A FEW DEFICIENCIES IN THE TEACHING OF MATHEMATICS.

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I need offer no apology for the title of this paper for if there were nothing to improve in our teaching of mathematics we would not be gathered here this afternoon. We are all interested in the discovery of the defects which really exist and in the means suggested for their improvement. The history of the teaching of mathematics in America is full of this onward movement toward better instruction. There has been a gradual change in the mathematical material used in the school and in the method of its presentation. Methods have been modified; material has been stricken out and material added, until now we are teaching twice as much in each branch as was taught a hundred years ago. And we are teaching it better and more thoroughly. It is the general testimony that a century ago the students who undertook the higher mathematics were so poorly prepared that they were less able than our students to grapple with the difficulties that presented themselves. In those days arithmetic, algebra, and geometry were college studies. The changing requirements of the day coupled with the increase and development of the college curriculum has resulted in marked modifications. A glance at the old texts will reveal the absence in them of many subjects which we consider of the utmost importance today. For instance in the geometries of fifty years ago original exercises for the student are not to be found, whereas today all good schools require such work of all pupils. Trigonometry such as Playfair's which was used extensively was largely geometrical. The ratio definitions of the trigonometric functions were not used until a much later date. It is only within fifteen years that there has appeared in our algebras an adequate consideration of convergency and divergency of indefinite series. The introduction of this subject is forcing out the study of continued fractions.

In this conflict for survival the less important must give way to that which is more essential. We are merely at one stage in this evolutionary process. We are being told by some that this

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or that should be left out and by others that this or that should be put in; some point to defects in our work and offer suggestions for their improvement. This afternoon I wish to point out some deficiencies in the mathematical training of the pupils who pass out from under our instruction, deficiencies which seem to me to be due to certain omissions in the scope and emphasis of that instruction.

Business men, manufacturers, college professors of mathematics and science, in fact all who have to deal with the boys and girls we have trained bring forward this indictment:—inaccuracy in the actual performance of all work in which mathematics is required. Say they your students are blunderers. They may know the theory but in practice they are blunderers. There can be no question but that in the main this charge is true. To test its truth you have but to set your senior class a series of calculations involving decimals and common fractions leading say to one desired result. But it is also to be said that inaccuracy in other things is a very common fault. A perfectly accurate man is hard to find and his worth is greater than that of rubies. Inaccuracy affects all lines of work. In many cases the result is a defective product which can nevertheless be used. A coat can be worn even though the cutter be inexact. The bread may be eaten though the proportions may differ slightly from the approved standard. The chair can be sold and used though there be deviations from the lines of greatest beauty and strength.

But note the difference. A mistake of the thousandth of an inch will cause the shaft to bind and the watch to stop. An error of one in a million will throw out a summer's work in a triangulation survey. One error in addition may cause the bookkeeper a night's work. A slight mistake in the Nautical Almanac may cause hundreds of lives and the loss of millions of dollars' worth of labor.

These last are cases where results depend upon the use of mathematics. Mathematics exists for the results to be obtained from its use. I know we hear a great deal about its value as a discipline and a mind trainer. I have no time and no desire to discuss that question here, and will only say in passing that it is my firm conviction that much of this talk of general mind training is not in accord with the facts of the case.

Mathematical study trains the mind to mathematical thought but to no other. A man may be a first class reasoner in geome-

try but a faulty reasoner in every other line of thought. No! mathematics exists in the school curriculum as well as everywhere else no more for the purpose of training the mind than does English exist for that purpose. The end of language is communication and preservation. Language is a failure when it does not communicate thought.

Mathematics fails then when it does not produce correct results. An algebraic equation is a mechanism which turns out results which are predetermined and which cannot be changed after the process is once started. The result must come right if the mechanism is properly handled. And it is this result we are after and not the motion of the crank. The point I wish to enforce is this:—even though a student understands the solution of an equation, if his results are not correct his work is of no value. I repeat of *no* value. He may have gotten some pleasure and profit out of the work but he has accomplished nothing—in the language of the physicist he has done no work.

It is at such a point as this where I feel assured many teachers encourage inaccuracy. The student gives evidence that he understands the theory but his results are incorrect; it is the practice of many teachers to mark say 50 or 60 or 70 or 80 or 90 for the correct method. Such work is really valueless and deserves no credit. The student says, "can I get no credit for the know how?" "I say no, unless you can do something." The physics teacher will accept no work of that kind. The work must be within the possible degree of accuracy obtainable under the circumstances.

In my opinion we wrong the pupil when we allow him to feel that 90 per cent work is all right. The 90-per-cent-accuracy man can occupy no permanent place or be successful in any of the hundred and one vocations of the day where mathematical precision is essential, and these pursuits are increasing every day. The ever increasing differentiation of scientific and engineering professions is opening splendid opportunities before the young men of to-day, pursuits unheard of a half a generation ago. The line of success in these is closed to the 90 per cent accuracy man and the sooner our school boys and girls are made aware of this fact the better for them. No! after methods are understood only correct work should be countenanced.

The other day I had an occasion to give a final examination in elementary algebra. Among the questions was one involving the solution of a system of simultaneous equations, another in-

volved the evaluation of a determinant, another the evaluation of a fraction involving radical expressions. A student whose paper showed that he understood the theory did not get correct results in any of these questions. His mark was zero and he was told that he could get no credit until he was able to get results. He is a student who aspires to the engineering profession. How long would such a blunderer last in such a profession? Last fall a pupil entered my class in beginning algebra with a report from his grammar school which showed a grade of 95 in arithmetic and yet that boy could not multiply or divide decimals and common fractions and get a correct result, and his father could not understand why he failed in his algebra work.

Another contributory cause to the inaccurate character of the products we turn out is the large number of subjects which we feel called upon to teach. I have not time to discuss this point more than to insist that we ought to teach less in quantity but more in the quality of work required. I will recur to this matter later.

Closely allied to accuracy is a common sense use of mathematics. Said Professor Woodworth at a recent meeting of electrical engineers, "The thing we want to get hold of most is good common sense mathematics." May I give a case or two in point? Last fall I gave to the class in algebra circular discs of cardboard. The diameter was to be measured with a foot rule divided to tenths of inches, estimating the nearest hundredth of an inch, and the circumference calculated.

Here is a sample of the work handed in with a perfectly straight face:

$$\begin{array}{r}
 3.1416 \\
 6.65 \\
 \hline
 157080 \\
 188496 \\
 188496 \\
 \hline
 20.891640
 \end{array}$$

The boy had simply used his figures in a mechanical way with no thought as to their meaning. Not a single member of the class could see anything absurd about the results. It took a long

time to convince them of two things. First, that the third figure of the measurement was doubtful. Second, that that should be taken into account in the calculation. By the device of coloring the doubtful figures this was at last accomplished:

In another exercise of similar character this result was presented.

.03 $\frac{2}{4}$ $\frac{1}{6}$

Students of our school mathematics get hold of the idea that the results of mathematical work must be exact and should be carried out to a large number of decimal places. They are therefore led into the mistake of thinking that exact results can be gotten from inexact data. And so when they have to apply their methods of calculation to the real work of mathematics they perform a large amount of useless and valueless work. But not only does this waste of time and effort follow but more fundamental errors arise. A few summers ago a mathematical professor in one of our leading universities founded a theory on the difference between two units in the tenth decimal place in a series of calculations performed upon five figured data. Another university professor sent back a thesis to be revised because the last figure in a result obtained from a long series of logarithmic calculations did not agree with his own result calculated by another method, when the figures concerned were clear beyond the possible accuracy of the tables used. Such use of figures is entirely unwarrantable and the results are absolutely worthless.

Almost no attention is paid to this important matter in our texts. True a shortened multiplication or division method is often given. But its use is seldom pointed out. The first time a student is brought up face to face with the question is in the physics laboratory. He has been taught that figures are sacred to the last one of a long string and he therefore cannot understand the use made of mathematics in physics. He either goes through a mass of useless and meaningless work or else becoming disgusted with the seeming inexactness of physical calculations becomes careless, thinks that any result will do. Of course it is now devolved upon the physics teacher to correct his mathematical ideas and make them usable. This, I say is a task which should not be put upon him. The pupil should have been taught common sense mathematics in the first place.

Of course one reason for the present condition is this that the pupil is not brought into intimate contact with real problems in which mathematics is required. In a division of twenty-five just

entering our first year work there were but two who could measure a rectangular block of wood to tenths of inches and compute the volume. Not a single one could cut a strip of cardboard so that it should have a given area. A strip $1\frac{1}{2}$ by 10 inches was given. It was required to cut a strip of area 12 square inches and this was the way it was attempted:

Only ten could be cut off.

Our texts abound in scholastic, academic problems that are artificial and unnatural. Here is one taken from one of the best arithmetics with which I am acquainted.

"What is the radius of a circle whose area is 153.9384 square rods?" The answer is exact, of course, using the value of π given in the example at the top of the page. How often do I hear this remark from pupils: "But that answer cannot be correct because it does not come out even." Deliver me from this pep-tonized pabulum which is fed out to students as mathematics. Most of our popular texts on trigonometry are quite humorous in the seriousness with which they compute angles to the tenth of a second when the measured sides are given in three figures and the angles read but to minutes. A problem had better be left out if it cannot be given in a rational, sensible, real way. For, and I wish to repeat this, the interpretation of mathematical formulæ and the ability to express relations in mathematical form is the essence of mathematical training.

A common sense use of figures in calculation should be taught early. By no means does this imply that a careful consideration of the theory of errors be attempted, for that would be impossible; but certain modes of procedure should be given and their reasonableness and inevitability enforced by suitable illustration. The impossibility of exact physical measurement should be revealed. I say revealed, for it is a revelation to the student.

A rectangular block of wood or metal in the hands of each pupil will furnish the material. Let him make the measurements himself and compute the volume.

First say to inches $3 \times 2 \times 2 = 12$. Use inch scale.

Next to tenths of an inch $2.7 \times 2.0 \times 2.2 = 11.88$. Use scale divided to tenths.

Then to hundredths of an inch $2.67 \times 2.01 \times 2.18 = 11.699406$.
Use scale divided to hundredths.

It may be then stated that the teacher has measured more accurately by use of a comparator with the result $2.673 \times 2.002 \times 2.181 = 11.671285626$.

Mark doubtful figures in red.

The student now sees the force in the practical rule; calculated results should show no more significant figures than the measured data. He sees or rather feels the need of a way of getting rid of so much useless calculation and the common method of contracted multiplication is evolved.

There should also be discussion of the closeness of approximation to be used in various circumstances. Measurement of farm areas and lengths, city lines and lots; the measurement of the carpenter and the machinist; the county surveyor and the U. S. Geodetic survey.

If our problems were more intimately connected with actual work we would have fewer pupils like one in our entering class who was puzzled over what to do with the 4 in this problem: One of the largest trees in Illinois is a cypress in Pulaski County. It was found to be 21.6 ft. in girth, 4 feet above the ground. What is its diameter?

All this work can be done in arithmetic and is decidedly more mathematical than the division of

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taken from the same arithmetic mentioned a few moments ago.

Elementary mathematics without a common sense interpretation of results is mathematics gone mad. Mathematics should be taught as it is used. Instruction in it should keep pace with the practice of those who daily use it in the serious work of the world. It must then be common sense and reasonable.

(To be continued)

There is a shortage of teachers in the various school districts in eastern Washington, some of the counties having places for as many as thirty competent teachers. This shortage also appears in northern Idaho and eastern Oregon, where new schoolhouses are being opened.

The high school at Rosalia, which ranks as one of the finest school structures in eastern Washington, has been opened with a large enrollment. The teaching force is limited, but will be enlarged as soon as teachers can be obtained by H. N. Showalter, county superintendent. E. A. Bryan, president of the state college, was one of the speakers. The structure cost \$25,000.