SCHOOL SCIENCE AND MATHEMATICS

VOL. IX. No. 4

CHICAGO, APRIL, 1909

WHOLE No. 69

FORCE AND TORQUE.*

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When your secretary courteously extended me an invitation to say something on this occasion, I was not inclined, either by taste or by superabundance of time and energy, to accept; yet I find myself always unable to forego an opportunity to advocate what appears to be the simplest point of view in scientific thought.

One thing is certain; you will not expect me to say anything new on a subject so old as dynamics. But there are other facts of importance besides those recently discovered; and sometimes very old truths need rediscovery. So important, indeed, are clear views on elementary mechanics that even if I make not a single suggestion of value to any one of you, I shall nevertheless feel justified in directing attention to a matter so vital in the pursuit of physical science.

The time is not very far past—if past it is at all—when the chief aim in the science of physics was to explain all physical phenomena in terms of mechanics. But I believe there is now a consensus of opinion, that what physical science seeks above all else is unity of view. Whether this unitary conception be dynamical or electrical, or something neither dynamical nor electrical, is a matter of secondary importance. The main search is after objective unity in our system of experience.

But even if unity should be secured by reducing ordinary inertia to an electrical phenomenon, this will not in the slightest degree affect the commanding position which dynamics now occupies in the science of physics.

In this dominant branch of natural philosophy, there is one idea about which, more and more, the entire subject of mechanics has come to center itself, namely, the idea of *force*.

*A lecture delivered before the C. A. S. & M. T., November 27, 1908.

Before this assembly, one need hardly recall that the "fundamental" quantities, so-called—length, mass, and time—are fundamental only by the sense that standards of these three quantities are at present more conveniently prepared and preserved than those of any other three.

If in the future it should become more practicable to employ another set of fundamental units, as *length*, *force*, and *time* are now used by the engineer, the change can be made without modifying a single dynamical law. The choice of these three units from which all others shall be derived is then a mere bit of opportunism and depends in no way upon anything fundamental in their nature.

Quite otherwise is it with the idea of force, the one idea which is involved in practically all the laws of mechanics, the one idea in terms of which nearly all mechanical explanations are conceived, the one idea whose introduction into modern thought is synonymous with the creation of the science of mechanics.

In emphasizing the fact that force has been the fundamental idea for two hundred and seventy years, and is likely to remain so for a long while, I am not unmindful of the brilliant attempt made by Hertz, at the very close of his too short life, to substitute for force a set of mechanical constraints (sometimes visible, sometimes invisible) from which by application of Gauss' theorem of Least Constraint, all the phenomena of nature may be reduced. Nor do I forget various attempts to make energy assume the present position of force in the foundation of dynamics. But I think it may be fairly said that none of these new systems have yet proved their superiority to the classical mechanics. At the same time, we must frankly admit that this may be merely another way of saying that the time is not yet ripe for them. And this admission we make all the more cheerfully, since our present system of mechanics which employs forces most freely, uses them, as Hertz suggests, only as idle pulleys. It remains yet to be shown that a system which does *not* contain these idle pulleys will work smoothly.

Unfortunately this term force is one of the most abused in the entire English vocabulary. The twenty-three widely different definitions of it contained in the great Oxford dictionary occupy no less than six columns of fine print.

My purpose is therefore, first of all, to direct your attention to the one single clear and definite sense in which the word

force is employed in physics. I was on the verge of saying *modern* physics, but when I recall the fact that we owe the idea to Galileo, that Newton, and Huygens, and D'Alembert, and Lagrange, and Helmholtz, and Kelvin have, in royal succession, employed the term in the same sense with Galileo, it becomes plain that the accurate use of the word is not confined to our own generation, or even to the last century, but covers the entire history of physics.

If certain instances of misuse do occur, it will be found either that they do not happen in the writings of men acknowledged as scientific authorities, or that as in the case of Lagrange, who is criticised by Hertz for defining force as a "tendency," the writer's words do not correspond with his practice.

Not infrequently does it happen that the clearest conception of an idea is obtained by going back to the master mind that first felt the need for that idea, and invented a term to connote it; in this case we go back to Galileo, who unmistakably laid the foundation of dynamics in his "Dialogue concerning two new sciences," published in 1638. He it was, who by the introduction of the experimental method, also made dynamics, from the very start, a branch of physics rather than of mathematics.

Previous to the time of the great Italian, men were familiar with the fact that certain bodies, such as "weights" and springs, produced certain pressures; and these pressures were detected by the muscular sense; or, as the Germans put it, the *Gegenstand* was recognized by its *Widerstand*.

But concerning the cause of this pressure and concerning the appropriate measure of this effect the predecessors of Galileo were completely in the dark. Whether, for instance, the pressure of a body held in the hand was determinative of the distance through which this body would fall in a given time or of the velocity it would acquire in so falling, or of the acceleration it would exhibit, was by no means clear, previous to the publication of the *Dialogues*. At this point Galileo made three great strides:

(1) He showed that in the case of any single heavy body, this quantity, which had hitherto been thought of as pressure, really determined the acceleration of the body when it fell.

If a body were held on a steep inclined plane where the pull along the plane was great, then the acceleration along the plane was also great when the body was released. If the slope of the

plane was slight, the pressure along the plane was slight, and the acceleration was slight. A body when supported vertically exerted its maximum pressure and experienced its maximum acceleration when released. Into velocity and distance entered other factors so that neither speed acquired nor space fallen were proportional to the pressure alone.

(2) But, secondly, Galileo found by experiment that the acceleration of gravity is the same for all bodies regardless of their size; from which fact he must have seen that the change of pressure which the hand experiences when a one pound weight is replaced by a ten pound piece is not therefore due to a change in *acceleration*. Thus without ever reaching the conception of mass as measured by inertia, he did prove that the vague something hitherto called pressure depends upon two separate factors—and that one of these is acceleration.

(3) The third of Galileo's achievements is a confession of ignorance. It was clear to him that friction was unavoidable, that all motions were in fact accelerated, and that the cause of this acceleration was in no case understood. Very well then, said he, let us confess our ignorance, let us preserve a becoming modesty, and say that whenever a body changes its velocity, we do not in general understand the cause, but *in our thought*, we will substitute for the cause something which we shall call "force," and this force we shall measure by the acceleration which it produces.

Force is therefore, merely a fictitious something with which we agree to replace an unknowable cause when we observe a body under acceleration.

This, as I conceive it, is the unique sense in which *force* has always been used by the physicist.

In the *Dialogues* it is Salviati, the character who represents Galileo himself, that is made to say, "It is in vain that we search after the causes of natural motions, concerning which each philosopher entertains a view which is different from that of every other philosopher." Newton at the very outset of the *Principia* repeats this idea when he says, "For I here design to give a mathematical notion of those forces without considering their physical causes and seats." * * * Wherefore the reader is not to imagine that by these words, I anywhere take upon me to define the kind of manner of any action, the causes or the physical reasons thereof.

The ancient dictum that "science is the search for causes and philosophy for the cause of causes" would therefore hardly meet the approval of either Galileo or Newton. Indeed, so far from considering force as a cause, Galileo would rather regard it as an effect, produced by unknown causes, but measured by the acceleration of the body. Science avoids causation, seeks classification, desires unity.

By confessing that the question of causation lies beyond the range of physics, Galileo delimited the subject in a manner which has enabled it to make tremendous strides. It is precisely by throwing aside, so far as possible, all metaphysical considerations that the physicist has been able to make trustworthy predictions and that the engineer has become a prophet.

The importance of a clear definition of force is not likely to be overestimated. The lack of such a definition is well illustrated by the history of the idea of "vital force," an idea which for want of definite description has never borne any fruit. The belief that there is something more involved in physiology than mere chemistry and physics is all but universal; just what this "something" is, however, and just how it may be appropriately described or measured, no one yet appears to know. Until this step is made, "vital force" will probably remain an idea too general and indefinite to be useful.

Mass.

The definition of force was completed and perfected when Huygens and Newton discovered the variability of gravity. Newton keenly appreciated the meaning of this discovery and on the first page of his *Principia* assigns the name of "Mass" to that factor of weight which does not vary when the acceleration of gravity changes—the factor which remains constant even when a body is removed from the earth to the orbit of the moon.

Newton's Third Law.

One other achievement of Newton completed the foundation of modern dynamics. Even before the birth of this great Englishman, Galileo had thoroughly grasped the Second Law of Motion, and had frequently employed it in the theory of projectiles. More than that, Galileo had correctly conceived the First Law of Motion, expressing it as merely a special case of the Second Law. However, the notion that there are always two forces involved in every dynamical transaction, had not dawned upon the Italian pioneer.

It remained for Newton to establish, and to describe as his

Third Law, the general statement that whether the action between two bodies results in acceleration or merely in a stress, two equal and oppositely directed forces are always involved.

In the case in which this interaction is a stress, i. e., in the case of mutually equilibrating forces without acceleration, there is no ambiguity or difficulty.

Poggendorff's Experiment.

When, however, the only action of a force is to produce acceleration, the reaction is not always so patent. The following experiment, which is described by Poggendorff in *Ann. der Physik*, 92, 179 (1854), and reproduced in Mach's *Mechanics*, exhibits the mass-reaction of an accelerated body in the most convincing manner. (*Experiment shown.*)

The Role of Analogy.

Pursuing the same subject, namely, the simplest point of view in these matters, let us pass for a moment to the dynamics of rotation. For here I venture to think, there has been an immense amount of time and energy needlessly wasted over problems which, when solved by translational methods, are exceedingly complex and which in the light of analogy are exceedingly simple.

When one recalls that in modern science any phenomenon is said to be explained only when it has been pigeon-holed with other phenomena which we already understand, it is not a little surprising that in the teaching of mechanics the use of analogy is so seldom employed. These ideas I wish to illustrate by the explanation of three fundamental phenomena.

The Gyroscope.

The first of these is the behavior of a rotating wheel when its axle is supported at one end only.

(*Experiment with bicycle wheel shown.*)

To make this matter clear, one has only to bear in mind that mass in translation is the strict analogue of moment of inertia in rotation; that linear displacement corresponds rigidly to angular displacement, and that the one remaining fundamental unit, time, is identical in both.

Then just as in the case of a particle moving with uniform circular motion, the centripetal force—the force which produces this motion—is at right angles to the direction of momentum, mv , is at right angles also to the axis about which the direction of momentum is rotating with an angular speed ω , and is numerically equal to the vector product of these two.

$$F = [mv\ w],$$

so also, in the case of the bicycle wheel, the torque which produces precession is at right angles to the axis of the wheel's angular momentum, $I\ \Omega$, at right angles also to the axis of precession, and numerically equal to their vector product

$$L = [I\ \Omega\ w]$$

In like manner, the centrifugal force which may be considered as the reaction of the centripetal is strictly an analogue of the torque produced by a precessing wheel to which may be considered as the reaction of the torque required to produce precession. Their numerical values are those of F and L , given above, with their signs changed.

Schlick's Ship Steadying Device.

The next illustration which I wish to consider is one identical with the bicycle wheel, except that here we shall consider the work done, while there we considered the torque exerted.

(Experiment with rolling-ship model shown.)

Let this square iron frame represent a transverse section of a vessel which is rolling, not pitching, in the sea. The vibration of the attached pendulum produces the roll of the vessel. A heavy wheel is placed amidships and is set spinning about a vertical axis.

Parallel to the keel of the ship is attached a rod which connects the lower end of the oscillating frame of the spinning fly-wheel with a dash-pot or friction-brake.

The angular momentum of the heavy fly wheel $I\ \Omega$, is about a vertical axis; the precession, w , is produced by the rolling; and is about a horizontal axis parallel to the keel.

It follows, therefore, after the analogy of centrifugal force, that the torque $[I\ \Omega\ w]$, exerted by the fly wheel is about an axis athwart-ship.

Now anything which opposes this torque will oppose the rolling of the ship; and if this torque is allowed to do work, this work will be at the expense of the energy of roll of the ship. Schlick therefore introduces a dash-pot filled, I believe, with water and glycerine to absorb the work done by this athwart-ship torque.

The same expression for moment of force, $I\ \Omega\ w$, may be derived from translational dynamics; but the route is longer, and, for most minds, vastly more difficult.

To show that the mere exertion of torque, without absorption

of energy will not diminish the roll, we have only to substitute, as I shall now do, for the dash-pot a spring which will return to the ship during *one* half the roll, practically all of the energy imparted to the spring during the *other* half of the roll. (*Experiment shown.*)

Brennan's Monorail Car.

A third illustration of the simplicity of view-point obtained by use of rigid analogy is furnished by the monorail car recently devised by Mr. Brennan in England.

This machine consists essentially of an iron car-frame upon which is mounted a horizontal, fore-and-aft axis. This fore-and-aft axis in turn carries a pair of vertical frames in each of which a heavy fly wheel is kept spinning about a horizontal axis.

These frames with their fly wheels have three distinguishing features:

(1) The entire load—frame, wheels, and all—is mounted on the fore-and-aft axis so that it can rotate freely, yet with its center of gravity in the axis, so that, of itself, it has no tendency to roll.

(2) These two wheels are maintained spinning, in opposite senses, about the axes transverse to their respective frames.

(3) These two frames can each precess about a vertical axis; but they are geared together in such a way that the precession of one is equal in amount but opposite in sense to that of the other. The result is that one wheel counterbalances the effect of the other except about a fore-and-aft (longitudinal) axis; and here they combined their effects. If now the car frame tends to roll about the single track as an axis (i. e. to upset) only one of two things can happen:

(a) The car frame may strike against the projecting axle of one of the fly wheels, which will not only prevent the car from upsetting farther, but will hurry on the precession by rolling friction of the shaft in such a way as to completely right the car.

(b) Or it may happen that the car frame will strike a projection on one of the wheel frames, by which the car will be prevented from farther upsetting and by which also precession will be caused, such that the live axle will a little later be thrown into action and the car righted exactly as in the first case.

Each side of the car frame is provided with the same kind of stops or lugs, so that whichever way the car starts to roll, it is at once prevented from rolling farther; and not only so, but the torque exerted by the car on the gyroscopic wheels is such as to hurry on the precession and to roll the car back until its center of gravity lies vertically over the track.

(*Monorail car shown.*)

Here again the righting torque of each fly wheel is measured by $I \Omega w$ where I is the moment of inertia, Ω the angular velocity of each wheel, and w the rate at which the wheel frame is precessing.

This expression is identical with that found in the case of the bicycle wheel, and in the ship-steadying device, and is identical in form with mvw , the ordinary expression for centrifugal force.

* * *

While most of what I have said is both old and obvious, I trust nevertheless, that I have brought from the background into the foreground the three following ideas:

- (1) Galileo's definition of force, presenting it not as a cause of motion, but as a measurable circumstance of actual motion.
- (2) Newton's idea of mass and mass-reaction, and
- (3) The power of exact analogue in dynamics.

The Cambridge Botanical Supply Co., Cambridge, Mass., have just issued their 1909 catalogue of botanical supplies. It is an up-to-date and complete catalogue, containing everything the teacher of botany needs for laboratory or lecture room work. Descriptions and photographs of the many special devices of their own make are given. There is also a very complete list of microscopical and lantern slides. The special feature of this issue is the listing of Perret's volcanology slides, for which they have the exclusive agency for this country and Canada. Fifty-six slides, made from actual photographs, showing every phase of volcanic eruptions, are listed. Every teacher of geology and physiography should be interested in these slides.

Chemistry teachers will be interested in an article entitled "Making Poison by the Ton," which appeared in the *Technical World Magazine* for January, 1909. It furnishes a good illustration of the use of the process of sublimation. The ore, a quartz rock, contains the arsenic combined with iron and sulphur, and by roasting the arsenic oxide is formed and this condenses in brick flues four feet wide, six feet high, and three hundred yards long. These flues terminate in tall stacks, but even this long trap does not prevent the escape of some arsenic to wreak vengeance on the vegetation of the surrounding country. One smelter near Everett, Wash., produces and ships to New York thirty-five tons of this poison each week.