	+0.00075747
180	7.1743940	5.3018242	6.5759677	-0.00004007
225	7.1259974	4.7540006n	5.1071093	-0.00078195
270	6.9818264	5.3057370n	6.3015531n	-0.00092845
315	6.8140506	4.1160058n	6.2647158n	-0.00053752
Σ_1	+0.00398125	-0.00000050	+0.00038656	-0.00000627
Σ_2	+0.00398114	-0.00000057	+0.00038622	-0.00000593

E	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$2 R_0 \frac{r}{a}$
0°	-0.00056525	+0.00006853	-0.00005434	0.00086952
45	-0.00039643	-0.00005480	-0.00001665	0.00114407
90	+0.00019888	-0.00012606	-0.00026956	0.00192565
135	+0.00107036	+0.00013221	-0.00048231	0.00304584
180	+0.00149415	+0.00029516	-0.00023402	0.00367816
225	+0.00108522	+0.00001279	-0.00000048	0.00310954
270	+0.00026184	-0.00015727	-0.00012395	0.00192246
315	-0.00036890	-0.00001193	-0.00018357	0.00109064
Σ_1	+0.00138962	+0.00008036	-0.00068187	+0.00839579
Σ_2	+0.00139025	+0.00007827	-0.00068301	+0.00839009

Table VIII. Neptune.

E	$\log R_0$	$\log S_0$	$\log W_0$	$R_0 \sin v + S_0 (\cos v + \cos E)$
0°	6.1608538	4.4740970	5.4884173n	+0.00000596
45	6.2445894	3.8488414n	4.6638037	+0.00014350
90	6.3882994	4.7004130n	5.8493726	+0.00023907
135	6.5139630	3.9991343	6.1440001	+0.00019164
180	6.5716138	4.9065336	6.1051515	-0.00001613
225	6.5324680	4.5451743	5.5269314	-0.00020686
270	6.3977492	4.6155196n	5.5968798n	-0.00024219
315	6.2283691	4.2800381n	5.6976862n	-0.00014154
Σ_1	+0.00101215	+0.00000189	+0.00012777	-0.00001329
Σ_2	+0.00101216	+0.00000189	+0.00012773	-0.00001326

E	$-R_0 \cos v + S_0 \left(\frac{r}{a \cos^2 \varphi} + 1 \right) \sin v$	$W_0 \cos u$	$W_0 \sin u$	$2 R_0 \frac{r}{a}$
0°	-0.00014483	+0.00002413	-0.00001913	0.00022279
45	-0.00010105	-0.00000441	-0.00000134	0.00029391
90	+0.00004641	-0.00002995	-0.00006404	0.00048902
135	+0.00026463	+0.00003683	-0.00013436	0.00075973
180	+0.00037292	+0.00009983	-0.00007914	0.00091802
225	+0.00027015	+0.00003362	-0.00000126	0.00079281
270	+0.00006594	-0.00003105	-0.00002447	0.00049978
315	-0.00009335	-0.00000323	-0.00004975	0.00028314
Σ_1	+0.00034044	+0.00006296	-0.00018678	+0.00212961
Σ_2	+0.00034038	+0.00006281	-0.00018671	+0.00212959

Melbourne, 1909 March 10.

C. F. Merfield.

Elements of comet 1909 a (Borrelly-Daniel).

A telegram has been received at this observatory from Professor *R. T. Crawford* at Berkeley, Cal., stating that the following elements of comet 1909 a have been computed by him from observations made on June 16, 18, 21.

$$\left. \begin{aligned} T &= 1909 \text{ June } 5.17 \text{ Greenw. M. T.} \\ \omega &= 4^\circ 59' \\ \Omega &= 306.19 \\ i &= 52.26 \\ q &= 0.846 \end{aligned} \right\} 1909.0$$

Harvard College Observatory, Cambridge, Mass., 1909 June 23.

E. C. Pickering.