A Model illustrating the Propagation of a Periodic Electric Current in a Telephone Cable, and the Simple Theory of its Operation

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of the magnet-carriage which he had mentioned, 0.03 mm., was that which was obtainable when working under observatory conditions. With regard to the material of the deflexion-bar, he pointed out that a sufficiently accurate temperature correction was always employed to correct for thermal expansion. There was no invar which was sufficiently non-magnetic to be used in accurate work with a magnetometer. With regard to the relation between pole-distance and length of magnet, he said that Dr. Börgen had carried out experiments on different magnets under different conditions, and had concluded that the pole-distance was a nearly constant fraction of the length of the magnet; but this was not altogether borne out by the present investigation.

XXVIII. *A Model illustrating the Propagation of a Periodic Electric Current in a Telephone Cable, and the Simple Theory of its Operation.* By J. A. FLEMING, M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London*.

[Plate XIV.]

THE propagation of alternating currents of electricity along linear conductors has been the subject of much mathematical investigation. Its importance in reference to the practical operation of submarine cables, telephone-lines, and more recently of the radiating antennae in Hertzian wave telegraphy, is well understood. Nevertheless, practical telegraph and telephone engineers have not always shown readiness to assimilate the ideas elaborated by theorists, and in some cases unsound theories of the phenomena have been promulgated. The assistance which practical men derive from considering the operation of a working model when appropriating physical ideas is considerable, and the author has found that for teaching purposes the model here described is of great use in making plain the meaning of mathematical expressions.

The simple theory of it may first be given. The object is to explain the propagation of a periodic or alternating electric

* Read June 10, 1904.

Fig. 1.

Fig. 2.

Fig. 2.

current in a telegraph or telephone cable having resistance, inductance, capacity, and leakage, or insulation conductance of a certain value per mile or knot.

The discussion necessarily proceeds on the assumption that the currents and electromotive forces with which we are concerned are of simple harmonic form, and accordingly the only mathematical symbolisation necessary is that required for representing a simple harmonic motion or simple sine function and its time-rate of change. This is most readily achieved by the use of the complex quantity to denote the maximum value and phase of the periodic quantities with which we are concerned. The representation of a vector quantity by the symbol $a + jb$, where a and b are lengths of lines and j is the *sign of perpendicularity* denoting that b is at right-angles to a is now so universally understood that no further explanation of it is necessary.

Consider, then, in the first place, an electric cable indefinitely extended in one direction, and at the terminated end a simple periodic electromotive force applied. Let the cable have a capacity C farads, an inductance L henrys, a copper resistance R ohms, and a dielectric or insulation resistance r ohms, all per unit of length.

If we take any element of length in the run of the cable, then there is a difference between the potential and the current at the two ends of this element. Let δv and δi be this difference, v and i being the potential and current at the generator end of the element. Then the symbolical expression for the manner in which this difference of potential and of current is created, is given by the familiar equations :

$$L\delta x \frac{di}{dt} + R\delta x i = \delta v, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$C\delta x \frac{dv}{dt} + K\delta x v = \delta i, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where K is written for $1/r$. Hence

$$L \frac{di}{dt} + Ri = \frac{dv}{dx}, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$C \frac{dv}{dt} + Kv = \frac{di}{dx} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If i and v both vary in a simple harmonic manner, and if I and V are their maximum values during the period; then, since the maximum values of di/dt and dv/dt considered as vectors are at right angles to the maximum values of i and v , we may write (3) and (4) in the form

$$\frac{dV}{dx} = (R + jpL)I, \quad . \quad . \quad . \quad (5)$$

$$\frac{dI}{dx} = (K + jpC)V, \quad . \quad . \quad . \quad (6)$$

where j is the sign of perpendicularity, and $p = 2\pi$ times the frequency n of the current.

Separating the variables by differentiation, we have

$$\frac{d^2V}{dx^2} = (R + jpL)(K + jpC)V, \quad . \quad . \quad . \quad (7)$$

$$\frac{d^2I}{dx^2} = (R + jpL)(K + jpC)I, \quad . \quad . \quad . \quad (8)$$

The above are the well-known vector equations for the propagation of a simple periodic current in a cable having the four constants L, R, C, K .

If we write P for $\sqrt{R + jpL} \cdot \sqrt{K + jpC}$, we have the equations

$$\frac{d^2V}{dx^2} = P^2V, \quad . \quad . \quad . \quad (9)$$

$$\frac{d^2I}{dx^2} = P^2I, \quad . \quad . \quad . \quad (10)$$

as the differential equations for the potential and current at any point in the cable.

An obvious solution of the above equations is

$$V = a\epsilon^{+Px} + b\epsilon^{-Px}, \quad . \quad . \quad . \quad (11)$$

$$I = \frac{P}{R + jpL} \left\{ a\epsilon^{+Px} - b\epsilon^{-Px} \right\}, \quad . \quad . \quad . \quad (12)$$

where a and b are constants of integration, and ϵ is the base of Napierian logarithms. The quantity P is clearly a complex quantity and can be expressed in the form $\alpha + j\beta$.

Hence if $\sqrt{R+jpL}\sqrt{K+jpC}=\alpha+j\beta$, we must have:—

$$\alpha^2 + \beta^2 = \sqrt{R^2 + p^2 L^2} \sqrt{K^2 + p^2 C^2}$$

and

$$\alpha^2 - \beta^2 = RK - p^2 LC,$$

or

$$2\alpha^2 = \sqrt{(R^2 + p^2 L^2)(K^2 + p^2 C^2)} + (RK - p^2 LC), \quad (13)$$

$$2\beta^2 = \sqrt{(R^2 + p^2 L^2)(K^2 + p^2 C^2)} - (RK - p^2 LC). \quad (14)$$

The above equations are well-known expressions. We shall call α and β the secondary constants of the cable.

Returning, then, to the case of the semi-infinite cable, we shall take the origin of coordinates at the end at which the simple periodic electromotive force of maximum value E is applied.

Hence for $x=0$ we have $V=E$, and for $x=\infty$ we have $V=0$.

Since, however, in obtaining the original differential equations, we reckoned x in the direction in which potential and current *increase* in value, we have to write $-x$ for x in the general solutions (11) and (12) before we can apply them to the above particular case. Making this change and determining the constants a and b from the terminal conditions, we arrive at the particular solutions of (9) and (10) applicable to the case of a semi-infinite cable having a periodic electromotive force E applied at one end. The potential V and the current I at any point in the cable at a distance x from the generator end is then given by the equations

$$V = E\epsilon^{-Px}, \quad (15)$$

$$I = E\epsilon^{-Px} \frac{\sqrt{K+jpC}}{\sqrt{R+jpL}} (16)$$

It should be clearly understood that in speaking of the potential or current at any point in the cable we mean the maximum value of the potential or current regarded as a vector.

Since $P=\alpha+j\beta$ and $\epsilon^{\pm j\beta x} = \cos \beta x \pm j \sin \beta x$, we can write (15) and (16) in the form

$$V = E\epsilon^{-\alpha x} (\cos \beta x - j \sin \beta x), \quad (17)$$

$$I = E \frac{\sqrt{K+jpC}}{\sqrt{R+jpL}} \epsilon^{-\alpha x} (\cos \beta x - j \sin \beta x). \quad . . (18)$$

Let us notice the physical signification of the above expressions. Equation 17 shows us that the potential V in magnitude and direction at any point at a distance x in the cable, reckoned from the generator end, is obtained by multiplying the magnitude of the terminal applied voltage E by two factors, (i.) by the attenuation factor e^{-ax} , and (ii.) by the phase factor $(\cos \beta x - j \sin \beta x)$. At any distance x from the origin, V is less than E in magnitude in the ratio of e^{-ax} to 1 and is shifted backwards through an angle βx relatively to E as regards phase.

This is easily seen to be the case if it is remembered that $\cos \theta + j \sin \theta$ is a rotating operator, and when it operates on a vector $a + jb$ it gives as a result a vector of the same size, viz. $\sqrt{a^2 + b^2}$, but shifted forwards through an angle θ . The size of the vector $(a + jb)(\cos \theta + j \sin \theta)$ is $\sqrt{a^2 + b^2}$, but its slope ϕ is such that $\phi = \tan^{-1} \frac{b}{a} + \theta$ where $\tan^{-1} \frac{b}{a}$ is the slope of the original vector $a + jb$. Hence the expression (16) merely means that the potential at any point in the cable whose distance from the origin is x is less than E in magnitude in an assigned ratio, and is shifted backward in phase by an assigned angle.

If x' is the distance from the generator end at which the maximum voltage has fallen to half the numerical value at the origin, then x' is given by the equation

$$\frac{1}{2} = e^{-ax'},$$

$$\text{or} \quad x' = \frac{0.7033}{a} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Again, since $\cos \beta x = \cos (\beta x + 2\pi)$ it is obvious that the wave-length of the potential wave is $2\pi/\beta$, and therefore the velocity of propagation of the potential wave is p/β .

These facts may be presented graphically as follows:—Take a line OX (see fig. 1, Pl. XIV.) to indicate the cable, and set up a perpendicular OE to represent in magnitude and direction, the voltage at the generator end. Then at equidistant points draw other lines decreasing in length in geometrical progression, and each shifted backwards or forwards in direction relatively to the preceding line by an equal angle. If we

suppose these lines to revolve with equal angular velocities round their ends as centres situated at equidistant intervals on the line OX; then their projections at the same instant on vertical lines drawn through their centres will represent at that instant the actual voltage at these points in the cable. The periodic change with time and distance may be represented by a working model made in the following manner:— On a long steel axle AA' are fastened a number of eccentric pulleys E₁, E₂, E₃, &c. (see fig. 2, Pl. XIV.). The eccentricities of these wheels decrease in geometric progression, and each eccentric is set in phase backward behind its preceding neighbour by an equal angle. These wheels are embraced by cords C₁, C₂, C₃, &c. attached to balls or blocks of metal P₁, P₂, P₃, &c. sliding on vertical rods R₁, R₂, R₃, &c. placed below each eccentric wheel. These cords are all of the same length.

When the axle carrying all the eccentrics is revolved by a handle W, the blocks P₁ &c. will rise and fall with a nearly simple harmonic motion, and at any instant all the blocks will be situated on a sinuous curve of continually decreasing amplitude. As the eccentric axle revolves the motion of the balls will depict the progression of a wave of potential along a cable having capacity, inductance, resistance, and leakage.

Returning, then, to the expressions (13) and (14) for the value of α and β , we may give these more convenient forms.

Taking the formulæ (13) and (14), add to, and subtract from, the quantity under the radical the term $2p^2CLKR$, and after rearranging terms we obtain

$$2\alpha^2 = \sqrt{(KR + p^2LC)^2 + p^2(LK - CR)^2} + (KR - p^2LC) \quad (20)$$

$$2\beta^2 = \sqrt{(KR + p^2LC)^2 + p^2(LK - CR)^2} - (KR - p^2LC) \quad (21)$$

If the cable constants have such relation that $LK - CR = 0$ or $L/R = C/K$, we have Mr. Oliver Heaviside's distortionless cable, and then obviously when this is the case

$$\alpha = \sqrt{KR}, \quad \beta = p \sqrt{CL}. \quad . \quad . \quad . \quad (22)$$

Hence we have for the abscissa of half voltage x' and the

potential wave-velocity w the values

$$\alpha' = \frac{0.7033}{\sqrt{KR}}; \quad w = 1/\sqrt{CL}. \quad . \quad . \quad . \quad (23)$$

Both these quantities are independent of the periodicity, and hence for this cable all potential waves travel at the same rate and are attenuated in the same ratio.

If, therefore, such a cable could be practically constructed with a small enough value for $\sqrt{KR}$, telephonic speech would be possible through it with results not possible in the case of non-distortionless cables.

The model shows clearly that the attenuation of the simple periodic wave depends on the value of α , that is on the variation of eccentricity of the wheels, and that the wave-length is determined by the value of β .

It is not then difficult even for a non-mathematician to comprehend that if a complex periodic electromotive force acts at one end, which may be resolved into the sum of a number of simple periodic electromotive forces, the result in general will be to produce corresponding simple periodic waves which travel at different speeds and degrade at various rates. Hence, in a comparatively short distance the initial complex wave form is so altered that it becomes unrecognizable. Herein, therefore, resides the cause of the practical limitations of telephony. The remedy for it rests in the alteration of the primary constants of the cable so as to bring about the necessary changes in the secondary constants.

There are, however, necessary practical limits to these changes. In overhead lines where capacity is relatively small, it is not difficult to introduce sufficient inductance (as Prof. Pupin has done) in the form of separated inductance-coils, and provided these are sufficiently close together within the limits of a wave-length the result is equivalent practically to continuous inductance. The same remedy for distortion could no doubt be applied to underground telephone trunk-lines, although the writer is not aware that it has been tried.

On the other hand, an addition to the inductance is not easily made in the case of a submarine cable. Any interference with the continuity of size or great increase in bulk or weight would hardly be permitted.

Hence, in the case of cables the favourite prescription for the defect has been that of creating a large dielectric conductance by artificial leaks. The effect of any such proposed added leakage can easily be foretold.

Taking the expressions for $2\alpha^2$ and $2\beta^2$ in (13) and (14) we may write them in the following form:

$$2\alpha^2 = pL \left(1 + \frac{R^2}{p^2 L^2}\right)^{\frac{1}{2}} pC \left(1 + \frac{K^2}{p^2 C^2}\right)^{\frac{1}{2}} + (RK - p^2 LC) \quad (24)$$

$$2\beta^2 = pL \left(1 + \frac{R^2}{p^2 L^2}\right)^{\frac{1}{2}} pC \left(1 + \frac{K^2}{p^2 C^2}\right)^{\frac{1}{2}} - (RK - p^2 LC). \quad (25)$$

If we employ any such frequency as 100 to 200, then p is a number of the order of 1000. In all actual well-insulated submarine or underground cables, the quantity K^2/C^2 per mile is a small quantity of the order of 10^{-4} , whilst R^2/L^2 is a large number of the order of 10^4 or 10^6 . Hence, if the frequency employed is anything of the order of that of the waves of the speaking voice, the quantity $K^2/p^2 C^2$ can be neglected, whilst the value of $R^2/p^2 L^2$ may approximate to unity, or say to a number such as 2, 3, or 4 near to unity. Accordingly, in practice we might have for the value of $\left(1 + \frac{R^2}{p^2 L^2}\right)^{\frac{1}{2}}$, reckoned per mile or per knot, a number of the order of, say, 2.

Hence, in this case, the values of α and β per mile would reduce to:—

$$2\alpha^2 = p^2 LC + RK \quad . \quad . \quad . \quad . \quad . \quad (26)$$

$$2\beta^2 = 3p^2 LC - RK \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Again, in the case of all well-insulated cables the value of RK is very small compared with $p^2 LC$ for such frequencies as above mentioned, and hence as a first rough approximation we can calculate the values of α and β for most actual submarine cables from the expressions

$$\alpha = \frac{p \sqrt{LC}}{\sqrt{2}}, \quad \beta = p \sqrt{\frac{3LC}{2}}.$$

In this case α is a function of the frequency, but the wave-velocity p/β is independent of the frequency. Accordingly there is attenuation depending on frequency. For any well-

insulated cable, however, we have the value of α nearly given by the equation,

$$2\alpha^2 = p^2 LC \left(1 + \frac{R^2}{L^2 L^2}\right)^{\frac{1}{2}} + RK - p^2 LC; \dots \quad (28)$$

and from this equation combined with (19) we can always calculate the distance for drop to half initial voltage or current, which may be roughly said to represent the limiting distance of good telephony.

Suppose, for instance, that we consider as an example any long submarine cable, say the Ireland-Newfoundland (1880) cable. The length (l) is 1423 knots = $264 \cdot 2 \times 10^6$ centims. The resistance per knot (R) = 4.161 ohms, the capacity per knot (C) = 0.315 microfarad, the insulation resistance per knot ($1/K$) = 400–500 megohms*.

The value of the inductance per knot (L) is not so definitely known. The copper core weighs 300 lbs. per knot, and is equivalent in section to a round wire of 0.34 centim. in diameter. As a first approximation, if we calculate the total inductance L from the formula for the inductance of a straight single wire,

$$L = 2l \left(\log_e \frac{4l}{d} - 1 \right) \dots \dots \dots (29)$$

we find $L = 11.5$ henrys, from which we deduce the average inductance per knot (L) to be 0.008 henry.

But the iron armour may increase this value 100 or at most 200 times. Hence the value of L/R per knot may be as low as 0.002, or perhaps as great as 0.02 or 0.4. On the other hand, the value of C/K per knot is 150 nearly, and hence the cable is very far from being distortionless.

If we calculate α corresponding to a value of $n = 170$, $p = 1000$, and an inductance 0.008 henry per knot, then we have $\alpha = 0.012$, and the distance for drop to half initial voltage is 60 knots. If we attempted to cure distortion by decreasing

* The above data are taken from Munro and Jamieson's 'Pocket-book of Electrical Rules and Tables,' 4th ed. p. 286. The term *naut* is used by some writers to denote a nautical or sea mile, a length of 2029 yards, whilst others employ the term *knot* for the above length. Strictly speaking a knot is the name for a speed, viz. one nautical mile per hour.

the dielectric resistance per knot to $1/150$ of a megohm, or about 6000 ohms, then we could make α independent of wavelength, but it would be increased in magnitude to a value 0.021, and the attenuation would be worse. On the other hand, if we could increase the inductance per knot to 0.8 of a henry, the dielectric resistance being only reduced to $2/3$ rd of a megohm per knot, the distortion would be cured, and the value of α reduced to 0.0021.

The distance for decay to half initial voltage would be raised to 350 knots, and a great improvement effected. This is only an illustration of the importance of inductance on which Mr. Oliver Heaviside has insisted for so many years. On the other hand, the difficulty is to apply the theoretical remedy practically in the case of submarine cables, and it is not easy to see how it is to be done.

Suppose, however, that we could make the resistance per knot 3 to 4 ohms, the capacity per knot 0.3 microfarad, the insulation resistance 300 megohms per knot, and the inductance say 300 henrys. We should then have $L/R = C/K = 100$. Hence we should have $\alpha = 0.0001$ and $\beta = 10^5$ corresponding to $p = 1000$. Accordingly the distance for decay to half voltage would be some 7000 knots. Such an achievement may be quite beyond the reach of practical work, but an approximation to it is perhaps not impossible. It cannot be done, however, merely by increasing the iron armour or bringing it nearer to the copper, because in that case the capacity is increased very considerably. The annulment of distortion by creating lateral leakage either through inductive or non-inductive leaks always leads to large values of α . In other words, to great attenuation. Hence, although the distortion of the complex potential or current wave may be cured, the decay of amplitude would extinguish audibility as far as telephonic transmission is concerned within quite moderate distances, and far less than that which could be obtained by the right application of inductance.

As a small contribution to this practical consideration of the problem, the model here described and method of discussing the theory may perhaps be useful.

Note added July 9th, 1904.

In reference to the above method of discussing the problem, it has been represented to the author that it would assist some mathematical readers if the steps by which the usual algebraical expressions for the potential and current at any point in the cable can be deduced from equations (17) and (18), were set out more at length.

The equations (17) and (18) are expressions showing the manner in which the potential and current at any distance x along the cable, considered as vectors, are related to the vectors representing the potential and current at the origin at the same instant.

If we desire to ascertain the state of affairs at any time t we have to consider these vectors as rotating round their extremities with a periodic time T . Accordingly the proper expression for the vector representing the potential at the origin at any time t from the beginning of the epoch will then be Ee^{jpt} , where $p = 2\pi/T$. Hence the corresponding expression for the vector representing the potential at any distance x along the cable at the same instant will be obtained by multiplying equation (17) by $e^{j\beta x}$, and will therefore be given by the equation

$$V = Ee^{jpt} e^{-(\alpha + j\beta)x}$$

$$\text{or by} \quad V = Ee^{-\alpha x} \{\cos(pt - \beta x) + j \sin(pt - \beta x)\}. \quad (17')$$

If, therefore, the numerical value of the potential at the origin at any instant is represented by the real part of Ee^{jpt} , viz. by $E \cos pt$, then the numerical or algebraic value of the potential at the same instant but at the distance x along the cable will be given by the real part of (17'), viz. by

$$v = Ee^{-\alpha x} \cos(pt - \beta x).$$

The above equation is the ordinary algebraic expression for the potential in a semi-infinite cable at any time t and any distance x from the origin. Under the above conditions the time t would be reckoned from the instant when the potential at the origin has its maximum value.

The last equation is the usual expression for a wave of

decreasing amplitude and wave-length $2\pi/\beta$ moving with velocity p/β .

Similar expressions can be obtained for the numerical or algebraic value of the current in the cable at any place and time.

DISCUSSION.

Mr. THOMAS H. BLAKESLEY congratulated Prof. Fleming upon the interesting result contained in his paper, that under a certain critical relationship among the properties of the cable relatively distortionless waves might take place whatever the frequency. But he thought some other of the consequences of this relationship might be more simply exhibited in the forms for Potential and Current :—

$$V = Ee^{-ax} \cos (\beta x - pt),$$

$$I = \frac{Ee^{-ax}}{q} \cos (\beta x - pt + m),$$

$$\text{where } q^2 = \frac{R}{K} \left\{ \frac{1 + \left(\frac{Lp}{R} \right)^2}{1 + \left(\frac{Cp}{K} \right)^2} \right\}^{\frac{1}{2}},$$

$$\tan 2m = \frac{(LK - RC)p}{RK + LCp^2}.$$

q is of the nature of resistance, and m is the angle of lag of the current behind the potential and is constant for all points of the cable.

It will be plainly seen that this lag angle disappears when the condition $LK - RC = 0$, but from the expression for q^2 it will be clear that this quantity will be independent of the frequency if $\frac{L}{R} = \frac{C}{K}$, which is again the same condition.

Thus the real degradation of current would be the same for all tones, and the conjunction of current and potential for all points, a very important matter, secured by this condition. But, as Prof. Fleming points out, it would not be easy in practice to secure the fulfilment of this condition. There is, however, something to be said for keeping both L and K small, as in fact they are in practice. For, looking at the

expression for q^2 , it will be noticed that it will become smaller as the frequency rises if $\frac{C}{K}$ is greater than $\frac{L}{R}$, that is if $CR > LK$, and this is emphatically true of cables in practice. Hence, though $e^{-\alpha x}$ grows less with rising frequency, the resistance q also becomes less; so that the degradation of current is not so great as that implied in the mere increase of α with frequency.

In operating upon the equations (13) and (14) to obtain the equations (20) and (21), it may be well to point out that the quantity

$$\sqrt{(R^2 + p^2 L^2)(K^2 + p^2 C^2)}$$

may be written in the two ways:—

$$\sqrt{(KR \pm p^2 LC)^2 + p^2(LK \mp CR)^2}.$$

Prof. Fleming has chosen the top signs in order to consider the relation $LK - CR = 0$.

But if the bottom signs are chosen we may contemplate the equality of KR and $p^2 LC$, in which case not merely does one of the squares under the radical sign disappear, but the values of α and of β coincide. The quantity $KR - LCp^2$ does in fact vanish when $K=0=L$, which more nearly coincides with fact than Prof. Fleming's critical condition. Perhaps it may be said that preference may be given to one or other of the two cases according as it is desired to transmit the voice as in telephony, or sound signals on the Morse system as in wireless telegraphy. In the latter case we have a great choice in the value of p which may be kept constant.

Mr. F. JACOB said the following corrections should be made to the data of the 1880 Anglo-American Cable. The length is 1852 nauts, the dielectric resistance of the cable as it lies in the sea is of the order of 20,000 megohms per naut, the figure 400 being at 75° F., and atmospheric pressure after 1 minute's electrification, but, on the other hand, the electrification or absorption of the dielectric masks all the effect of the high insulation value.

300 to 400 henrys per naut may be considered as absolutely beyond the realms of possibility, commercially or

otherwise. What is demanded is not of a higher order than one henry per naut.

The model is simply that of a wave motion with attenuation, and he failed to see how it helped the "practical man" to see the advantageous effect of adding inductance to an ordinary Line.

Dr. FLEMING, in reply, said that the relation between the electrical constants of the cable which gave distortionless transmission was discovered and announced twenty years or more ago by Mr. Oliver Heaviside. All that he (Dr. Fleming) had attempted to do was to simplify and illustrate the proofs of this condition. The employment of complex quantities in the discussion of alternating current problems led to a considerable simplification of the analysis. As in practice we were only concerned with the maximum or root-mean-square values of the potential and current, the introduction of the time-variable into the equations, as in the ordinary method of handling the problem, led to complicated expressions, the physical meaning of which was not readily apparent to most students. All the usual algebraic expressions for the current and potential at any point in the cable could, however, be readily obtained from the vector equations he had given.

As regards the practical realization of Heaviside's distortionless condition for submarine cables, the figures given by him (Dr. Fleming) were merely intended to indicate the remoteness of its possibility.

The correction furnished by Mr. Jacob for the insulation resistance of the dielectric when the cable was immersed did not affect the general conclusions, as if the insulation resistance of the cable was in fact greater than 300-400 megohms per naut, the departure from the distortionless condition would be then still greater.

On the other hand, he quite recognized that the so-called "absorption" of the dielectric introduced a factor not taken into account in the mathematical theory. As regards the practical superior limit of inductance which might be given to a cable, no one could say what invention might not bring forth. He was glad to hear that an inductance of one henry per naut might improve matters sensibly, because he had

devised a plan by which an approximation to this might be reached; but Mr. Jacob only confirmed the statement in his paper in expressing the opinion that an inductance of 300 to 400 henrys per naut was impracticable, at least at present and perhaps always.

The model was only intended as an assistance to those whose previous acquaintance with the subject was small, but it had other applications than to electrical problems, and might be used, for instance, to illustrate the decay of temperature waves in the earth's crust, penetration of current or magnetism into bodies, or in fact any class of oscillation covered by the equations under discussion.

Having regard to the fact that electricians of great practical experience in telephony had in the past advocated theories of the subject which were not capable of being reconciled with accepted scientific principles, he thought that an excuse might be made for going again over worn ground. There was still a great field open for advance in cable telephony, and improvements were only likely to be made by examining carefully how far theoretical remedies could be put into practice.

As regards the criticism that a knot is a measure of speed, the Author said that he had followed a custom adapted in most electrical manuals of calling a nautical or sea mile of 2029 yards a "knot," although he was aware that it was also employed as the name for a speed of one nautical mile per hour.