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ON A THEOREM OF RANKINE RELATING TO THE
ECONOMY OF SINGLE ACTING EXPANSION
ENGINES, FIRST PUBLISHED IN 1851.

By PROFESSOR W. P. TROWBRIDGE.

[Read (by title) at the meeting of the American Society Mechanical Engineers, at Philadelphia.]

The object of the theorem above referred to was, in the words of the author, "to investigate and explain the method of determining the rate of expansion, and consequently the dimensions and proportions of a Cornish engine which, with a given maximum pressure of steam in the cylinder, at a given velocity, shall perform a given amount of work at the least possible pecuniary cost, taking into account the expense of fuel and the interest of capital required for the construction."

He further states, as a fundamental proposition, that "by increasing the ratio of expansion in a Cornish engine the quantity of steam required to perform a given duty is diminished and the cost of fuel and of the boilers is lowered. But, at the same time, as the cylinders and every part of the engine must be made larger to admit of greater expansion the cost of the engine is increased."

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“It thus becomes a problem of maxima and minima to determine what ratio of expansion ought to be adopted, under given circumstances, in order that the sum of the actual cost of fuel and interest of the capital employed in construction may be the least possible as compared with the work done.”

It is somewhat remarkable that this theorem is nowhere alluded to in the author's subsequent elaborate treatise on the steam engine; while on the other hand he was the first to formulate in that treatise, definitely, the true theory of the efficiency of the steam in the cylinder of an engine, and to explain how the combined efficiencies of the boiler, engine and mechanism are to be ascertained from a purely mechanical and theoretical point of view. During the thirty years that have elapsed since the publication of the theorem, its value or importance, if either can be attached to it, have been ignored not only by all eminent writers on the steam engine, but by those most interested, the designers and users of steam engines.

It is reasonable to suppose, from these circumstances, either that there is an inherent fallacy in the theorem itself, or that engineers and business men have found it to be, even if true as a mathematical proposition, inapplicable in designing engines, or in using them.

An apology ought to be offered for asking the further time and attention of this Society to a subject upon which there can be very little difference of opinion among well informed engineers, as far as its theoretical aspects are concerned; but in the recent renewal of the discussion, upon the basis of this old theorem of Rankine, such positive grounds are taken in regard to the necessity of a radical change of views and of practice among engineers concerning the proper method of ascertaining the conditions of maximum economy in the application of steam-power, that a thorough criticism of the new methods seems to be called for. In order to do this it is necessary to enunciate the theorem in its mathematical form.

Let p_e represent the mean effective pressure in the cylinder of an engine per square foot of piston, and V_2 the volume in cubic feet generated by the piston in one hour; then $p_e V_2$ will represent in foot-pounds the work per hour of the weight of steam admitted to the cylinder. Let W represent this weight and h^1 the cost of producing it, including the cost of fuel and the hourly interest on the cost of the boilers; then $\frac{h^1}{W} = h$ will be the cost of producing one pound hourly.

Let K^1 represent the total interest on the cost of the engine, A the area of the piston, and K the interest per hour of the cost of the engine per square foot of the piston, then

$$K^1 = K. A.$$

The assumption made is, that the work in foot-pounds divided by the cost of the work must be a maximum for greatest economy; that is, if y be this ratio

$$y = \frac{p_0 V_2}{h. W. + K.A.} \text{ must be a maximum.}$$

Let l represent the length of stroke, n the number of revolutions per hour, r the ratio of expansion, then

$$W = \frac{A.l.n}{r} . D$$

D being the weight of a cubic foot of steam at the initial pressure.

The above expression then becomes

$$\begin{aligned} y &= \frac{p_0 V_2}{h . \frac{A.l.n}{r} . D + K.A.} \\ &= \frac{A.l.n \left[\frac{p_1}{r-1} \left(\frac{r}{r} - \frac{1}{r^r} \right) - p_2 \right]}{h . \frac{A.l.n}{r v_1} + K.A.} \end{aligned}$$

r only being supposed variable in the second member, and in which p_1 is the initial pressure r , the exponent of v in the equation of the adiabatic curve of expansion, $p v^r = p_1 v_1^r = \text{constant}$, and p_2 the back pressure.

The result of the operation for finding the maximum of y by the calculus is an equation between the values of V_2 , K , h , A and r , as follows, supposing y to be 10

$$h \frac{A.l.n}{v_1} (p_1 r^{-10} - p_2) = K.A [p_1 10 - 10 r^{-10}]$$

This result and the former expressions will be recognized as those given in a paper read at the Hartford meeting of the Society by Messrs. Wolff and Denton, as being substantially the same as those

of Rankine. In this expression v_1 represents the value of one pound of steam at the initial pressure p_1 , and $\frac{1}{v_1} = D$, D being the weight of a cubic foot of steam at the same pressure.

The usual mode of stating the efficiency of the steam in the cylinder of an engine is

$$E = \frac{p_2 V_2}{H \cdot \frac{A \ln}{r} D \cdot \times 772}$$

In this expression no element of cost enters, but the denominator represents the number of foot-pounds of work which enters, *as heat*, into the cylinder each hour.

H being the heat of evaporation, $\frac{A \ln}{r} D$ the weight W , and 772 the dynamic equivalent of a unit of heat. The maximum of this expression corresponds to such a degree of expansion by the adiabatic curve that the terminal pressure in the cylinder is just equal to the back pressure p_2 . The maximum of y will correspond to a less degree of expansion, or to more steam used in proportion to the work developed.

The work $p_2 V_2$ for any degree of expansion may be represented by a rectangle of which the base is V_2 and the altitude p_2 . The denominator may also be represented by a rectangle having the same base, but with a much greater altitude. The efficiency E is the ratio of these rectangles, or what is the same thing, the ratio of the altitudes of the rectangles. A somewhat similar process may be applied to the ratio giving the value of y , as explained by Rankine and worked out for varying values of the mean effective pressures and for various engines and boilers by Messrs. Wolff and Denton in the paper above referred to.

The graphical process thus introduced by Rankine shows at once the ratio of expansion which gives the maximum of y , or the rate of expansion which gives the greatest number of foot-pounds for the least amount of money, according to the author, *when his assumed premises are accepted*.

In regard to the maximum of E , the mechanical efficiency, a discussion by Rankine, found in his work on the steam engine, is a legitimate and correct one, but the theorem which introduces the cost is, at

least when applied to modern steam engines, fallacious, inasmuch as the *premises assumed are not true*. It is a very ingenious discussion, considered as a mere mathematical problem, admitting all the assumptions; but if these assumptions are not correct it must be considered, as it is practically, mere play of mathematics without practical value. In this presentation of the matter, from a purely theoretical standpoint, the secondary influences of cylinder condensation, clearance, etc., are not considered.

The first false assumption is in the total cost of making steam per hour, $h.W$. h is a constant, and is the cost of evaporating one pound of steam, including interest on cost of boiler; and when multiplied by the weight W it is assumed that the cost of a boiler constructed to evaporate this quantity of steam varies with the quantity of steam produced.

Whereas it is well known that the *same boiler* will produce more or less steam depending simply on the draft, and upon other circumstances not introduced. No exact relation of a general nature exists between the cost of a boiler and the quantity of steam it produces. Moreover, the chimney, or other appliance for producing draft, is an essential part of the boiler, and its costs should be included in the cost of making steam. The relation of cost to the quantity of steam produced is thus still more uncertain.

In the discussion of Rankine's theorem, by Messrs. Wolff and Denton, and by Prof. Thurston, who adopts and endorses their discussion, the quantity h is made to include also hourly wages of firemen, coal-passers, interest, a redemption annuity, or sinking fund, for the whole *supposed* lifetime of the boiler, etc.

This accumulated quantity h , multiplied by the weight of steam used at different grades of expansion, is taken at the hourly cost of making the quantity of steam W . It is hardly necessary to dwell on the fallacy of these assumptions, as the error must be apparent from the above statement, which will hardly be denied by those conversant with steam apparatus.

The second false assumption is in making the total cost of an engine proportional to the area of the piston.

The value of K in the formula is the interest on the cost of the engine divided by the area of the piston. It is well known that the cost of modern steam engines does not vary as the piston area, nor as the volume of the cylinder.

The following statement, taken from the yet unpublished records of the Tenth Census, has been furnished me by Mr. Charles H. Fitch, special agent of the census, who has been engaged under my direction in determining the statistics of engines and boilers.

CYLINDER CAPACITY AND COST.

"The following table exhibits comparatively the relations of cylinder capacity, weight and cost in steam engines of the horizontal plain slide valve type, rated at from eight to one hundred horse-power :

Cylinder Capacity of Engine.	Ratio of stroke to diameter.	Price per cubic foot of cylinder.	Weight of engine per cubic foot of cylinder (lbs).
0.19	2.00	\$1,910 00	8,684
0.27	1.71	1,392 00	6,296
0.46	2.00	1,113 00	7,500
0.59	1.77	952 00	7,118
0.72	1.60	872 00	6,944
0.90	2.00	749 00	6,264
1.31	1.66	707 00	6,183
1.58	2.00	644 00	5,860
1.79	1.43	633 00	5,168
2.14	1.71	569 00	5,023
2.79	1.50	497 00	4,337
3.48	1.87	451 00	4,253
3.53	1.33	509 00	5,241
4.41	1.66	448 00	4,736
4.35	1.20	517 00	4,638
5.44	1.50	459 00	4,964

"It is obvious that no formula can truly exhibit the relative changes of price and capacity, which does not consider the actual sizes of engines.

compared. The examples cited present an unusual degree of uniformity, a line of engines rated at uniform piston speed, and of similar excellent workmanship and finish.

“Between different styles and qualities of engines there can be no very definite comparison. For the same cylinder capacity a finely built, closely fitted engine may cost twice as much as an engine of inferior workmanship.”

Rankine evidently assumed that the cost is proportional to the volume of the cylinder or to piston area, because this is the only assumption that will enable an engineer to *design an engine with a given power*, the steam being cut off at the point of the stroke determined by his theorem.

He gives an example by designing an engine of 100 HP, having first found the proper ratio of expansion for a cylinder having a unit area of piston and a given speed. He finds the cost of a 100 HP engine of the same speed to be by his process £6018, an assumed cost of £250 per square foot of piston. Of course, if his 100 HP engine, with all its appliances, was found, when actually constructed, to cost more or less than £6018 the whole process becomes invalidated.

It would seem that nothing more need be said to exhibit the fallacy of the assumptions or premises on which the mathematical work of the whole discussion is based. But we may place these fallacies on broader grounds.

There cannot be, from the nature of finance and pure mechanics, any exact mathematical relation between abstract mechanical laws and financial operations.

The former are invariable and immutable, the latter are dependent upon bargain and sale, the efficacy of human labor or upon human necessities, and sometimes on human follies.

No more conspicuous illustration of these truths, it seems to me, could be given than this plausible, but false application of mathematics, and especially of the calculus, to a problem in which the cost of an apparatus and its mechanical performance are introduced as elements of a formula which is claimed to be general in its nature and practically correct in its applications.

Looking at Rankine's theorem from another point of view it is claimed that although it is not applicable *for designing engines*, yet after an engine and boiler have been completed, and the actual cost of the whole plant is known, it is then practicable to determine for that

engine the point of cut-off which will give the most power for the money already and actually expended.

Assuming, for instance, the principle that the power divided by its cost should be a maximum, the same mathematical expression

$$y = \frac{p_s V_2}{h.W + K.A}$$

is merely restricted to an apparatus of which the cost is known.

Taking this view of the problem the absurdity of including in any manner in the variable term of the denominator ($h.W$) the interest on the cost of the boiler, firemen's wages, insurance, depreciation of value from use, repairs, etc., is evident at once; because none of these items vary with the quantity of steam used.

Moreover, it is not necessary to estimate the interest on the cost of the engine K by the square foot of piston, but the formula, admitting for the sake of argument, its validity, should be

$$y = \frac{p_s V_2}{h.W + K}$$

in which $h.W$ includes only those items of cost which vary with the quantity of steam used, and in which K includes all the constant hourly expenditures of the plant, including boiler and engine. We should thus have to include in h only the cost of making one pound of steam per hour estimated from the price of coal, while K would necessarily embrace the following items.

1. Hourly interest on cost of boiler.
2. " " " engine.
3. " " " chimney.
4. " " " boiler room and engine room.
5. Wages of Engineer and fireman per hour.
6. Insurance per hour.
7. Taxes per hour.
8. A portion of the wages of general foreman and manager per hour.
9. Estimated repairs per hour.
10. Sinking fund for redemption of cost of boiler and engine, estimated to last (blank) years.

Everything, in fact, must be included which can reasonably be charged to the cost of the power, as distinguished from expenditures for utilization of power and other expenses connected with it.

In this mathematical formula for which it has recently been claimed that it furnishes "*the only correct solution which has ever been presented*" and that it is of "*immediate practical use*," we find all of these quantities.

Let us see how they are to be determined numerically :

Interest—Who shall establish the rate ?

Cost of engine and boiler—What shall it include ?

Boiler and engine room and setting—How shall the cost be separated from that of the rest of the plant, in a manufacturing establishment, or in a steamship from other parts of the structure ?

Wages of engineers and firemen—What shall be done if these wages should happen to change from time to time ?

Insurance—When a whole establishment is insured what part shall be charged to power ?

Salaries of general foreman and manager—What part shall be charged to power ?

Repairs—What amount shall be *estimated* ?

Sinking Fund—How *estimated* ?

Who can estimate the life of a boiler or an engine ?

All or nearly all of the above quantities can be assumed only by mere guesswork—there is not a shadow of a guide or a rule for most of them, not even the results of experience which so often aid in the solution of practical problems.

If the quantities in the formula are arrived at by mere guess, and without experience to guide the person who guesses, of what value are the results ?

A good deal has been said in this connection about the "business man's cut-off" as distinguished from the scientific man's cut-off. This wretched process is like that of finding the correct time, so admirably described by John Phoenix in his burlesque exploring expedition from San Francisco to the "Mission," one of its suburbs. The party, after attempting for a whole night to determine the true time by observations of the stars with a transit, which could not be properly adjusted, concluded that it would be just as well to step into a corner grocery and get the time from the grocer's clock.

There is one other important consideration which should not be lost sight of in applying this method to a given plant, viz., that not until all these questions of cost have been successively guessed at and thus established, and not until after the boiler and engine have been erected,

ready for running, will the manufacturer or the steamship owner know what power he has at his disposal. The power developed at the "business man's" economical point of cut-off depends on these items of cost, and he must accept what his guesswork has given him.

Possibly the power thus available will not run his works or his ship, and he may find that in attempting this mode of economy he is a greater loser in the end for want of sufficient power or from having too much power. The moment dollars and cents enter into a problem it ceases to be a mechanical, and becomes a financial one; and in such problems there must always be two sides to the account.

A horse-power as power has no standard or intrinsic value; its value depends entirely upon its serving fully and precisely the objects for which it is employed.

A definite amount of power just sufficient for performing a given work has a certain value, where one-half or indeed any less power might be of no value, and more power might involve loss.

Steam-power is like a manufactured article, and the question of preventing waste in the manufacture, by high grades of expansion and costly engines, is one in which all the conditions and circumstances of the use of the power must be considered. It cannot be narrowed down to a mere ratio between the cost of the engine and the cost of fuel as is claimed in the new method.

The attempt to apply this new method to marine engines in public vessels, where no questions of interest and of profit and loss can enter, is not the least of the absurdities that have marked the discussion of the subject.

The introduction of the sinking fund idea as a part of the cost involves another absurdity, because the operation of a sinking fund is to reduce the principal of the debt, and the value of the quantity K , in the formula assumed as constant, is not a constant under this assumption, but a variable quantity, practically a vanishing quantity. Thus the validity of the mathematical discussion is destroyed, even if there were no other elements of unsoundness in the process. It seems to me unfortunate that crude ideas on this subject should have been put forth as the results of exact scientific investigation, and a theorem practically abandoned by its author, invoked to persuade business men that our most eminent and successful engineers of the present day have been practically ignorant of correct principles regarding the economical use of steam.

Nothing tends more to bring science into disrepute with those engaged in industrial pursuits than the hasty announcement in the name of science of conclusions or results, as truths, which are merely personal and unverified speculations, or false deductions from assumed and unproved premises.

THE SPECIFIC HEAT OF PLATINUM, AND THE USE OF THIS METAL IN THE PYROMETER.

By J. C. HOADLEY.

In a paper which I had the honor to present to the American Society of Mechanical Engineers at the meeting held at Hartford in May, 1881, I gave, as the best estimate of the mean specific heat of platinum which I was able to form—based on the data to be found in Clarke's "Constants of Nature"—0.0333, equal to one-thirtieth of the specific heat of water under standard conditions; and apparently uniform, or nearly so, at all temperatures ordinarily attainable. At the same time I stated that a careful study of the original memoirs might disclose such reasons for assigning greater weight to the results obtained by some experimenters than to the varying results obtained by others, as to lead to a probable mean result much more trustworthy than that of my first crude approximation.

Such a study of the original memoirs, and a careful discussion of the several experiments therein described, for which I had neither the requisite facilities nor the necessary leisure, was made during his summer vacation by Mr. Silas W. Holman, Instructor in Physics at the Massachusetts Institute of Technology, who generously placed at my disposal the interesting and valuable fruits of his labors.

Mr. Holman first clearly points out the distinction between the terms "true specific heat" and "mean specific heat." The *true* specific heat of any substance at t° , is the amount of heat, expressed in heat units, required to raise the unit weight of the substance from any temperature t° to $t^\circ + 1^\circ$, or more properly from t° to $t^\circ + dt$, whence we may write the true specific heat at $t^\circ = \frac{dh}{dt}$ = first differential coefficient of the heat with regard to the temperature (of course